

**GEOMETRY**

Wednesday, June 11, 2025 — 9:15 a.m. to 12:15 p.m., only

Student Name: \_\_\_\_\_

Mr. Sibol ☺

School Name: \_\_\_\_\_

JMAP

**The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.**

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III, and IV** directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

**Notice ...**

**A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.**

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

## Part I

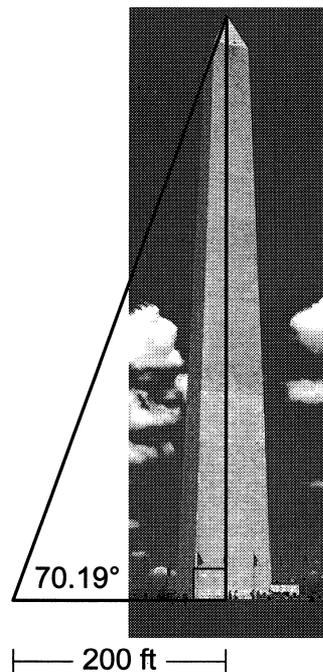
Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for  
computations.

- 1 The perimeter of a triangle is 18. What is the perimeter of a similar triangle after a dilation with a scale factor of 3?

(1) 6  
(2) 18  
(3) 54  
(4) 162

- 2 The Washington Monument, shown below, is in Washington, D.C. At a point on the ground 200 feet from the center of the base of the monument, the angle of elevation to the top of the monument is  $70.19^\circ$ .



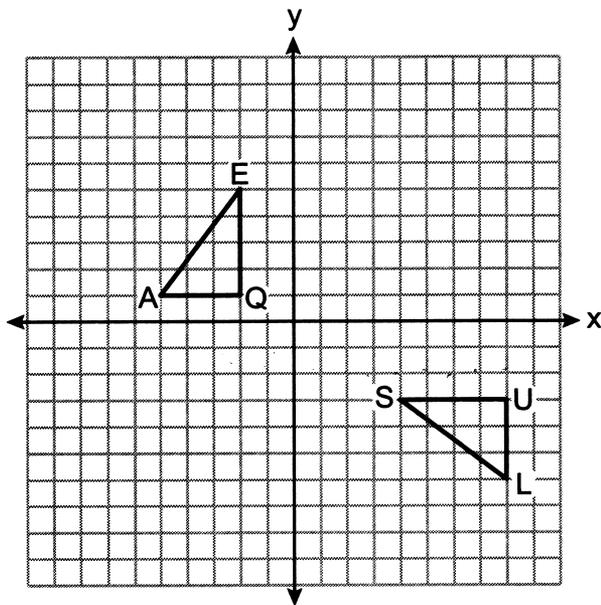
$$\tan 70.19 = \frac{h}{200}$$
$$h \approx 555$$

What is the height of the monument, to the nearest foot?

(1) 188  
(2) 213  
(3) 555  
(4) 590

Use this space for  
computations.

3 On the set of axes below,  $\triangle EQA$  and  $\triangle SUL$  are graphed.



Which sequence of transformations shows that  $\triangle EQA \cong \triangle SUL$ ?

- (1) Rotate  $\triangle EQA$   $90^\circ$  counterclockwise about the origin and then translate 9 units right and 1 unit down.
- (2) Rotate  $\triangle EQA$   $90^\circ$  counterclockwise about the origin and then reflect over the line  $x = 4$ .
- (3) Reflect  $\triangle EQA$  over the  $x$ -axis and then rotate  $90^\circ$  clockwise about the origin.
- (4) Translate  $\triangle EQA$  10 units right and then reflect over the line  $x = -1$ .

4 If two sides of a triangle have lengths of 2 and 8, the length of the third side could be

- (1) 10
- (2) 7
- (3) 6
- (4) 4

$$2 + 7 > 8$$

5 A regular octagon is rotated about its center. Which angle measure will carry the octagon onto itself?

Use this space for computations.

- (1)  $36^\circ$   
 (2)  $90^\circ$

- (3)  $144^\circ$   
 (4)  $160^\circ$

$$\frac{360}{8} = 45$$

90 is a multiple of 45

6 An equation of a circle is  $x^2 + y^2 - 6x + 2y = 14$ . What are the coordinates of the center and the length of the radius of this circle?

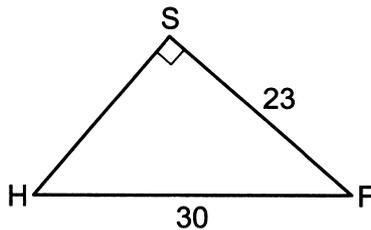
- (1)  $(-3, 1)$  and  $r = 5$   
 (2)  $(3, -1)$  and  $r = 5$

- (3)  $(-3, 1)$  and  $r = \sqrt{24}$   
 (4)  $(3, -1)$  and  $r = \sqrt{24}$

$$x^2 - 6x + 9 + y^2 + 2y + 1 = 14 + 10$$

$$(x - 3)^2 + (y + 1)^2 = 24$$

7 In  $\triangle HSF$  below,  $m\angle S = 90^\circ$ ,  $HF = 30$ , and  $FS = 23$ .



$$\cos F = \frac{23}{30}$$

$$F \approx 40$$

What is  $m\angle F$ , to the nearest degree?

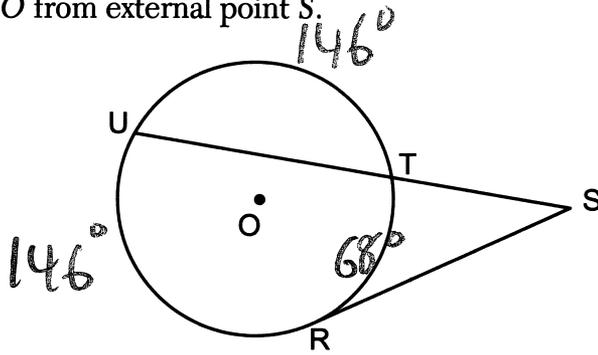
- (1)  $53^\circ$   
 (2)  $50^\circ$

- (3)  $40^\circ$   
 (4)  $37^\circ$



Use this space for computations.

10 In the diagram below, tangent  $\overline{SR}$  and secant  $\overline{STU}$  are drawn to circle  $O$  from external point  $S$ .



$$\frac{146 - 68}{2} = 39$$

If  $\widehat{TU} \cong \widehat{RU}$  and  $m\widehat{TR} = 68^\circ$ , what is  $m\angle S$ ?

- (1)  $22^\circ$                       (3)  $39^\circ$   
(2)  $34^\circ$                       (4)  $78^\circ$

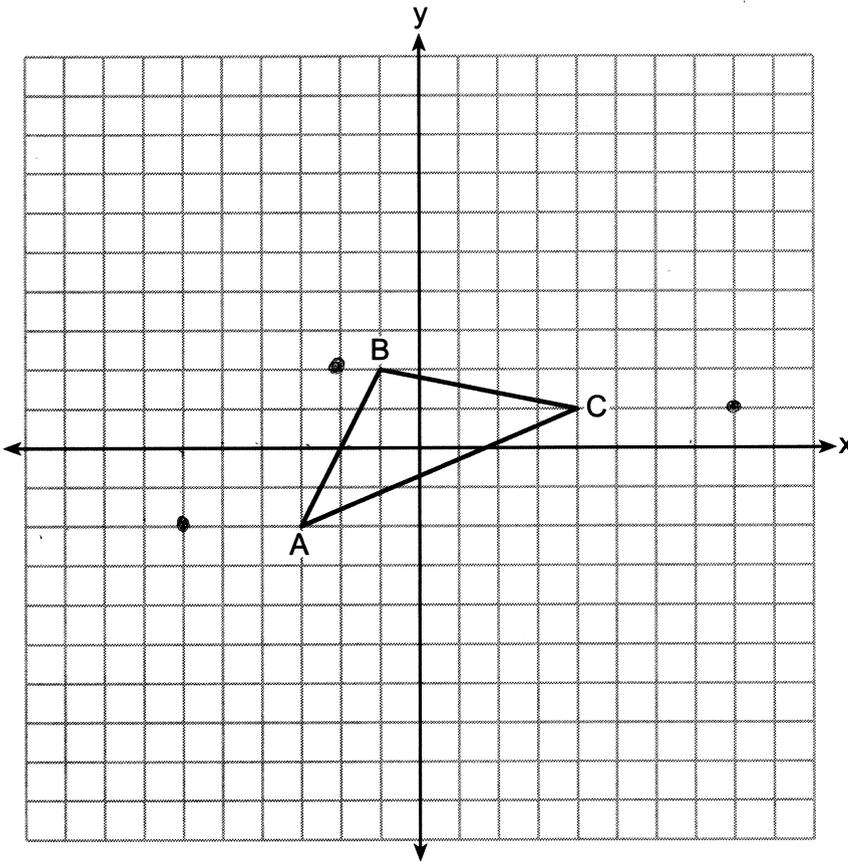
11 Triangle  $RST$  has  $m\angle S = 33^\circ$ ,  $RS = 7$ , and  $ST = 12$ . What is the area of  $\triangle RST$ , to the nearest tenth?

- (1) 22.9                      (3) 35.2  
(2) 27.3                      (4) 45.7

$$\frac{1}{2} (7)(12) \sin 33 \approx 22.9$$

Use this space for computations.

- 12 Triangle  $ABC$ , with vertices whose coordinates are  $A(-3,-2)$ ,  $B(-1,2)$ , and  $C(4,1)$ , is graphed on the set of axes below.



Triangle  $A'B'C'$ , whose vertices have coordinates  $A'(-6,-2)$ ,  $B'(-2,2)$ , and  $C'(8,1)$ , is the image of  $\triangle ABC$ . The transformation that maps  $\triangle ABC$  onto  $\triangle A'B'C'$  is a

- (1) dilation  
(2) translation  
(3) vertical stretch  
(4) horizontal stretch

- $m=4$   
13 Which equation represents a line parallel to the line represented by  $y = 4x + 6$  and passing through the point  $(-3,2)$ ?

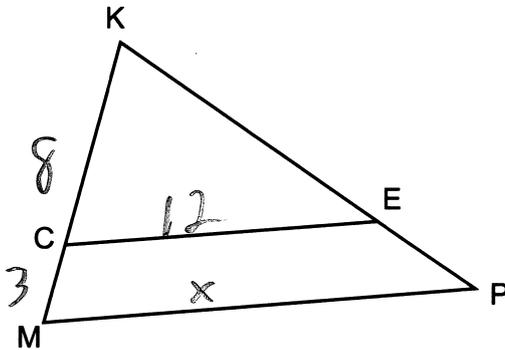
- (1)  $y - 2 = 4(x + 3)$   
(2)  $y + 3 = 4(x - 2)$   
(3)  $y - 2 = -\frac{1}{4}(x + 3)$   
(4)  $y + 3 = -\frac{1}{4}(x - 2)$

Use this space for computations.

14 Which two-dimensional figure is always formed when a plane intersects a right cylinder perpendicular to its base?

- (1) circle  
(2) triangle  
(3) rhombus  
(4) rectangle

15 In  $\triangle KMP$  below,  $\overline{CE}$  is drawn parallel to  $\overline{MP}$ .



$$\frac{8}{12} = \frac{11}{x}$$
$$\frac{8x}{8} = \frac{132}{8}$$

If  $KC = 8$ ,  $CM = 3$ , and  $CE = 12$ , what is the length of  $\overline{MP}$ ?

$$x = 16.5$$

- (1) 24  
(2) 16.5  
(3) 15  
(4) 4.5

16 A parallelogram must be a rectangle if its diagonals

- (1) are perpendicular  
(2) bisect each other  
(3) bisect its angles  
(4) are congruent

Use this space for computations.

17 Point  $O$  divides  $\overline{COA}$  such that  $CO:OA = 1:4$ .

If  $C$  has coordinates  $(-2, -9)$  and  $A$  has coordinates  $(3, 6)$ , the coordinates of  $O$  are

(1)  $(2, 3)$

(3)  $(0, -3)$

(2)  $(1, 0)$

(4)  $(-1, -6)$

$$-2 + \frac{1}{5}(3 - (-2)) = -1$$

$$-9 + \frac{1}{5}(6 - (-9)) = -6$$

18 A spherical balloon is fully inflated with helium to a diameter of 1.7 feet. If helium costs \$0.80 per cubic foot, what is the cost to completely fill the balloon with helium?

(1) \$2.06

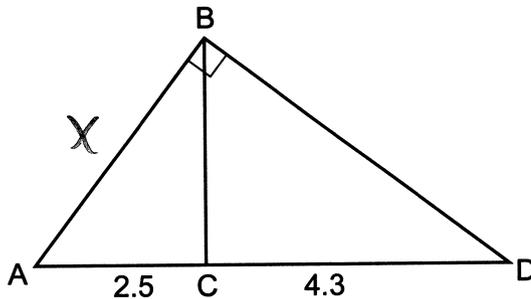
(3) \$3.22

(2) \$2.42

(4) \$16.46

$$\frac{4}{3} \pi \left(\frac{1.7}{2}\right)^3 \cdot 0.8 \approx 2.06$$

19 In right triangle  $ABD$  below, altitude  $\overline{BC}$  is drawn to hypotenuse  $\overline{AD}$ ,  $AC = 2.5$ , and  $CD = 4.3$ .



$$x^2 = 2.5 \cdot 6.8$$

$$x \approx 4.1$$

What is the length of  $\overline{BA}$ , to the nearest tenth?

(1) 3.3

(3) 4.1

(2) 3.4

(4) 5.4

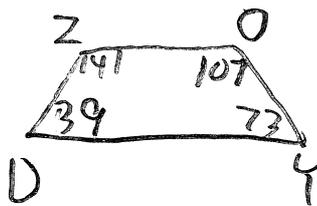
20 Trapezoid  $ZOYD$  has parallel sides  $\overline{ZO}$  and  $\overline{DY}$ . If  $m\angle Z = 141^\circ$  and  $m\angle Y = 73^\circ$ , what is  $m\angle D$ ?

(1)  $39^\circ$

(2)  $73^\circ$

(3)  $107^\circ$

(4)  $141^\circ$



Use this space for computations.

21 Triangle  $ABC$  is translated 5 units to the left and 2 units up to map onto  $\triangle PQR$ . Which statement is *not* always true?

(1)  $\triangle PQR \cong \triangle ABC$

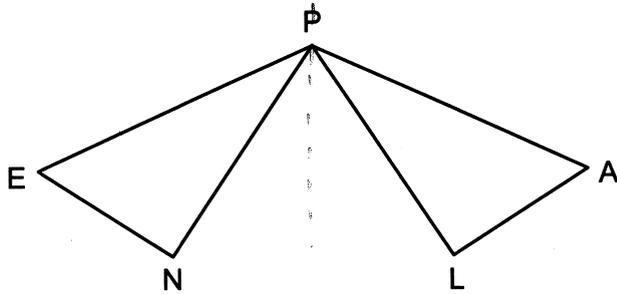
(3)  $BQ = \sqrt{29}$

(2)  $\angle A \cong \angle Q$

(4)  $RQ = CB$

Use this space for computations.

22 In the diagram below, congruent triangles  $PEN$  and  $PAL$  are drawn.



Which rigid motion maps  $\triangle PEN$  onto  $\triangle PAL$ ?

- (1) a point reflection of  $\triangle PEN$  through  $P$
- (2) a reflection of  $\triangle PEN$  over the angle bisector of  $\angle EPA$
- (3) a rotation of  $\triangle PEN$  about point  $P$ , mapping  $\overline{PE}$  onto  $\overline{PA}$
- (4) a translation of  $\triangle PEN$  along  $\overline{EA}$ , mapping point  $E$  onto  $A$

23 A cone has a height of 8 inches and volume of 75.4 cubic inches.

What is the diameter of the cone, to the nearest inch?

- (1) 9
- (2) 2
- (3) 3
- (4) 6

$$75.4 = \frac{1}{3} \pi r^2 \cdot 8$$
$$r \approx 3$$

$$5(2) - 2(0) = 10$$

24 The line represented by the equation  $5x - 2y = 10$  is transformed by a dilation centered at  $(2,0)$  with a scale factor of 2. The image of the line

- (1) is the original line
- (2) passes through the point  $(4,0)$
- (3) passes through the point  $(0,-10)$
- (4) is perpendicular to the original line

## Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

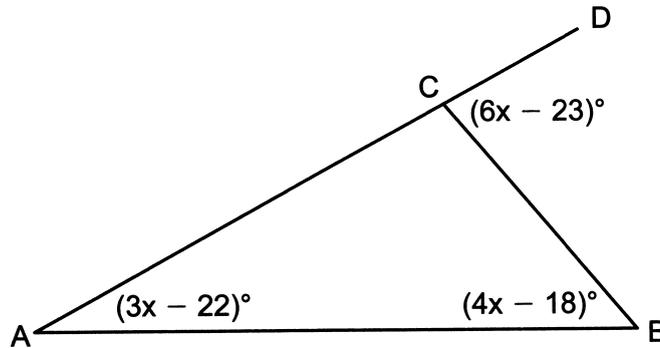
- 25 In the year 2020, the village of Depew, New York had an area of 5.1 square miles and a population of 15,069. In the same year, the village of Lancaster, New York had an area of 2.7 square miles and a population of 10,087.

Which village had the larger population density in 2020? Justify your answer.

Lancaster

$$\frac{10087}{2.7} > \frac{15069}{5.1}$$

- 26 In  $\triangle ABC$  below,  $\overline{AC}$  is extended through  $C$  to  $D$ ,  $m\angle A = (3x - 22)^\circ$ ,  $m\angle B = (4x - 18)^\circ$ , and  $m\angle BCD = (6x - 23)^\circ$ .

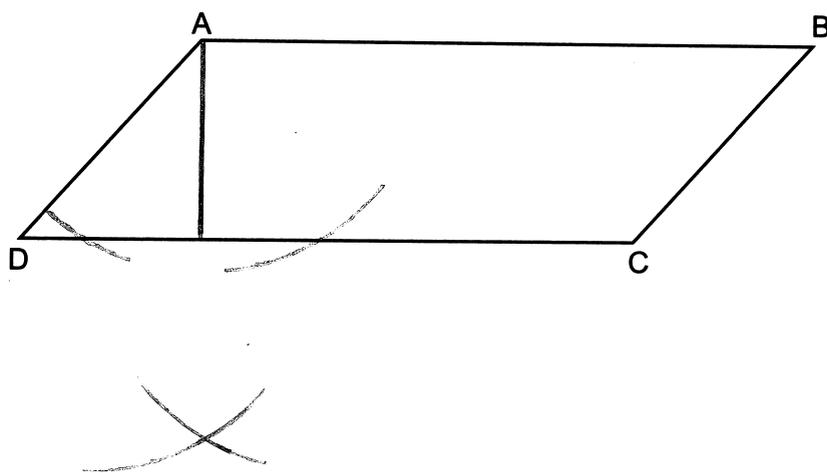


Determine and state  $m\angle ACB$ .

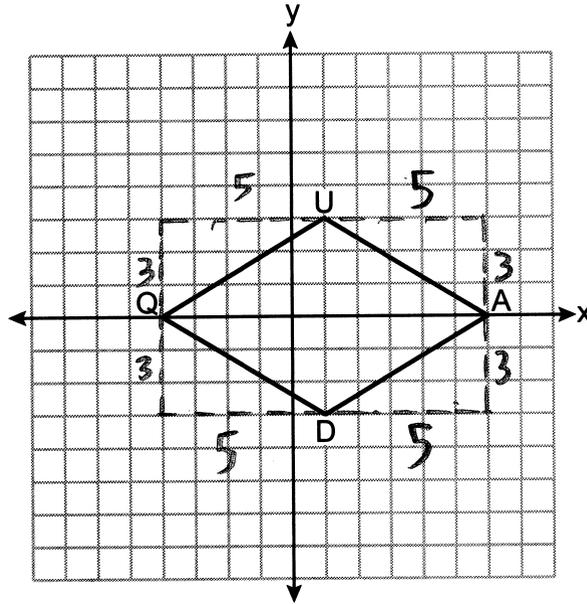
$$\begin{aligned} 3x - 22 + 4x - 18 &= 6x - 23 \\ 7x - 40 &= 6x - 23 \\ x &= 17 \end{aligned}$$

$$\begin{aligned} &180 - (6(17) - 23) \\ &180 - (102 - 23) \\ &180 - 79 \\ &101^\circ \end{aligned}$$

27 Parallelogram  $ABCD$  is shown below. Using a compass and straightedge, construct the altitude from point  $A$  to side  $\overline{DC}$ . [Leave all construction marks.]



28 Quadrilateral  $QUAD$  is graphed on the set of axes below.



Determine and state the area of quadrilateral  $QUAD$ .

$$\begin{aligned} & (10 \times 6) - 4\left(\frac{1}{2}(5)(3)\right) \\ & 60 - 30 \\ & 30 \end{aligned}$$

29 In a right triangle, the acute angles have the relationship  $\sin(3x - 7)^\circ = \cos(x + 1)^\circ$ .

Determine and state the value of  $x$ .

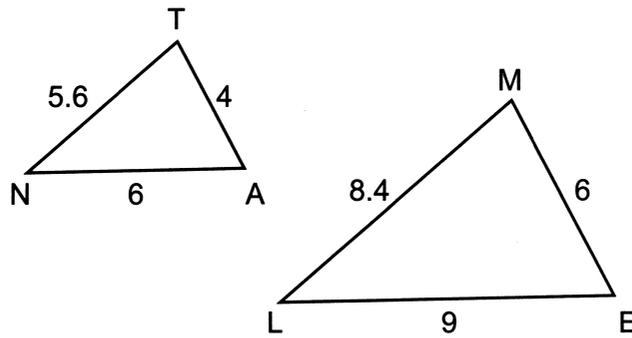
$$3x - 7 + x + 1 = 90$$

$$4x - 6 = 90$$

$$4x = 96$$

$$x = 24$$

31 In triangles  $ANT$  and  $ELM$  below,  $AN = 6$ ,  $NT = 5.6$ ,  $TA = 4$ ,  $EL = 9$ ,  $LM = 8.4$ , and  $ME = 6$ .



Explain why  $\triangle ANT \sim \triangle ELM$ .

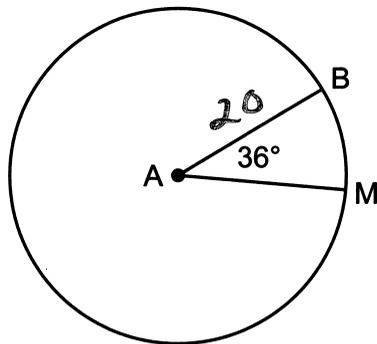
$$\frac{5.6}{6} = \frac{8.4}{9} \rightarrow 50.4 = 50.4$$

$$\frac{4}{6} = \frac{6}{9} \rightarrow 36 = 36$$

$$\frac{4}{5.6} = \frac{6}{8.4} \rightarrow 33.6 = 33.6$$

The corresponding sides are proportional.

30 In circle A below,  $m\angle BAM = 36^\circ$ .



If  $AB = 20$ , determine and state the length of  $\widehat{MB}$ .  
[Leave your answer in terms of  $\pi$ .]

$$\frac{36}{360} (\pi)(40) = 4\pi$$

### Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

**32** A store sells colored craft sand in the three different containers below.

Container 1: A square prism with a base length of 4 inches and a height of 7.5 inches.

Container 2: A cylinder with a diameter of 5 inches and a height of 6 inches.

Container 3: A cone with a diameter of 7.5 inches and a height of 8.5 inches.

If the containers are filled to the top, which container will hold the most sand?

Justify your answer.

$$1) 4 \times 4 \times 7.5 = 120$$

$$2) \pi (2.5)^2 (6) \approx 118$$

$$3) \frac{1}{3} \pi (3.75)^2 (8.5) \approx 125$$

The cone

33 Quadrilateral *MIKE* has vertices with coordinates  $M(-1,-3)$ ,  $I(-3,3)$ ,  $K(5,4)$ , and  $E(7,-2)$ .

Prove *MIKE* is a parallelogram, and prove *MIKE* is *not* a rhombus.

[The use of the set of axes below is optional.]

$$m_{MI} = \frac{-3-3}{-1-3} = \frac{-6}{-2} = 3$$

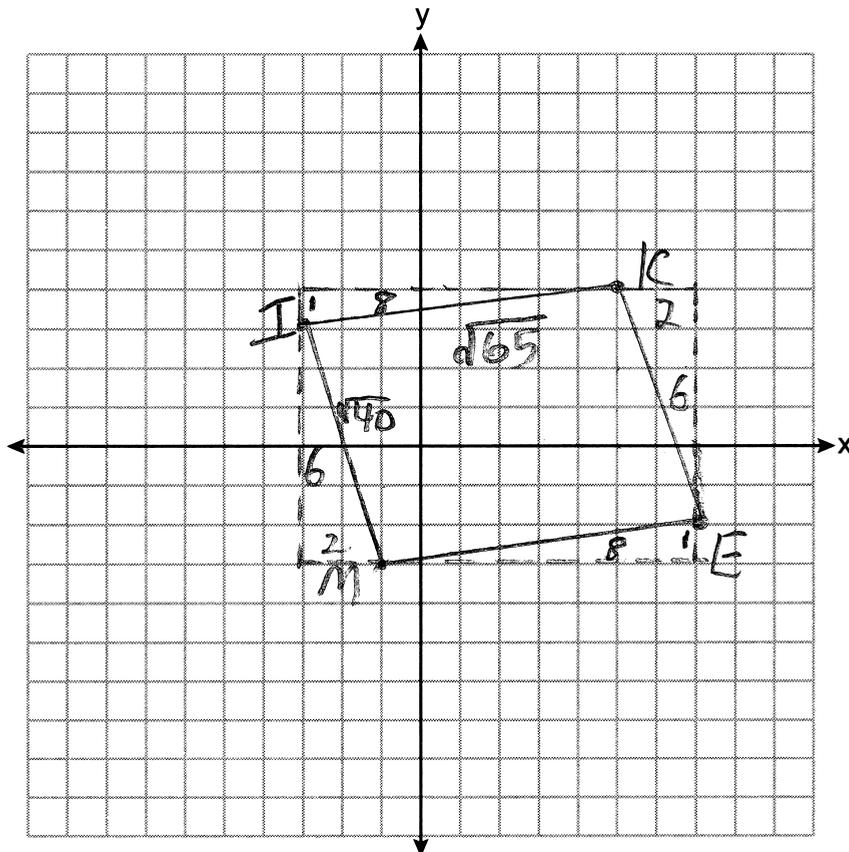
$$m_{KE} = \frac{4-2}{5-7} = \frac{2}{-2} = -1$$

$$m_{IK} = \frac{4-3}{5-3} = \frac{1}{2}$$

$$m_{ME} = \frac{-2-3}{7-1} = \frac{-5}{6}$$

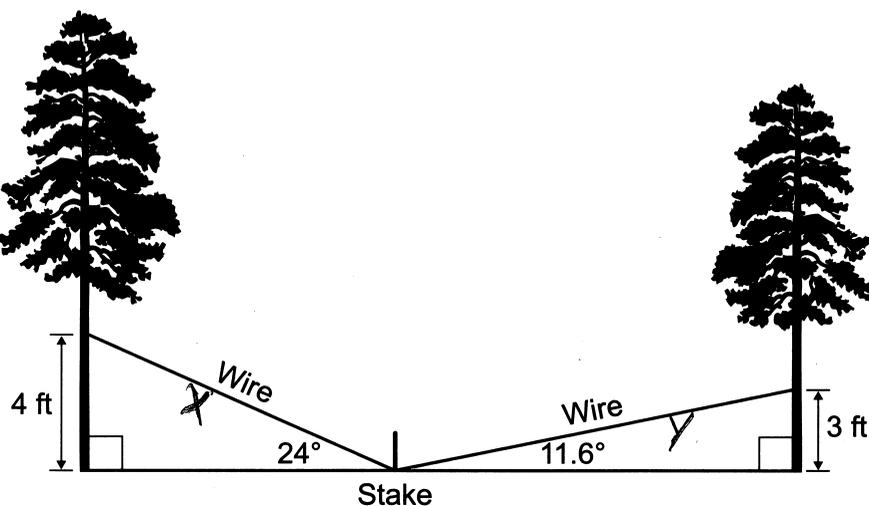
Because opposite sides of Quadrilateral *MIKE* have the same slope, the sides are parallel. The opposite sides of a parallelogram are parallel, so *MIKE* is a parallelogram.

$MI = \sqrt{6^2 + 2^2} = \sqrt{40}$   
 $IK = \sqrt{8^2 + 1^2} = \sqrt{65}$



Because two sides of Quadrilateral *MIKE* have different lengths, they are not congruent. All the sides of rhombus are congruent, so *MIKE* is not a rhombus.

34 A park ranger needs to secure two different trees with wire. A wire is to be attached from a stake in the ground to each tree. The wire is attached at two different heights and two different angles of elevation, as indicated in the model below.



The park ranger has 20 feet of wire. Does the park ranger have enough wire to secure both trees? Justify your answer.

$$\sin 24 = \frac{4}{x}$$

$$x \approx 9.8$$

$$\sin 11.6 = \frac{3}{y}$$

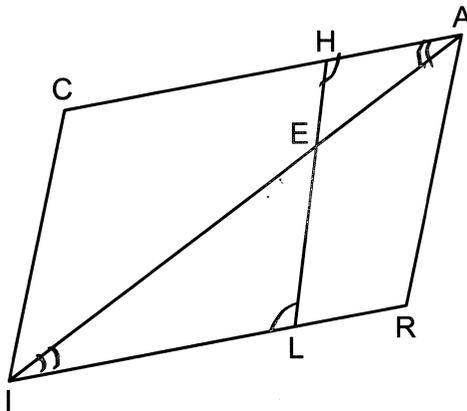
$$y \approx 14.9$$

No.  $9.8 + 14.9 > 20$

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

35 Given: Quadrilateral  $CARI$  with  $\overline{CA} \cong \overline{RI}$  and  $\overline{CI} \cong \overline{RA}$ , and  $\overline{AEI}$  and  $\overline{LEH}$  are drawn



Prove:  $HA \cdot EL = LI \cdot EH$

Statement	Reason
① Quad $CARI$ with $\overline{CA} \cong \overline{RI}$ & $\overline{CI} \cong \overline{RA}$ & $\overline{AEI}$ & $\overline{LEH}$ are drawn	① Given
② Quad $CARI$ is a parallelogram	② Opposite sides of a parallelogram are congruent
③ $\overline{CA} \parallel \overline{RI}$ & $\overline{CI} \parallel \overline{RA}$ are $\parallel$	③ Opposite sides of a parallelogram are parallel
④ $\angle HAE \cong \angle LIE$ & $\angle HAE \cong \angle LIE$	④ Parallel line cut by a transversal create congruent alternate interior angles
⑤ $\triangle HAE \sim \triangle LIE$	⑤ AA
⑥ $\frac{HA}{EH} = \frac{LI}{EL}$	⑥ Corresponding sides of similar triangles are proportional
⑦ $HA \cdot EL = LI \cdot EH$	⑦ The product of the means equals the product of the extremes.