A2.A.76: Angle Sum and Difference Identities 4: Apply the angle sum and difference formulas for trigonometric functions

1 If A and B are positive acute angles, $\sin A = \frac{5}{13}$, and $\cos B = \frac{4}{5}$, what is the value of $\sin(A + B)$?

2 If $\sin x = \frac{12}{13}$, $\cos y = \frac{3}{5}$, and x and y are acute angles, the value of $\cos(x - y)$ is

3 If $\sin A = \frac{3}{5}$, $\sin B = \frac{5}{13}$, and angles A and B are acute angles, what is the value of $\cos(A - B)$?

4 If A and B are both acute angles, $\sin A = \frac{5}{13}$ and $\sin B = \frac{4}{5}$, then $\sin(A - B)$ is

5 If $\cos x = \frac{12}{13}$ and $\sin y = \frac{4}{5}$, then $\sin(x - y)$ equals

6 Given angle A in Quadrant I with $\sin A = \frac{12}{13}$ and angle B in Quadrant II with $\cos B = -\frac{3}{5}$, what is the value of $\cos(A - B)$?

7 If $\sin A = \frac{4}{5}$, $\tan B = \frac{5}{12}$, and angles A and B are in Quadrant I, what is the value of $\sin(A + B)$?

8 If $\tan A = 8$ and $\tan B = \frac{1}{2}$, what is the value of $\tan(A + B)$?

9 If $\tan A = \frac{2}{3}$ and $\tan B = \frac{1}{2}$, what is the value of $\tan(A + B)$?

10 If $\tan A = \frac{2}{3}$ and $\sin B = \frac{5}{\sqrt{41}}$ and angles A and B are in Quadrant I, find the value of $\tan(A + B)$.

11 If $\tan A = \frac{2}{3}$ and $\tan B = 3$, express $\tan(A - B)$ as a fraction in simplest form.

12 When $\sin x = -\frac{8}{17}$ and x lies in Quadrant III and $\cos y = -\frac{4}{5}$ and y lies in Quadrant II, what is $\cos(x - y)$?

13 Using the formula for $\cos(x - y)$, find the exact value of $\cos 15°$ in radical form if $m\angle x = 45$ and $m\angle y = 30$.

14 Express as a single fraction the exact value of $\sin 75°$.

15 If $\sin x = \frac{4}{5}$, where $0° < x < 90°$, find the value of $\cos(x + 180°)$.

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Answer Section

1 ANS:
$\frac{56}{65}$

If $\sin A = \frac{5}{13}$ and A is a positive acute angle, $\cos A = \frac{12}{13}$. If $\cos B = \frac{4}{5}$ and B is a positive acute angle, $\sin B = \frac{3}{5}$.

$\sin(A + B) = \frac{5}{13} \cdot \frac{4}{5} + \frac{12}{13} \cdot \frac{3}{5} = \frac{56}{65}$

REF: 060312b

2 ANS:
$\frac{63}{65}$

If $\sin x = \frac{12}{13}$ and x is an acute angle, $\cos x = \frac{5}{13}$. If $\cos y = \frac{3}{5}$ and y is an acute angle, $\sin y = \frac{4}{5}$.

$\cos(x - y) = \frac{5}{13} \cdot \frac{3}{5} + \frac{12}{13} \cdot \frac{4}{5} = \frac{63}{65}$.

REF: 080316b

3 ANS:
$\frac{63}{65}$

REF: 018620siii

4 ANS:
$-\frac{33}{65}$

REF: 069020siii

5 ANS:
$-\frac{33}{65}$

REF: 089432siii

6 ANS:
$\frac{33}{65}$

$\cos(A - B) = \left(\frac{5}{13}\right)\left(-\frac{3}{5}\right) + \left(\frac{12}{13}\right)\left(\frac{4}{5}\right) = -\frac{15}{65} + \frac{48}{65} = \frac{33}{65}$

REF: 011214a2

7 ANS:

$\frac{63}{65}$

If $\sin A = \frac{4}{5}$ and angle A is in Quadrant I, $\cos A = \frac{3}{5}$. If $\tan B = \frac{5}{12}$, and angle B is in Quadrant I, then

$\tan^2 B + 1 = \sec^2 B$

$(\frac{5}{12})^2 + 1 = \sec^2 B$

$\frac{25}{144} + 1 = \sec^2 B$

$\frac{169}{144} = \sec^2 B$

$\frac{13}{12} = \sec B$

$\frac{12}{13} = \cos B$

and

$\cos^2 B + \sin^2 B = 1$

$(\frac{12}{13})^2 + \sin^2 B = 1$

$\frac{144}{169} + \sin^2 B = 1$

$\sin^2 B = \frac{25}{169}$

$\sin B = \frac{5}{13}$

. Then $\sin(A + B) = \frac{4}{5} \cdot \frac{12}{13} + \frac{3}{5} \cdot \frac{5}{13} = \frac{63}{65}$.

REF: 080409b

8 ANS:

$-\frac{17}{6}$

REF: 069525siii

9 ANS:

$\frac{7}{4}$

REF: 010018siii

10 ANS:

$\frac{23}{2}$ $\cos^2 B + \sin^2 B = 1$ $\tan B = \frac{\sin B}{\cos B} = \frac{\frac{5}{\sqrt{41}}}{\frac{4}{\sqrt{41}}} = \frac{5}{4}$

$$\cos^2 B + \left(\frac{5}{\sqrt{41}}\right)^2 = 1$$

$$\cos^2 B + \frac{25}{41} = \frac{41}{41}$$

$$\cos^2 B = \frac{16}{41}$$

$$\cos B = \frac{4}{\sqrt{41}}$$

$$\tan(A+B) = \frac{\frac{2}{3} + \frac{5}{4}}{1 - \left(\frac{2}{3}\right)\left(\frac{5}{4}\right)} = \frac{\frac{8+15}{12}}{\frac{12}{12} - \frac{10}{12}} = \frac{\frac{23}{12}}{\frac{2}{12}} = \frac{23}{2}$$

REF: 081037a2

11 ANS:

$-\frac{7}{9}$

REF: 088405b

12 ANS:

$\frac{36}{85}$

REF: 060141siii

13 ANS:

$\frac{\sqrt{6} + \sqrt{2}}{4}$

REF: 089041siii

14 ANS:

$$\sin(45+30) = \sin 45 \cos 30 + \cos 45 \sin 30$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

REF: 061136a2

15 ANS:

$-\frac{3}{5}$. If $\sin x = \frac{4}{5}$ and $0° < x < 90°$, then $\cos x = \frac{3}{5}$.

$$\begin{aligned}\cos(A + B) &= \cos x \cos 180° - \sin x \sin 180° \\ &= \frac{3}{5}(-1) - \frac{4}{5}(0) \\ &= -\frac{3}{5}\end{aligned}$$

REF: 080126b