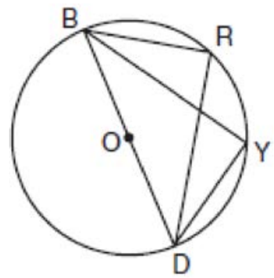
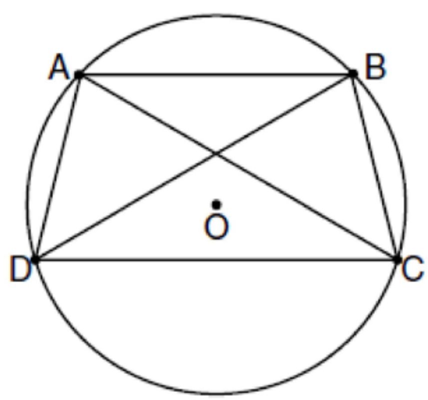
G.G.27: Circle Proofs: Write a proof arguing from a given hypothesis to a given conclusion

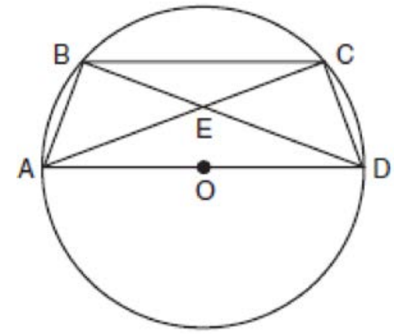
1 In the accompanying diagram, $m\overparen{BR} = 70$, $m\overparen{YD} = 70$, and $\overline{BOD}$ is the diameter of circle O. Write an explanation or a proof that shows ΔRBD and ΔYBD are congruent.



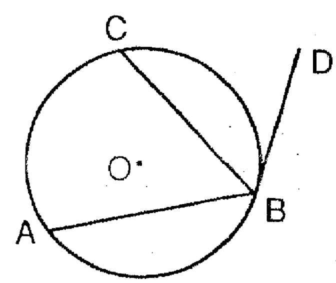
2 In the diagram below, quadrilateral $ABCD$ is inscribed in circle O, $\overline{AB} \parallel \overline{DC}$, and diagonals $\overline{AC}$ and $\overline{BD}$ are drawn. Prove that $\Delta ACD \cong \Delta BDC$.



3 In the accompanying diagram of circle O, $\overline{AD}$ is a diameter with $\overline{AD}$ parallel to chord $\overline{BC}$, chords $\overline{AB}$ and $\overline{CD}$ are drawn, and chords $\overline{BD}$ and $\overline{AC}$ intersect at E.
Prove: $\overline{BE} \cong \overline{CE}$

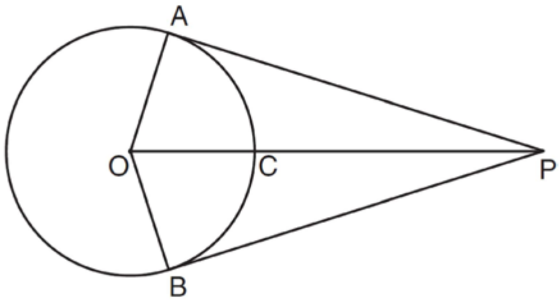


4 Given: circle O, $\overline{DB}$ is tangent to the circle at B, $\overline{BC}$ and $\overline{BA}$ are chords, and C is the midpoint of $\overparen{AB}$.

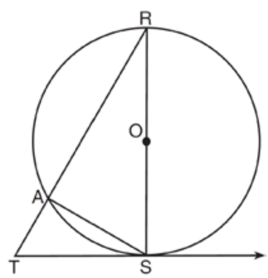


Prove: $\angle ABC \cong \angle CBD$

5 In the diagram below, $\overline{PA}$ and $\overline{PB}$ are tangent to circle O, $\overline{OA}$ and $\overline{OB}$ are radii, and $\overline{OP}$ intersects the circle at C. Prove: $\angle AOP \cong \angle BOP$



6 In the diagram of circle O below, diameter $\overline{RS}$, chord $\overline{AS}$, tangent $\overrightarrow{TS}$, and secant $\overline{TAR}$ are drawn.



Complete the following proof to show $(RS)^2 = RA \cdot RT$

Statements	Reasons
1. circle O, diameter $\overline{RS}$, chord $\overline{AS}$, tangent $\overrightarrow{TS}$, and secant $\overline{TAR}$	1. Given
2. $\overline{RS} \perp \overrightarrow{TS}$	2. ____________________
3. $\angle RST$ is a right angle	3. $\perp$ lines form right angles
4. $\angle RAS$ is a right angle	4. ____________________
5. $\angle RST \cong \angle RAS$	5. ____________________
6. $\angle R \cong \angle R$	6. Reflexive property
7. $\triangle RST \sim \triangle RAS$	7. ____________________
8. $\frac{RS}{RA} = \frac{RT}{RS}$	8. ____________________
9. $(RS)^2 = RA \bullet RT$	9. ____________________

G.G.27: Circle Proofs: Write a proof arguing from a given hypothesis to a given conclusion
Answer Section

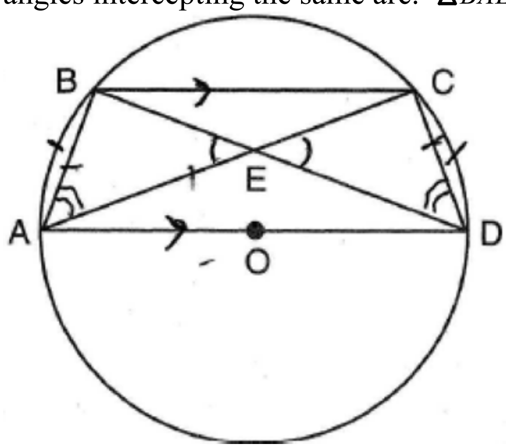
1 ANS:
The measure of an inscribed angle is half that of its intercepted arc. Therefore $m\angle RDB = m\angle YDB = 35$. Because $\angle BRD$ and $\angle DYB$ intercept a semicircle, they are both right angles. $\overline{BD} \cong \overline{DB}$ from the reflexive property. Therefore $\Delta RBD \cong \Delta YDB$ because of AAS. Alternatively, because congruent chords intersect congruent arcs, $\overline{BR} \cong \overline{YD}$ and HL applies.

REF: 010732b

2 ANS:
Because $\overline{AB} \parallel \overline{DC}$, $\overset{\frown}{AD} \cong \overset{\frown}{BC}$ since parallel chords intersect congruent arcs. $\angle BDC \cong \angle ACD$ because inscribed angles that intercept congruent arcs are congruent. $\overline{AD} \cong \overline{BC}$ since congruent chords intersect congruent arcs. $\angle DAC \cong \angle DBC$ because inscribed angles that intercept the same arc are congruent. Therefore, $\Delta ACD \cong \Delta BDC$ because of AAS.

REF: fall0838ge

3 ANS:
$\overset{\frown}{AB} \cong \overset{\frown}{DC}$ because parallel lines intercept congruent arcs. $\overline{AB} \cong \overline{DC}$ because congruent chords intercept congruent arcs. $\angle BEA$ and $\angle CED$ are congruent vertical angles. $\angle BAC$ and $\angle CDB$ are congruent inscribed angles intercepting the same arc. $\Delta BAE \cong \Delta CDE$ because of AAS. $\overline{BE} \cong \overline{CE}$ because of CPCTC.



REF: 060934b

4 ANS:
$\overset{\frown}{AC} \cong \overset{\frown}{CB}$ because of the definition of midpoint; $\angle ABC \cong \frac{1}{2}\overset{\frown}{AC}$ as the measure of an inscribed angle is one-half the measure of its intercepted arc; $\angle CBD \cong \frac{1}{2}\overset{\frown}{BC}$ as the measure of an angle formed by a tangent and a chord that intersect at the point of tangency is one-half the measure of the intercepted arc; $\frac{1}{2}\overset{\frown}{AC} \cong \frac{1}{2}\overset{\frown}{CB}$ because of the multiplication property of equalities; and $\angle ABC \cong \angle CBD$ because of substitution.

REF: 019439siii

5 ANS:
$\overline{OA} \cong \overline{OB}$ because all radii are equal. $\overline{OP} \cong \overline{OP}$ because of the reflexive property. $\overline{OA} \perp \overline{PA}$ and $\overline{OB} \perp \overline{PB}$ because tangents to a circle are perpendicular to a radius at a point on a circle. $\angle PAO$ and $\angle PBO$ are right angles because of the definition of perpendicular. $\angle PAO \cong \angle PBO$ because all right angles are congruent. $\triangle AOP \cong \triangle BOP$ because of HL. $\angle AOP \cong \angle BOP$ because of CPCTC.

REF: 061138ge

6 ANS:
2. The diameter of a circle is $\perp$ to a tangent at the point of tangency. 4. An angle inscribed in a semicircle is a right angle. 5. All right angles are congruent. 7. AA. 8. Corresponding sides of congruent triangles are in proportion. 9. The product of the means equals the product of the extremes.

REF: 011438ge