A2.A.75: Law of Sines - The Ambiguous Case 2: Determine the solution(s) from the SSA situation (ambiguous case)

1 In $\triangle ABC$, $m\angle A = 74$, $a = 59.2$, and $c = 60.3$. What are the two possible values for $m\angle C$, to the *nearest tenth*?

2 How many distinct triangles can be formed if $m\angle A = 35$, $a = 10$, and $b = 13$?

3 How many distinct triangles can be formed if $m\angle A = 30$, side $b = 12$, and side $a = 8$?

4 What is the total number of distinct triangles that can be constructed if $AC = 13$, $BC = 8$, and $m\angle A = 36$?

5 If the measure of $\angle A = 40°$, $a = 5$, and $b = 6$, how many different triangles can be constucted?

6 In $\triangle DEF$, $d = 5$, $e = 8$, and $m\angle D = 32$. How many distinct triangles can be drawn given these measurements?

7 How many distinct triangles can be constructed if $m\angle A = 30$, side $a = \sqrt{34}$, and side $b = 12$?

8 Sam is designing a triangular piece for a metal sculpture. He tells Martha that two of the sides of the piece are 40 inches and 15 inches, and the angle opposite the 40-inch side measures 120°. Martha decides to sketch the piece that Sam described. How many different triangles can she sketch that match Sam's description?

9 An architect commissions a contractor to produce a triangular window. The architect describes the window as $\triangle ABC$, where $m\angle A = 50$, $BC = 10$ inches, and $AB = 12$ inches. How many distinct triangles can the contractor construct using these dimensions?

10 Sam needs to cut a triangle out of a sheet of paper. The only requirements that Sam must follow are that one of the angles must be 60°, the side opposite the 60° angle must be 40 centimeters, and one of the other sides must be 15 centimeters. How many different triangles can Sam make?

A2.A.75: Law of Sines - The Ambiguous Case 2: Determine the solution(s) from the SSA situation (ambiguous case)
Answer Section

1 ANS:
78.3 and 101.7

$$\frac{59.2}{\sin 74} = \frac{60.3}{\sin C} \quad 180 - 78.3 = 101.7$$

$$C \approx 78.3$$

REF: 081006a2

2 ANS:
2

$$\frac{10}{\sin 35} = \frac{13}{\sin B} \quad . \quad 35 + 48 < 180$$

$$B \approx 48, 132 \quad 35 + 132 < 180$$

REF: 011113a2

3 ANS:
2

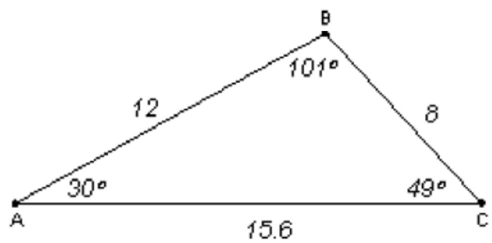
$$\frac{8}{\sin 30^\circ} = \frac{12}{\sin C}$$

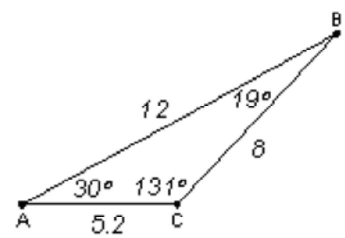
$$C \approx 49^\circ$$

$$or\ C \approx 131^\circ\ (180^\circ - 49^\circ)$$

$$49^\circ + 30^\circ < 180^\circ\ \triangle$$

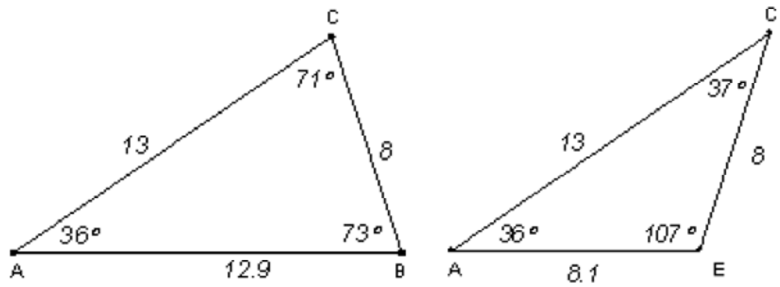
$$131^\circ + 30^\circ < 180^\circ\ \triangle$$





REF: 080414b

4 ANS:
2



$$\frac{8}{\sin 36°} = \frac{13}{\sin B}$$
$$B \approx 73°$$
$$or\ B \approx 107°\ (180° - 73°)$$
$$73° + 36° < 180°\ \triangle$$
$$107° + 36° < 180°\ \triangle$$

REF: 080519b

5 ANS:
2

$$\frac{5}{\sin 40} = \frac{6}{\sin B} \quad . \quad 50.5 + 40 < 180$$
$$B = 50.5 \text{ or } 129.5 \quad 129.5 + 40 < 180$$

REF: 061011b

6 ANS:
2

$$\frac{5}{\sin 32} = \frac{8}{\sin E} \quad 57.98 + 32 < 180$$
$$E \approx 57.98 \quad (180 - 57.98) + 32 < 180$$

REF: 011419a2

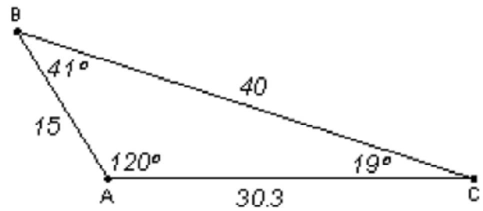
7 ANS:
none

$$\frac{\sqrt{34}}{\sin 30} = \frac{12}{\sin B}$$
$$B = \sin^{-1} \frac{12 \sin 30}{\sqrt{34}}$$
$$\approx \sin^{-1} \frac{6}{5.8}$$

REF: 011523a2

8 ANS:
1
The triangle has an obtuse angle of 120°, and may not have a second obtuse angle. Check if one triangle is possible.

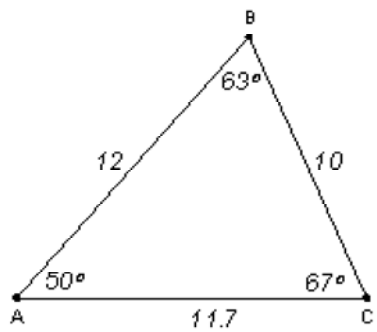
$$\frac{40}{\sin 120°} = \frac{15}{\sin C}$$
$$C \approx 19°$$
$$or\ C \approx 161°\ (180° - 19°)$$
$$19° + 120° < 180°\ \ \triangle$$
$$161° + 120° > 180°\ \ \sim\triangle$$

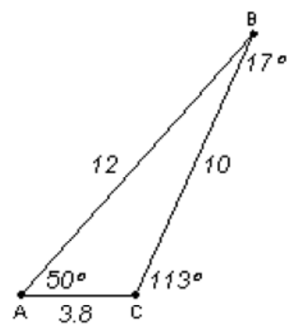


REF: 060416b

9 ANS:
2

$$\frac{10}{\sin 50°} = \frac{12}{\sin C}$$
$$C \approx 67°$$
$$or\ C \approx 113°\ (180° - 67°)$$
$$67° + 50° < 180°\ \ \triangle$$
$$113° + 50° < 180°\ \ \triangle$$

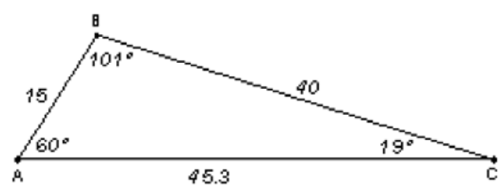




REF: 080311b

10 ANS:
1

$$\frac{40}{\sin 60°} = \frac{15}{\sin C}$$
$$C \approx 19°$$
$$or\ C \approx 161°\ (180° - 19°)$$
$$19° + 60° < 180°\ \ \triangle$$
$$161° + 60° > 180°\ \ \sim\triangle$$



REF: 060620b