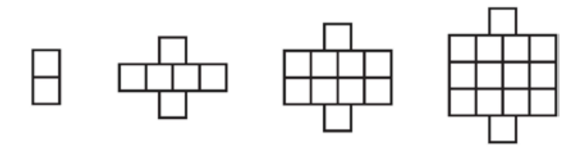
F.BF.A.2: Sequences 2

1 Which representation yields the same outcome as the sequence defined recursively below?

$$a_1 = 3$$
$$a_n = -4 + a_{n-1}$$

1) $3, 7, 11, 15, 19, \ldots$
2) $3, -1, -5, -9, -13, \ldots$
3) $a_n = 4n - 1$
4) $a_n = 4 - n$

2 A pattern of blocks is shown below.



If the pattern of blocks continues, which formula(s) could be used to determine the number of blocks in the *n*th term?

I	II	III
$a_n = n + 4$	$a_1 = 2$ $a_n = a_{n-1} + 4$	$a_n = 4n - 2$

1) I and II
2) I and III
3) II and III
4) III, only

3 In 2014, the cost to mail a letter was 49¢ for up to one ounce. Every additional ounce cost 21¢. Which recursive function could be used to determine the cost of a 3-ounce letter, in cents?

1) $a_1 = 49;\ a_n = a_{n-1} + 21$
2) $a_1 = 0;\ a_n = 49a_{n-1} + 21$
3) $a_1 = 21;\ a_n = a_{n-1} + 49$
4) $a_1 = 0;\ a_n = 21a_{n-1} + 49$

4 At her job, Pat earns \$25,000 the first year and receives a raise of \$1000 each year. The explicit formula for the *n*th term of this sequence is $a_n = 25{,}000 + (n-1)1000$. Which rule best represents the equivalent recursive formula?

1) $a_n = 24{,}000 + 1000n$
2) $a_n = 25{,}000 + 1000n$
3) $a_1 = 25{,}000,\ a_n = a_{n-1} + 1000$
4) $a_1 = 25{,}000,\ a_n = a_{n+1} + 1000$

5 Savannah just got contact lenses. Her doctor said she can wear them 2 hours the first day, and can then increase the length of time by 30 minutes each day. If this pattern continues, which formula would *not* be appropriate to determine the length of time, in either minutes or hours, she could wear her contact lenses on the *n*th day?

1) $a_1 = 120$
$a_n = a_{n-1} + 30$
2) $a_n = 90 + 30n$
3) $a_1 = 2$
$a_n = a_{n-1} + 0.5$
4) $a_n = 2.5 + 0.5n$

6 The sequence $a_1 = 6, a_n = 3a_{n-1}$ can also be written as

1) $a_n = 6 \cdot 3^n$
2) $a_n = 6 \cdot 3^{n+1}$
3) $a_n = 2 \cdot 3^n$
4) $a_n = 2 \cdot 3^{n+1}$

7 Which function could be used to represent the sequence $8, 20, 50, 125, 312.5, \dots$, given that $a_1 = 8$?

1) $a_n = a_{n-1} + a_1$
2) $a_n = 2.5(a_{n-1})$
3) $a_n = a_1 + 1.5(a_{n-1})$
4) $a_n = (a_1)(a_{n-1})$

8 A recursive formula for the sequence $18, 9, 4.5, \dots$ is

1) $g_1 = 18$
$g_n = \frac{1}{2} g_{n-1}$
2) $g_n = 18\left(\frac{1}{2}\right)^{n-1}$
3) $g_1 = 18$
$g_n = 2g_{n-1}$
4) $g_n = 18(2)^{n-1}$

9 The first four terms of the sequence with $a_1 = 40$ and $a_n = \frac{3}{4} a_{n-1}$ are

1) $30, 22, 17, 13$
2) $40, 30, 22\frac{1}{2}, 16\frac{7}{8}$
3) $40, 30, 22, 17$
4) $30, 22\frac{1}{2}, 16\frac{7}{8}, 12\frac{21}{33}$

10 A recursive formula for the sequence $40, 30, 22.5, \dots$ is

1) $g_n = 40\left(\frac{3}{4}\right)^n$
2) $g_1 = 40$
$g_n = g_{n-1} - 10$
3) $g_n = 40\left(\frac{3}{4}\right)^{n-1}$
4) $g_1 = 40$
$g_n = \frac{3}{4} g_{n-1}$

11 A recursive formula for the sequence $64, 48, 36, \dots$ is

1) $a_n = 64(0.75)^{n-1}$
2) $a_1 = 64$
$a_n = a_{n-1} - 16$
3) $a_n = 64 + (n-1)(-16)$
4) $a_1 = 64$
$a_n = 0.75a_{n-1}$

12 What is the fourth term of the sequence defined by

$$a_1 = 3xy^5$$
$$a_n = \left(\frac{2x}{y}\right)a_{n-1}?$$

1) $12x^3y^3$
2) $24x^2y^4$
3) $24x^4y^2$
4) $48x^5y$

13 An initial investment of $5000 in an account earns 3.5% annual interest. Which function correctly represents a recursive model of the investment after n years?

1) $A = 5000(0.035)^n$
2) $a_0 = 5000$
 $a_n = a_{n-1}(0.035)$
3) $A = 5000(1.035)^n$
4) $a_0 = 5000$
 $a_n = a_{n-1}(1.035)$

14 The population of Jamesburg for the years 2010-2013, respectively, was reported as follows:
250,000 250,937 251,878 252,822
How can this sequence be recursively modeled?

1) $j_n = 250{,}000(1.00375)^{n-1}$
2) $j_n = 250{,}000 + 937^{(n-1)}$
3) $j_1 = 250{,}000$
 $j_n = 1.00375 j_{n-1}$
4) $j_1 = 250{,}000$
 $j_n = j_{n-1} + 937$

15 In 2010, the population of New York State was approximately 19,378,000 with an annual growth rate of 1.5%. Assuming the growth rate is maintained for a large number of years, which equation can be used to predict the population of New York State t years after 2010?

1) $P_t = 19{,}378{,}000(1.5)^t$
2) $P_0 = 19{,}378{,}000$
 $P_t = 19{,}378{,}000 + 1.015P_{t-1}$
3) $P_t = 19{,}378{,}000(1.015)^{t-1}$
4) $P_0 = 19{,}378{,}000$
 $P_t = 1.015P_{t-1}$

16 The average depreciation rate of a new boat is approximately 8% per year. If a new boat is purchased at a price of $75,000, which model is a recursive formula representing the value of the boat n years after it was purchased?

1) $a_n = 75{,}000(0.08)^n$
2) $a_0 = 75{,}000$
 $a_n = (0.92)^n$
3) $a_n = 75{,}000(1.08)^n$
4) $a_0 = 75{,}000$
 $a_n = 0.92(a_{n-1})$

17 After Roger's surgery, his doctor administered pain medication in the following amounts in milligrams over four days.

Day (n)	1	2	3	4
Dosage (m)	2000	1680	1411.2	1185.4

How can this sequence best be modeled recursively?

1) $m_1 = 2000$
 $m_n = m_{n-1} - 320$
2) $m_n = 2000(0.84)^{n-1}$
3) $m_1 = 2000$
 $m_n = (0.84)m_{n-1}$
4) $m_n = 2000(0.84)^{n+1}$

18 A tree farm initially has 150 trees. Each year, 20% of the trees are cut down and 80 seedlings are planted. Which recursive formula models the number of trees, a_n, after n years?

1) $a_1 = 150$
$a_n = a_{n-1}(0.2) + 80$

2) $a_1 = 150$
$a_n = a_{n-1}(0.8) + 80$

3) $a_n = 150(0.2)^n + 80$

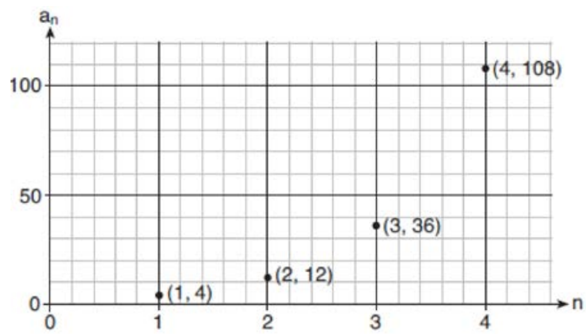
4) $a_n = 150(0.8)^n + 80$

19 The formula below can be used to model which scenario?

$$a_1 = 3000$$
$$a_n = 0.80a_{n-1}$$

1) The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it.
2) The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it.
3) A bank account starts with a deposit of $3000, and each year it grows by 80%.
4) The initial value of a specialty toy is $3000, and its value each of the following years is 20% less.

20 Write a recursive formula, a_n, to describe the sequence graphed below.



21 Write a recursive formula for the sequence $6, 9, 13.5, 20.25, \ldots$

22 Write a recursive formula for the sequence $189, 63, 21, 7, \ldots$.

23 The explicit formula $a_n = 6 + 6n$ represents the number of seats in each row in a movie theater, where n represents the row number. Rewrite this formula in recursive form.

24 The population, in millions of people, of the United States can be represented by the recursive formula below, where a_0 represents the population in 1910 and n represents the number of years since 1910.

$$a_0 = 92.2$$
$$a_n = 1.015a_{n-1}$$

Identify the percentage of the annual rate of growth from the equation $a_n = 1.015a_{n-1}$. Write an exponential function, P, where $P(t)$ represents the United States population in millions of people, and t is the number of years since 1910. According to this model, determine algebraically the number of years it takes for the population of the United States to be approximately 300 million people. Round your answer to the *nearest year*.

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Answer Section

1 ANS: 2
$d = -4$

REF: 012321ai
2 ANS: 3 REF: 061522ai
3 ANS: 1 REF: 011708ai
4 ANS: 3 REF: 011824aii
5 ANS: 4
$a_1 = 2.5 + 0.5(1) = 3$

REF: 011916aii
6 ANS: 3 REF: 081618aii
7 ANS: 2 REF: 011919ai
8 ANS: 1
(2) is not recursive

REF: 081608aii
9 ANS: 2
$\frac{3}{4}(40) = 30; \frac{3}{4}(30) = 22.5; \frac{3}{4}(22.5) = 16.875$

REF: 081608a2
10 ANS: 4
(1) and (3) are not recursive

REF: 012013aii
11 ANS: 4
1) is a correct formula, but not recursive

REF: 082216aii
12 ANS: 3
$a_4 = 3xy^5\left(\frac{2x}{y}\right)^3 = 3xy^5\left(\frac{8x^3}{y^3}\right) = 24x^4y^2$

REF: 061512a2
13 ANS: 4 REF: 062412aii
14 ANS: 3 REF: 061623aii
15 ANS: 4 REF: 081624aii
16 ANS: 4 REF: 081810aii
17 ANS: 3 REF: 081909aii
18 ANS: 2 REF: 012321aii

19 ANS: 4
The scenario represents a decreasing geometric sequence with a common ratio of 0.80.

REF: 061610aii

20 ANS:
$a_1 = 4$

$a_n = 3a_{n-1}$

REF: 081931aii

21 ANS:
$\frac{9}{6} = 1.5 \quad a_1 = 6$

$a_n = 1.5 \cdot a_{n-1}$

REF: 061931aii

22 ANS:
$\frac{63}{189} = \frac{1}{3} \quad a_1 = 189$

$a_n = \frac{1}{3} a_{n-1}$

REF: 062329aii

23 ANS:
$a_1 = 12$

$a_n = a_{n-1} + 6$

REF: 012430aii

24 ANS:
1.5%; $P(t) = 92.2(1.015)^t$;

$$\frac{300}{92.2} = (1.015)^t$$

$$\log\frac{300}{92.2} = t\log(1.015)$$

$$\frac{\log\frac{300}{92.2}}{\log(1.015)} = t$$

$$t \approx 79$$

REF: 062237aii