*A2.A.73: Solve for an unknown side or angle, using the Law of Sines or the Law of Cosines*

1. 060817b, P.I. A2.A.73
    Al is standing 50 yards from a maple tree and 30 yards from an oak tree in the park. His position is shown in the accompanying diagram. If he is looking at the maple tree, he needs to turn his head 120° to look at the oak tree.

    Maple tree

    50 yd

    Al's location 120°

    30 yd Oak tree
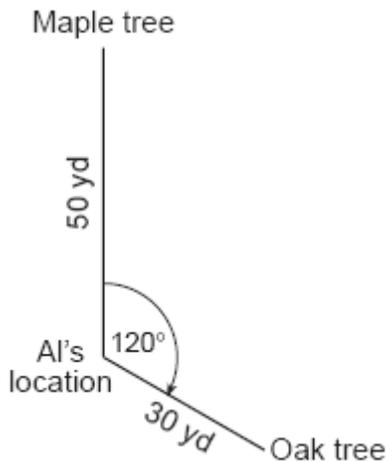
    How many yards apart are the two trees?

    [A] 75   [B] 65.2   [C] 58.3   [D] 70

2. 010227b, P.I. A2.A.73
    Two straight roads, Elm Street and Pine Street, intersect creating a 40° angle, as shown in the accompanying diagram. John's house (*J*) is on Elm Street and is 3.2 miles from the point of intersection. Mary's house (*M*) is on Pine Street and is 5.6 miles from the intersection. Find, to the *nearest tenth of a mile*, the direct distance between the two houses.

    Pine Street
    J
    40°
    M
    Elm Street
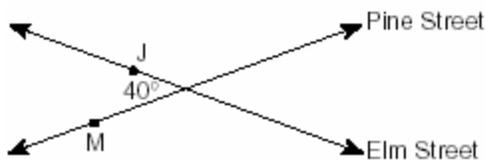
3. 060232b, P.I. A2.A.73
    Kieran is traveling from city *A* to city *B*. As the accompanying map indicates, Kieran could drive directly from *A* to *B* along County Route 21 at an average speed of 55 miles per hour or travel on the interstates, 45 miles along I-85 and 20 miles along I-64. The two interstates intersect at an angle of 150° at *C* and have a speed limit of 65 miles per hour. How much time will Kieran save by traveling along the interstates at an average speed of 65 miles per hour?

    I-85
    45 miles   C
    A   150° I-64
                              20 miles
    County Route 21
                              B
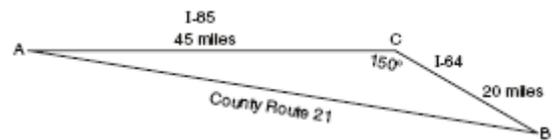
4. 080329b, P.I. A2.A.73
    A ship at sea is 70 miles from one radio transmitter and 130 miles from another. The angle between the signals sent to the ship by the transmitters is 117.4°. Find the distance between the two transmitters, to the *nearest mile*.

5. 010528b, P.I. A2.A.73
    To measure the distance through a mountain for a proposed tunnel, surveyors chose points *A* and *B* at each end of the proposed tunnel and a point *C* near the mountain. They determined that *AC* = 3,800 meters, *BC* = 2,900 meters, and $m\angle ACB = 110$. Draw a diagram to illustrate this situation and find the length of the tunnel, to the *nearest meter*.

6. fall9929b, P.I. A2.A.73
    The Vietnam Veteran's Memorial in Washington, DC consists of two walls of black, polished granite, each 246.75 feet long, which meet at an angle of 125.2°. If extended, the west wall would reach to the Lincoln Memorial, 900 feet away from the end of the wall and the east wall would reach to the Washington Monument, 3,500 feet away from the end of the wall. Find the distance between the Lincoln Memorial and the Washington Monument to the *nearest foot*.

7. 060127b, P.I. A2.A.73
A wooden frame is to be constructed in the form of an isosceles trapezoid, with diagonals acting as braces to strengthen the frame. The sides of the frame each measure 5.30 feet, and the longer base measures 12.70 feet. If the angles between the sides and the longer base each measure 68.4°, find the length of one brace to the *nearest tenth of a foot.*

8. 060434b, P.I. A2.A.73
A surveyor is mapping a triangular plot of land. He measures two of the sides and the angle formed by these two sides and finds that the lengths are 400 yards and 200 yards and the included angle is 50°. What is the measure of the third side of the plot of land, to the *nearest yard*? What is the area of this plot of land, to the *nearest square yard*?

9. 010929b, P.I. A2.A.73
In $\triangle ABC$, $a = 24$, $b = 36$, and $c = 30$. Find $m\angle A$ to the *nearest tenth of a degree.*

10. 080828b, P.I. A2.A.73
During a training exercise in the Mojave Desert, two military vehicles left the base camp at the same time, one traveling at an average speed of 25 miles per hour and the other at an average speed of 50 miles per hour. Each vehicle traveled along a level, straight route. If the exercise requires the two vehicles to be 65 miles apart after traveling for 1 hour, what must the angle between the two routes be, to the *nearest degree*?

11. 060933b, P.I. A2.A.73
The accompanying diagram shows a triangular plot of land located in Moira's garden.
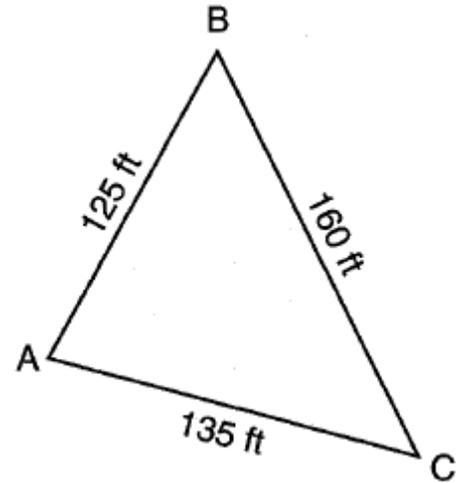
Find the area of the plot of land, and round your answer to the *nearest hundred square feet.*

12. 080734b, P.I. A2.A.73
A farmer has a triangular field with sides of 240 feet, 300 feet, and 360 feet. He wants to apply fertilizer to the field. If one 40-pound bag of fertilizer covers 6,000 square feet, how many bags must he buy to cover the field?

13. 060333b, P.I. A2.A.73
A farmer has determined that a crop of strawberries yields a yearly profit of $1.50 per square yard. If strawberries are planted on a triangular piece of land whose sides are 50 yards, 75 yards, and 100 yards, how much profit, to the *nearest hundred dollars*, would the farmer expect to make from this piece of land during the next harvest?

14. 060634b, P.I. A2.A.73
A triangular plot of land has sides that measure 5 meters, 7 meters, and 10 meters. What is the area of this plot of land, to the *nearest tenth of a square meter*?

A2.A.73: Solve for an unknown side or angle, using the Law of Sines or the Law of Cosines

[1]   D

[4] 3.8, and the Law of Cosines is used.
[3] Appropriate work is shown, but one computational or rounding error is made.
[2] Appropriate work is shown, but one computational error and one rounding error are made.
or [2] The Law of Cosines is shown, but sine is used instead of cosine, such as
$x^2 = 3.2^2 + 5.6^2 - 2(3.2)(5.6)(\sin 40),$ but an appropriate answer is determined, based on that error.
[1] Substitution into the Law of Cosines is used, but no further work is shown.
or [1] 3.8, but no work is shown.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[2]   incorrect procedure.

[4] 0.15 hour or 9 minutes or an appropriately rounded answer, and appropriate work is shown, such as using the Law of Cosines.
[3] Appropriate work is shown, but one computational or rounding error is made.
[2] The correct distance along County Route 21 is found, but no further work or incorrect work is shown.
or [2] Appropriate work is shown, but one computational and one rounding error are made.
[1] The Pythagorean theorem is used to find the distance along County Route 21, and this distance is used to compare travel times.
or [1] 0.15 hour or an equivalent answer, but no work is shown.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[3]   incorrect procedure.

[4] 174, and appropriate work is shown, such as the use of the Law of Cosines.
[3] Appropriate work is shown, but one computational or rounding error is made.
[2] Appropriate work is shown, but two or more computational or rounding errors are made.
or [2] One conceptual error is made when applying the Law of Cosines, but an appropriate answer is found.
[1] Correct substitution is made into the Law of Cosines, but no further correct work is shown.
or [1] 174, but no work is shown.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[4]   incorrect procedure.

[4] 5,513 and a correct diagram is drawn, and appropriate work is shown, such as using the Law of Cosines.

[3] Appropriate work is shown, but one computational or rounding error is made.

or [3] 5,513, and appropriate work is shown, but no diagram is drawn.

or [3] Appropriate work is shown, but the calculations are performed in radians, resulting in an answer of 6,698.

or [3] An incorrect diagram is drawn, but an appropriate solution is found using the Law of Cosines.

[2] Appropriate work is shown, but two or more computational or rounding errors are made.

or [2] Appropriate work is shown, but one conceptual error is made.

or [2] Appropriate work is shown, but an incorrect substitution is made into the Law of Cosines, but an appropriate solution is found.

[1] Appropriate work is shown, but one conceptual error and one computational or rounding error are made.

or [1] Correct substitution is made into the Law of Cosines, but no further correct work is shown.

or [1] A correctly labeled diagram is drawn, but no further correct work is shown.

or [1] 5,513, but no work is shown and no diagram is drawn.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[5]

[4] 4506 and shows an appropriate application of the Law of Cosines such as

$$a^2 = 1146.75^2 + 3746.75^2 - 2(1146.75)(3746.75)\cos 125.2°.$$

[3] Makes an appropriate application of the Law of Cosines, but does not add the wall length to the distances. (answer 4086)

or [3] Uses the appropriate method, but makes a minor mathematical or rounding error.

or [3] 4506 showing a correct diagram but showing no Law of Cosines.

[2] Correctly uses the Law of Cosines, with or without the wall added, but does not find the square root.

or [2] Uses Law of Cosines without the wall added and makes a minor mathematical mistake.

or [2] Finds the distance between the ends of the two walls (answer 438) using the Law of Sines or Cosines.

[1] Obtains the correct answer of 4506, but does not show any work.

or [1] Sets up diagram with correct sides (1146.75 and 3746.75) and angle, but does not solve problem.

[0] Response is completely incorrect, irrelevant, or incoherent; or is a correct response that was obtained by an obviously

[6] incorrect procedure

[4] 11.8, and an appropriate application of the Law of Cosines is shown.

[3] Appropriate work is shown, but one computational or rounding error is made.

or [3] The Law of Cosines is correctly applied, but the square root is not found.

[2] The Law of Cosines is applied correctly, and correct substitutions are shown, but no further work is shown.

or [2] Appropriate work is shown, but more than one computational error is made.

[1] The diagram is set up with the correct sides and angles, and the Law of Cosines is written, but substitution is not made.

or [1] The diagram is set up with the correct sides and angles, but no further work is shown.

or [1] 11.8, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[7] incorrect procedure.

[6] 312 and 30,642, and appropriate work is shown, such as using the Law of Cosines and the area formula.

[5] Appropriate work is shown, but one computational or rounding error is made.

[4] Appropriate work is shown, but two or more computational or rounding errors are made.

or [4] Appropriate work is shown, but one conceptual error is made.

or [4] Appropriate work is shown, but the square root is not computed to find the length of the third side, but an appropriate area is found.

or [4] The length of the third side is found correctly, but no further correct work is shown.

[3] Appropriate work is shown, but one conceptual error and one computational or rounding error are made.

[2] The area of the triangle is found correctly, but no further correct work is shown.

or [2] 312 and 30,642, but no work is shown.

[1] Appropriate work is shown to find the area of the triangle, but one computational or rounding error is made, and no further correct work is shown.

or [1] 312 or 30,642, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[8] incorrect procedure.

[4] 41.4, and appropriate work is shown.
[3] Appropriate work is shown, but one computational or rounding error is made.
[2] Appropriate work is shown, but two or more computational or rounding errors are made.
or [2] Appropriate work is shown, but one conceptual error is made.
[1] Appropriate work is shown, but one conceptual error and one computational or rounding error are made.
or [1] A correct substitution is made into the Law of Cosines, but no further correct work is shown.
or [1] 41.4, but no work is shown.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[9]   incorrect procedure.
_____

[4] 116, and appropriate work is shown, such as the use of the Law of Cosines.
[3] Appropriate work is shown, but one computational or rounding error is made.
[2] Appropriate work is shown, but two or more computational or rounding errors are made.
or [2] Appropriate work is shown, but one conceptual error is made.
or [2] Correct substitution is made into the Law of Cosines, but no further correct work is shown.
[1] Appropriate work is shown, but one conceptual error and one computational or rounding error are made.
or [1] A complete and correctly labeled diagram is drawn, but no further correct work is shown.
or [1] 116, but no work is shown.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[10]   incorrect procedure.
_____

[6] 8,200, and appropriate work is shown, such as using the Law of Cosines or Hero(n)'s formula.
[5] Appropriate work is shown, but one computational or rounding error is made.
[4] Appropriate work is shown, but two or more computational or rounding errors are made.
[3] Appropriate work is shown, but one conceptual error is made.
or [3] The Law of Cosines is used to find an angle, but no further correct work is shown.
[2] Appropriate work is shown, but one conceptual error and one computational or rounding error are made.
[1] A correct substitution is made into the Law of Cosines, but no further correct work is shown.
or [1] 8,200, but no work is shown.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[11]   incorrect procedure.
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[6] 6, and appropriate work is shown, such as determining the area of the field, using Heron's formula or using the Law of Cosines to determine one angle of the triangle, followed by $A = \frac{1}{2}ab\sin C,$ and then $A \div 6000.$

[5] Appropriate work is shown, but one computational or rounding error is made.

[4] Appropriate work is shown, but two or more computational or rounding errors are made.

or [4] Appropriate work is shown to find the area of the triangle, but the number of bags of fertilizer is not found.

[3] Appropriate work is shown, but one conceptual error is made.

or [3] The Law of Cosines is used to find an angle, and substitution is made into the correct area equation, but no further correct work is shown.

[2] Appropriate work is shown, but one conceptual error and one computational or rounding error are made.

or [2] The Law of Cosines is used to find an angle, but no further correct work is shown.

[1] Correct substitution is made into the Law of Cosines, but no further correct work is shown.

or [1] 6, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[12]   incorrect procedure.

[6] 2,700, and appropriate work is shown, such as using the Law of Cosines and finding the area of the triangle.

[5] Appropriate work is shown, but one computational or rounding error is made.

[4] Appropriate work is shown, but more than one computational or rounding error is made.

or [4] Appropriate work is shown, and the area of the triangle is determined correctly, but the dollar amount is not determined or is determined incorrectly.

or [4] The Law of Cosines is used correctly to determine an angle, but an incorrect procedure is used to find the area, but an appropriate dollar amount is found.

or [4] The Law of Cosines is used incorrectly to determine an angle, but a correct procedure is used to find the area, and an appropriate dollar amount is found.

[3] The Law of Cosines is used correctly to determine an angle, but an incorrect procedure is used to find the area, and the dollar amount is not determined or is determined incorrectly.

[2] The Law of Cosines is used correctly to determine an angle, but no further correct work is shown.

[1] A correct equation using the Law of Cosines is written, but no further correct work is shown.

or [1] 2,700, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[13]   incorrect procedure.

[6] 16.2, and appropriate work is shown, such as using the Law of Cosines to find one angle, and then using $K = \dfrac{1}{2}ab\sin C$ or Hero(n)'s formula, $A = \sqrt{s(s-a)(s-b)(s-c)}$, to find the area.

[5] Appropriate work is shown, but one computational or rounding error is made.

[4] Appropriate work is shown, but two or more computational or rounding errors are made.

[3] Appropriate work is shown, but one conceptual error is made, but an appropriate area is found.

or [3] The Law of Cosines is used to find a correct measure for one of the angles of the triangle, but no further correct work is shown.

[2] Appropriate work is shown, but one conceptual error and one computational or rounding error are made.

[1] Correct substitution is made into the Law of Cosines, but no further correct work is shown.

or [1] 16.2, but no work is shown.

[0] Right triangle trigonometry is used inappropriately.

or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an

[14] obviously incorrect procedure.