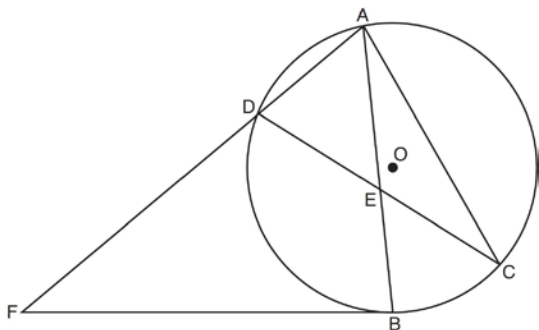


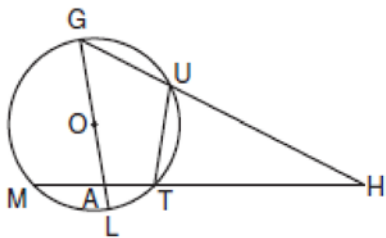
**G.G.51: Arcs Determined by Angles 6: Investigate theorems about the arcs determined by angles intersecting a circle when the vertex is on the circle**

- 1 Chords  $\overline{AB}$  and  $\overline{CD}$  intersect at  $E$  in circle  $O$ , as shown in the diagram below. Secant  $\overline{FDA}$  and tangent  $\overline{FB}$  are drawn to circle  $O$  from external point  $F$  and chord  $\overline{AC}$  is drawn. The  $m\widehat{DA} = 56$ ,  $m\widehat{DB} = 112$ , and the ratio of  $m\widehat{AC} : m\widehat{CB} = 3 : 1$ .

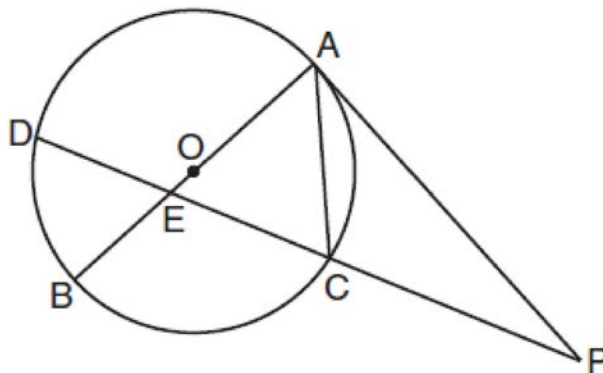


Determine  $m\angle CEB$ . Determine  $m\angle F$ . Determine  $m\angle DAC$ .

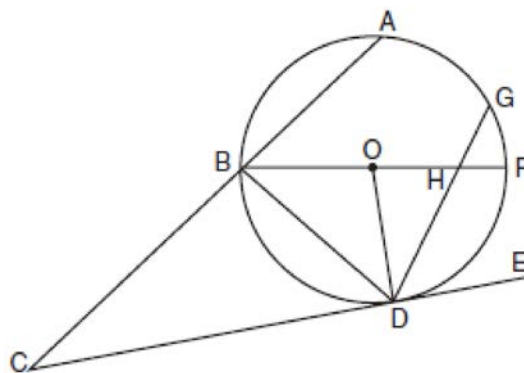
- 2 Given circle  $O$  with diameter  $\overline{GOAL}$ ; secants  $\overline{HUG}$  and  $\overline{HTAM}$  intersect at point  $H$ ;  $m\widehat{GM} : m\widehat{ML} : m\widehat{LT} = 7 : 3 : 2$ ; and chord  $\overline{GU} \cong$  chord  $\overline{UT}$ . Find the ratio of  $m\angle UGL$  to  $m\angle H$ .



- 3 In the accompanying diagram,  $\overline{PA}$  is tangent to circle  $O$  at  $A$ , chord  $\overline{AC}$  and secant  $\overline{PCED}$  are drawn, and chords  $\overline{AOB}$  and  $\overline{CD}$  intersect at  $E$ . If  $m\widehat{AD} = 130$  and  $m\angle BAC = 50$ , find  $m\angle P$ ,  $m\angle BEC$ , and  $m\angle PCA$ .



- 4 In the accompanying diagram, circle  $O$  has radius  $\overline{OD}$ , diameter  $\overline{BOHF}$ , secant  $\overline{CBA}$ , and chords  $\overline{DHG}$  and  $\overline{BD}$ ;  $\overline{CE}$  is tangent to circle  $O$  at  $D$ ;  $m\widehat{DF} = 80$ ; and  $m\widehat{BA} : m\widehat{AG} : m\widehat{GF} = 3 : 2 : 1$ . Find  $m\widehat{GF}$ ,  $m\angle BHD$ ,  $m\angle BDG$ ,  $m\angle GDE$ ,  $m\angle C$ , and  $m\angle BOD$ .



**G.G.51: Arcs Determined by Angles 6: Investigate theorems about the arcs determined by angles intersecting a circle when the vertex is on the circle**

**Answer Section**

1 ANS:

$$52, 40, 80. \quad 360 - (56 + 112) = 192. \quad \frac{192 - 112}{2} = 40. \quad \frac{112 + 48}{2} = 80$$

$$\frac{1}{4} \times 192 = 48$$

$$\frac{56 + 48}{2} = 52$$

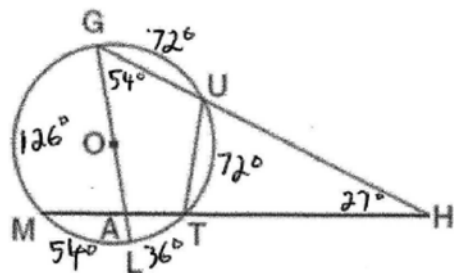
REF: 081238ge

2 ANS:

2:1.  $\widehat{GM}$  and  $\widehat{ML}$  form a semi-circle and measure  $126^\circ$  ( $\frac{7}{10} \times 180$ ) and  $54^\circ$  ( $\frac{3}{10} \times 180$ ), respectively.

$\widehat{LT}$  measures  $36^\circ$ .  $\widehat{GM}$  and  $\widehat{ML}$  form a semi-circle and measure  $126^\circ$ .  $\widehat{GUT}$  measures  $144^\circ$  ( $180 - 36$ ). Equal chords intercept equal arcs. Because chord  $\overline{GU} \cong$  chord  $\overline{UT}$ ,  $\widehat{GU}$  and  $\widehat{UT}$  each measures  $72^\circ$  ( $\frac{144}{2}$ ).  $m\widehat{UTL} = 108$  ( $72 + 36$ ). The measure of an inscribed angle is half that of its

intercepted arc. So  $m\angle UGL = 54$ . The angle formed by a tangent and a secant is equal to half the difference between the intercepted arcs.  $\frac{126 - 72}{2} = 27$ . The ratio of  $m\angle UGL$  to  $m\angle H$  is 54:27, or 2:1.



REF: 080333b

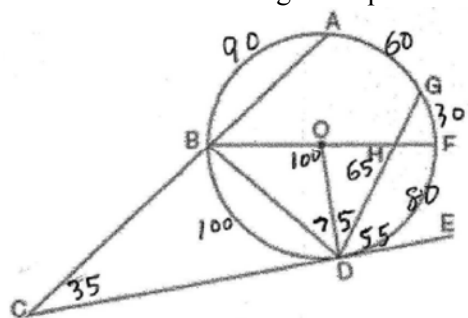
3 ANS:

25, 115, 115

REF: 011033b

4 ANS:

30, 65, 75, 55, 35, 100.  $\widehat{BA}$ ,  $\widehat{AG}$  and  $\widehat{GF}$  form a semi-circle and measure  $90^\circ$  ( $\frac{3}{6} \times 180$ ),  $60^\circ$  ( $\frac{2}{6} \times 180$ ) and  $30^\circ$  ( $\frac{1}{6} \times 180$ ), respectively. The measure of an inscribed angle is half that of its intercepted arc. So  $m\angle BDG = 75$  ( $\frac{90+60}{2}$ ) and  $m\angle HBD = 40$  ( $\frac{80}{2}$ ). Therefore  $m\angle BHD = 65$  ( $180 - (75 + 40)$ ). The angle formed by a tangent and a chord is half the intercepted arc. Since the intercepted arc is  $110^\circ$  ( $80 + 30$ ),  $m\angle GDE = 55$ . Given diameter  $\overline{BOHF}$  and  $m\widehat{DF} = 80$ ,  $m\widehat{BD} = 100$ . The angle formed by a tangent and a secant is equal to half the difference between the intercepted arcs, so  $m\angle C = \frac{(60 + 30 + 80) - 100}{2} = 35$ . The measure of a central angle is equal to the measure of the arc it intercepts, so  $m\angle BOD = 100$ .



REF: 080633b