NAME:_____

A2.A.68: Solve trigonometric equations for all values of the variable from 0 to 360 degrees

1. 080610b, P.I. A2.A.68
A solution set of the equation $5\sin\theta + 3 = 3$ contains all multiples of

[A] 45° [B] 180° [C] 135° [D] 90°

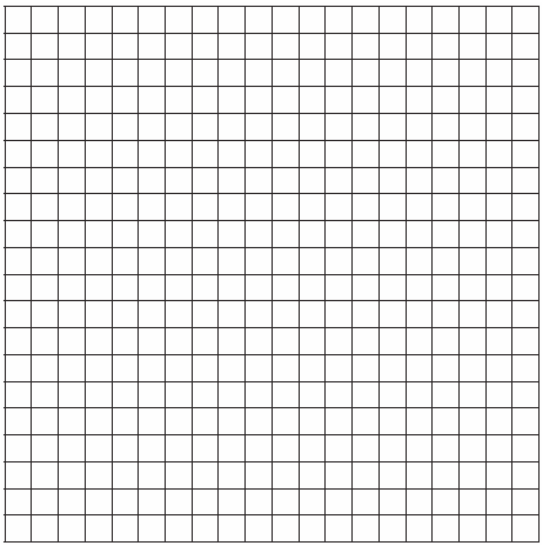
2. 010523b, P.I. A2.A.68
Solve the following equation algebraically for all values of θ in the interval $0° \le \theta \le 180°$.

$$2\sin\theta - 1 = 0$$

3. 060319b, P.I. A2.A.68
What value of x in the interval $0° \le x \le 180°$ satisfies the equation $\sqrt{3}\tan x + 1 = 0$?

[A] 150° [B] 30° [C] -30° [D] 60°

4. 060731b, P.I. A2.A.68
The average annual snowfall in a certain region is modeled by the function $S(t) = 20 + 10\cos(\frac{\pi}{5}t)$, where S represents the annual snowfall, in inches, and t represents the number of years since 1970. What is the minimum annual snowfall, in inches, for this region? In which years between 1970 and 2000 did the minimum amount of snow fall? [The use of the grid is optional.]



5. 010630b, P.I. A2.A.68
An architect is using a computer program to design the entrance of a railroad tunnel. The outline of the opening is modeled by the function $f(x) = 8\sin x + 2$, in the interval $0 \le x \le \pi$, where x is expressed in radians. Solve algebraically for all values of x in the interval $0 \le x \le \pi$, where the height of the opening, f(x), is 6. Express your answer in terms of π. If the x-axis represents the base of the tunnel, what is the maximum height of the entrance of the tunnel?

6. 010727b, P.I. A2.A.68
Find, to the *nearest degree*, all values of θ in the interval $0° \le \theta \le 180°$ that satisfy the equation $8\cos^2\theta - 2\cos\theta - 1 = 0$.

7. 080432b, P.I. A2.A.68
Solve algebraically for all values of θ in the interval $0° \le \theta \le 360°$ that satisfy the equation $\frac{\sin^2\theta}{1+\cos\theta} = 1$.

8. 010317b, P.I. A2.A.68
If $(\sec x - 2)(2\sec x - 1) = 0$, then x terminates in

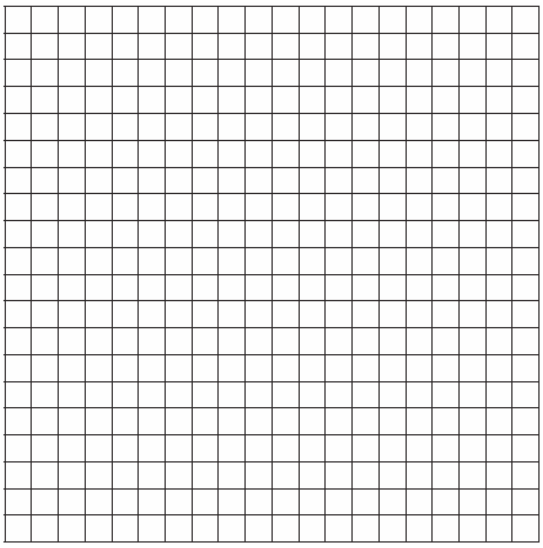
[A] Quadrant I, only

[B] Quadrants I and II, only

[C] Quadrants I and IV, only

[D] Quadrants I, II, III, and IV

9. 080833b, P.I. A2.A.68
Find all values of x in the interval $0° \le x < 360°$ that satisfy the equation $3\cos 2x = \cos x + 2$. Express your answers to the *nearest degree*. [The use of the grid is optional.]



10. 060932b, P.I. A2.A.68
Solve the equation $\cos\theta = 2 + 3\cos 2\theta$ for all values of θ to the *nearest tenth of a degree*, in the interval $0° \le \theta < 360°$.

11. 060131b, P.I. A2.A.68
In the interval $0° \le A \le 360°$, solve for all values of A in the equation $\cos 2A = -3\sin A - 1$.

12. 060530b, P.I. A2.A.68
Find, to the *nearest degree*, all values of θ in the interval $0° < \theta < 360°$ that satisfy the equation $3\cos 2\theta + \sin\theta - 1 = 0$.

13. 060829b, P.I. A2.A.68
Find all values of θ in the interval $0° \le \theta < 360°$ that satisfy the equation $3\cos 2\theta + 2\sin\theta + 1 = 0$, and round all answers to the *nearest hundredth of a degree*. [Only an algebraic solution can receive full credit.]

14. 010829b, P.I. A2.A.68
Find all values of x in the interval $0° < x < 360°$ that satisfy the equation $3\cos x + \sin 2x = 0$.

15. 060427b, P.I. A2.A.68
Navigators aboard ships and airplanes use nautical miles to measure distance. The length of a nautical mile varies with latitude. The length of a nautical mile, L, in feet, on the latitude line θ is given by the formula $L = 6{,}077 - 31\cos 2\theta$. Find, to the *nearest degree*, the angle θ, $0 \le \theta \le 90°$, at which the length of a nautical mile is approximately 6,076 feet.

16. 010832b, P.I. A2.A.68
The horizontal distance, in feet, that a golf ball travels when hit can be determined by the formula $d = \frac{v^2 \sin 2\theta}{g}$, where v equals initial velocity, in feet per second; g equals acceleration due to gravity; θ equals the initial angle, in degrees, that the path of the ball makes with the ground; and d equals the horizontal distance, in feet, that the ball will travel. A golfer hits the ball with an initial velocity of 180 feet per second and it travels a distance of 840 feet. If g = 32 feet per second per second, what is the smallest initial angle the path of the ball makes with the ground, to the *nearest degree*?

A2.A.68: Solve trigonometric equations for all values of the variable from 0 to 360 degrees

[1] B

[2] 30 and 150, and appropriate work is shown.
[1] Appropriate work is shown, but one computational error is made.
or [1] Appropriate work is shown, but one conceptual error is made.
or [1] Appropriate work is shown, but only 30 or 150 is found.
or [1] 30 and 150, but no work is shown.
[0] 30 or 150, but no work is shown.
or [0] The value of $\sin\theta$ is shown to be $\frac{1}{2}$.
or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an
[2] obviously incorrect procedure.

[3] A

[4] 10 and 1975, 1985, and 1995, and appropriate work is shown or an appropriate explanation is written.
[3] Appropriate work is shown, but one computational or graphing error is made.
or [3] 10, and appropriate work is shown, but only two of the years are found.
[2] Appropriate work is shown, but two or more computational or graphing errors are made.
or [2] Appropriate work is shown, but one conceptual error is made, such as graphing an incorrect function.
or [2] An incorrect equation of equal difficulty is solved appropriately.
or [2] 1975, 1985, and 1995, and appropriate work is shown or an appropriate explanation is written, but the minimum snowfall is not found.
[2] 10, and appropriate work is shown, but only one of the years is found.
[1] Appropriate work is shown, but one conceptual error and one computational or graphing error are made.
or [1] 10, and appropriate work is shown or an appropriate explanation is written, but the years are not found.
or [1] 10 and 1975, 1985, and 1995, but no work is shown.
[0] 10 or 1975, 1985, and 1995, but no work is shown.
or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an
[4] obviously incorrect procedure.

[4] $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ and 10, and appropriate work is shown.
[3] Appropriate work is shown, but one computational error is made.
or [3] $x = 0.52$ and $x = 2.62$ or $x = 30°$ and $x = 150°$ and 10, and appropriate work is shown.
or [3] $\frac{\pi}{6}$ and $\frac{5\pi}{6}$, and appropriate work is shown, but the maximum height is missing.
[2] Appropriate work is shown, but two or more computational errors are made.
or [2] Appropriate work is shown, but one conceptual error is made.
or [2] $x = 0.52$ and $x = 2.62$ or $x = 30°$ and $x = 150°$, and appropriate work is shown, but the maximum height is missing.
or [2] $\frac{\pi}{6}$ or $\frac{5\pi}{6}$ and 10, and appropriate work is shown.
[1] Appropriate work is shown, but one conceptual error and one computational error are made.
or [1] 30° or 150° and 10, and appropriate work is shown.
or [1] $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ and 10, but no work is shown.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure. [5]

[4] 60 and 104, and appropriate work is shown either algebraically or graphically.
[3] Appropriate work is shown, but one computational or rounding error is made.
or [3] Appropriate work is shown, but only one correct angle is found.
or [3] 60 and 104, and appropriate work is shown, but additional angles outside the interval are found.
[2] Appropriate work is shown, but two or more computational or rounding errors are made.
or [2] Appropriate work is shown, but one conceptual error is made.
or [2] $\cos\theta = -\frac{1}{4}$ and $\cos\theta = -\frac{1}{2}$, but no further correct work is shown.
[1] Appropriate work is shown, but one conceptual error and one computational or rounding error are made.
or [1] 60 and 104, but no work is shown.
[0] 60 or 104, but no work is shown.
or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure. [6]

[4] 90 and 270, and appropriate work is shown, such as solving $\sin^2\theta = 1 + \cos\theta$.
[3] Appropriate work is shown, but one computational error is made or the answers are expressed in radians.
or [3] Appropriate work is shown, but 180 is not rejected as a solution.
or [3] Appropriate work is shown, but only one solution is found.
[2] Appropriate work is shown, but two or more computational errors are made.
or [2] Appropriate work is shown, but one conceptual error is made.
or [2] An incorrect trigonometric substitution is made, but the equation is solved appropriately.
or [2] A trigonometric equation set equal to zero is written, but no further correct work is shown.
or [2] 90 and 270, but a graphic solution is provided.
[1] The equation $\sin^2\theta - \cos\theta - 1 = 0$ is found, but no further correct work is shown.
or [1] A graphic solution is provided, and one computational or graphing error is made.
or [1] 90 and 270, but no work is shown.
[0] 90 or 270, but no work is shown.
or [0] 90, 180, and 270, but no work is shown.
or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an
[7] obviously incorrect procedure.

[8] C

[6] 0, 146, and 214, and appropriate work is shown.
[5] Appropriate work is shown, but one computational, rounding, factoring, or graphing error is made.
or [5] Appropriate work is shown, and the equation is solved for 0 and 146, but 214 is not found.
or [5] Appropriate work is shown to find the correct solutions, but 360 is included.
[4] Appropriate work is shown, but two or more computational, rounding, factoring, or graphing errors are made.
or [4] Appropriate work is shown, but the equation is solved for 0, 146, and 360.
[3] Appropriate work is shown, but one conceptual error is made.
or [3] Appropriate work is shown, and the equation is factored correctly, but no further correct work is shown.
[2] Appropriate work is shown, but one conceptual error and one computational, rounding, factoring, or graphing error are made.
or [2] $6\cos^2 x - \cos x - 5 = 0$ is written, but no further correct work is shown.
[1] $2\cos^2 x - 1$ is substituted for $\cos 2x$, but no further correct work is shown.
or [1] 0, 146, and 214, but no work is shown.
[0] 0 or 146 or 214, but no work is shown.
or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an
[9] obviously incorrect procedure.

[4] 60, 109.5, 250.5, 300, and appropriate work is shown algebraically or graphically.
[3] Appropriate work is shown, but one computational, factoring, graphing, or rounding error is made.
or [3] Appropriate work is shown, but only three correct values of θ are found.
[2] Appropriate work is shown, but two or more computational, factoring, graphing, or rounding errors are made.
or [2] Appropriate work is shown, but one conceptual error is made.
or [2] Appropriate work is shown, but only two correct values of θ are found.
[1] Appropriate work is shown, but one conceptual error and one computational, factoring, graphing, or rounding error are made.
or [1] Appropriate substitutions are made and the equation is written in standard form, but no further correct work is shown.
or [1] 60, 109.5, 250.5, 300, but no work is shown.
[0] 60 or 109.5 or 250.5 or 300, but no work is shown.
or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an
[10] obviously incorrect procedure.

[4] 210° and 330°, and appropriate work is shown.
[3] Correct substitution and factoring are shown, with at least the reference angle of 30° found.
or [3] Correct substitution is shown, and the equation is put in standard form and factored correctly, but an incorrect reference angle is used to find appropriate answers.
or [3] An incorrect quadratic equation is solved correctly, and appropriate angles are determined.
[2] Correct substitution is shown, and the equation is put in standard form and factored correctly, but no angles are found.
[1] Correct substitution is shown, but the equation is not factored or is factored incorrectly.
or [1] 210° and 330°, but no work is shown.
[0] 210° or 330° or 30°, but no work is shown.
or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an
[11] obviously incorrect procedure.

[4] 42, 138, 210, and 330, and appropriate work is shown, such as substituting for cos 2θ and solving the resulting quadratic equation.
[3] Appropriate work is shown, but one computational or rounding error is made.
or [3] Incorrect substitution is made for cos 2θ, such as $1-\sin^2\theta$, but all further work is appropriate.
[2] Appropriate work is shown, but two or more computational errors are made.
or [2] Appropriate work is shown, but one conceptual error is made.
or [2] Correct substitution is made, and appropriate work is shown to obtain the values of sin θ, but the values of θ are not found.
or [2] A quadratic equation in terms of sin θ is written in standard form, but no further correct work is shown.
[1] Appropriate work is shown, but one conceptual error and one computational error are made.
or [1] Correct substitution is made for cos 2θ, but no further correct work is shown.
or [1] 42, 138, 210, and 330, but no work is shown. [All four answers must be identified to receive this credit.]
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[12]

[4] 90, 221.81, and 318.19, and appropriate work is shown, such as solving the equation $3\sin^2\theta-\sin\theta-2=0$.
[3] Appropriate work is shown, but one computational or rounding error is made.
or [3] The equation is solved correctly for θ, but only one or two of the solutions are found.
[2] Appropriate work is shown, but two or more computational or rounding errors are made.
or [2] Appropriate work is shown, but one conceptual error is made.
or [2] 90, 221.81, and 3.18.19, and appropriate work is shown, but a graphic method is used.
or [2] Appropriate work is shown to find the values of $\sin\theta$, but no further correct work is shown.
[1] Appropriate work is shown, but one conceptual error and one computational or rounding error are made.
or [1] A correct quadratic equation in standard form is written, but no further correct work is shown.
or [1] 90, 221.81, and 318.19, but no work is shown.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[13]

[4] 90° and 270°, and appropriate work is shown, such as solving the equation $3\cos x + 2\sin x \cos x = 0$ or sketching a graph and finding the x-intercepts.
[3] Appropriate work is shown, but one computational, factoring, or graphing error is made.
or [3] Appropriate work is shown, but the answers are expressed in radian measure.
[2] Appropriate work is shown, but two or more computational, factoring, or graphing errors are made.
or [2] Appropriate work is shown, but one conceptual error is made,
or [2] An appropriate graph is sketched, but no further correct work is shown.
[1] Appropriate work is shown, but one conceptual error and one computational, factoring, or graphing error are made.
or [1] Correct substitution is made for sin 2x, but no further correct work is shown.
or [1] 90° and 270°, but no work is shown.
[0] 90° or 270°, but no work is shown.
or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
[14]

[4] 44, and appropriate work is shown, such as solving the equation $6{,}076 = 6{,}077 - 31\cos 2\theta$.
[3] Appropriate work is shown, but one computational or rounding error is made.
[2] Appropriate work is shown, but two or more computational or rounding errors are made.
or [2] Appropriate work is shown, but one conceptual error is made.
or [2] An incorrect equation of equal difficulty is solved appropriately.
[1] Appropriate work is shown, but one conceptual error and one computational error are made.
or [1] 44, but no work is shown.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
[15]

[4] 28, and appropriate work is shown, such as substituting into the given equation or solving the equation graphically.
[3] Appropriate work is shown, but one computational, rounding, or graphing error is made.
or [3] Appropriate work is shown, but 56, the value of 2θ, is given as the answer.
[2] Appropriate work is shown but two or more computational, rounding, or graphing errors are made.
or [2] Appropriate work is shown, but one conceptual error is made.
[1] Appropriate work is shown, but one conceptual error and one computational, rounding, or graphing error are made.
or [1] Appropriate work is shown to find the value of sin 2θ, but no further correct work is shown.
or [1] 28, but no work is shown.
[0] A.zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
[16]