The University of the State of New York

222D HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Monday, January 19, 1920—9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry. The minimum time requirement is two recitations a week for a school year or four recitations a week for half a school year.

Name the author of the textbook you have used in your study of solid geometry.

Answer eight questions, including four from group I, two from group II and two from group III.

Group I

Answer four questions from this group.

1 Prove that the intersections of two parallel planes with a third plane are parallel.

2 Prove that the sum of the face angles of a convex polyhedral angle is less than four right angles.

3 Prove that the volume of a triangular prism is equal to the product of its base and altitude.

4 Prove that the line connecting the center of a sphere and the center of a small circle of the sphere is perpendicular to the plane of the circle.

5 Prove that the surface of a sphere is equal to the product of the diameter by the circumference of a great circle of the sphere.

Group II

Answer two questions from this group.

6 A right circular cylinder is circumscribed about a sphere. Show that (*a*) the surface of the sphere is equivalent to $\frac{2}{3}$ of the total surface of the cylinder, (*b*) the volume of the sphere is $\frac{2}{3}$ of the volume of the cylinder. [No authorities (reasons) are required in answering this question.]

7 In the pyramid A-BCD, prove that the lines joining in order the mid points of BC, AC, AD and BD form a parallelogram.

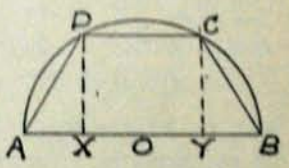
8 Prove that if perpendiculars are let fall upon the faces of a dihedral angle from any point within the angle, the plane of the perpendiculars is perpendicular to the edge of the dihedral angle.

Group III

Answer two questions from this group.

9 The total surface (T) of a regular tetrahedron is $100\sqrt{3}$ square units. Find the altitude (H) and the volume (V) of the solid.

10 The frustum of a regular pyramid has square bases 8″ and 4″ respectively on a side, and an altitude of 15″. Find the altitude of an equivalent pyramid whose base is a mid section of the frustum.

11 A lune whose angle is 40° is equivalent to a zone on the same sphere. Find the ratio of the altitude of the zone to the radius of the sphere.

12 Half of a regular hexagon inscribed (as shown in the drawing) in a semicircle whose radius is 12, is revolved about the diameter of the semicircle as an axis. Find the surface and volume generated by the semipolygon.