The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Tuesday, August 20, 2024 — 12:30 to 3:30 p.m., only

Student Name: Mr. Sibol

School Name: JMAP

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice ...
A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

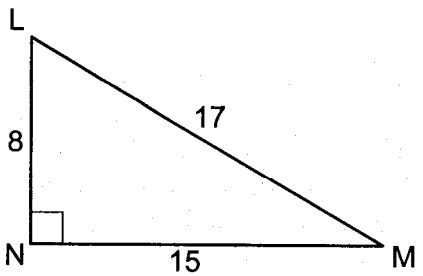
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 In right triangle LMN below, $LN = 8$, $MN = 15$, and $LM = 17$.



If triangle LMN is translated such that it maps onto triangle XYZ, which statement is always true?

(1) $XY = 15$
(2) $YZ = 17$
(3) $m\angle Z = 90°$
(4) $m\angle X = 90°$

2 Directed line segment KC has endpoints $K(-4,-2)$ and $C(1,8)$. Point E divides $\overline{KC}$ such that $KE{:}EC$ is 3:2. What are the coordinates of point E?

(1) $(-1,4)$
(2) $(-2,2)$
(3) $(-3,0)$
(4) $(0,6)$

$-4+\frac{3}{5}(1--4) = -4+3 = -1$

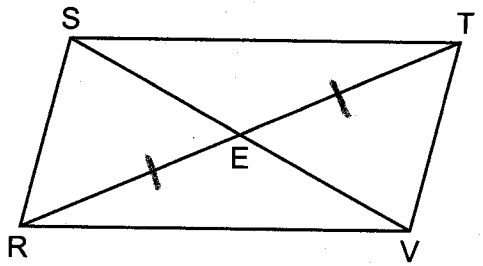
$-2+\frac{3}{5}(8--2) = -2+6 = 4$

3 In right triangle DAN, $m\angle A = 90°$. Which statement must always be true?

(1) $\cos D = \cos N$
(2) $\cos D = \sin N$
(3) $\sin A = \cos N$
(4) $\cos A = \tan N$

Use this space for computations.

4 In the diagram below of parallelogram $RSTV$, diagonals $\overline{SV}$ and $\overline{RT}$ intersect at E.



Which statement is always true?

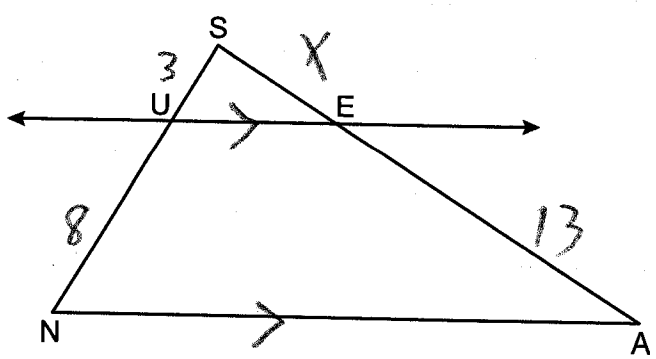
(1) $\overline{SR} \cong \overline{RV}$

(2) $\overline{RT} \cong \overline{SV}$

(3) $\overline{SE} \cong \overline{RE}$

(4) $\overline{RE} \cong \overline{TE}$

5 In $\triangle SNA$ below, $\overline{UE} \parallel \overline{NA}$.



If $SU = 3$, $SN = 11$, and $EA = 13$, what is the length of $\overline{SE}$, to the *nearest tenth*?

(1) 2.5

(2) 3.5

(3) 4.9

(4) 17.9

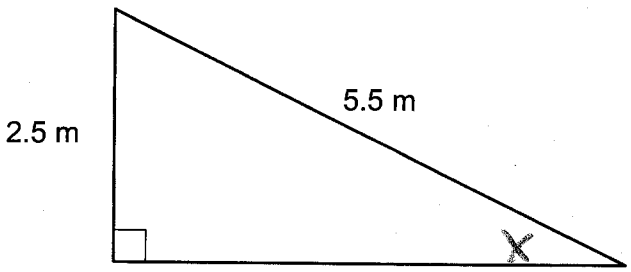
$\frac{x}{13} = \frac{3}{8}$

$\frac{8x}{8} = \frac{39}{8}$

$x \approx 4.9$

Use this space for computations.

6 Many roofs are slanted to prevent the buildup of snow. As modeled below, the length of a roof is 5.5 meters and it rises to a height of 2.5 meters.



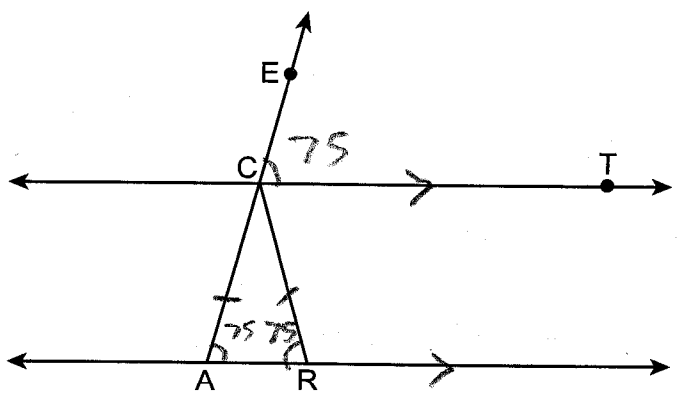
$\sin x = \frac{2.5}{5.5}$

$x \approx 27$

The angle of elevation of the roof, to the *nearest degree*, is

(1) 24°
(2) 25°
(3) 27°
(4) 28°

7 In the diagram below, $\overleftrightarrow{CT} \parallel \overleftrightarrow{AR}$, and $\overleftrightarrow{ACE}$ and $\overline{RC}$ are drawn such that $\overline{AC} \cong \overline{RC}$.



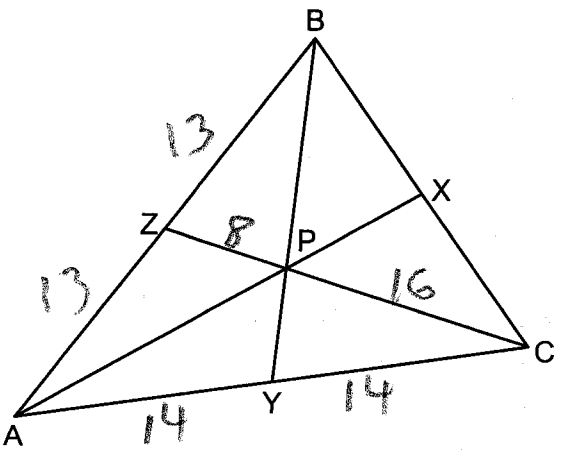
If $m\angle ECT = 75°$, what is $m\angle ACR$?

(1) 30°
(2) 60°
(3) 75°
(4) 105°

$180 - 2(75) = 30$

Use this space for computations.

8 In the diagram below, $\triangle ABC$ has medians $\overline{AX}$, $\overline{BY}$, and $\overline{CZ}$ that intersect at point P.



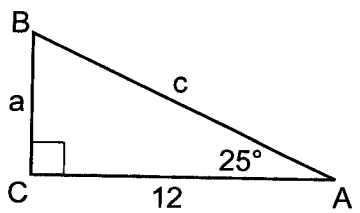
$\frac{x}{16} = \frac{1}{2}$

$x = 8$

If $AB = 26$, $AC = 28$, and $PC = 16$, what is the perimeter of $\triangle CZA$?

(1) 57
(2) 65
(3) 70
(4) 73

9 In right triangle ABC below, $m\angle C = 90°$, $AC = 12$, and $m\angle A = 25°$.



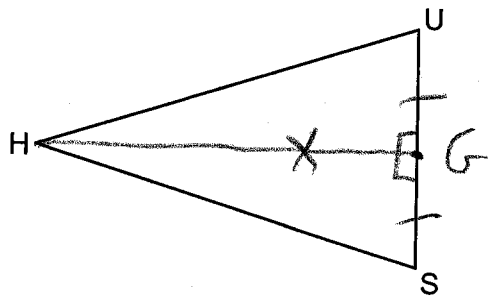
Which equation is correct for $\triangle ABC$?

(1) $a = \frac{12}{\tan 25°}$
(2) $a = 12 \tan 25°$
(3) $c = \frac{12}{\tan 25°}$
(4) $c = 12 \tan 25°$

$\tan 25 = \frac{a}{12}$

10 Triangle HUS is shown below.

Use this space for computations.



If point G is located on $\overline{US}$ and $\overline{HG}$ is drawn, which additional information is sufficient to prove $\triangle HUG \cong \triangle HSG$ by SAS?

(1) $\overline{HG}$ bisects $\overline{US}$

(2) $\overline{HG}$ is an altitude

(3) $\overline{HG}$ bisects $\angle UHS$

(4) $\overline{HG}$ is the perpendicular bisector of $\overline{US}$

11 The area of the base of a cone is 9π square inches. The volume of the cone is 36π cubic inches. What is the height of the cone in inches?

(1) 12

(2) 8

(3) 3

(4) 4

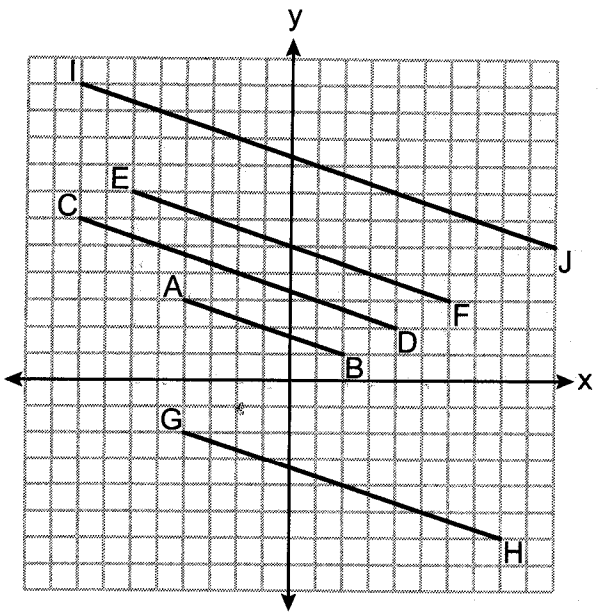
$$36\pi = \frac{9\pi h}{3}$$

$$108 = 9h$$

$$12 = h$$

12 On the set of axes below, $\overline{AB}$, $\overline{CD}$, $\overline{EF}$, $\overline{GH}$, and $\overline{IJ}$ are drawn.

Use this space for computations.



Which segment is the image of $\overline{AB}$ after a dilation with a scale factor of 2 centered at $(-2,-1)$?

(1) $\overline{CD}$

(2) $\overline{EF}$

(3) $\overline{GH}$

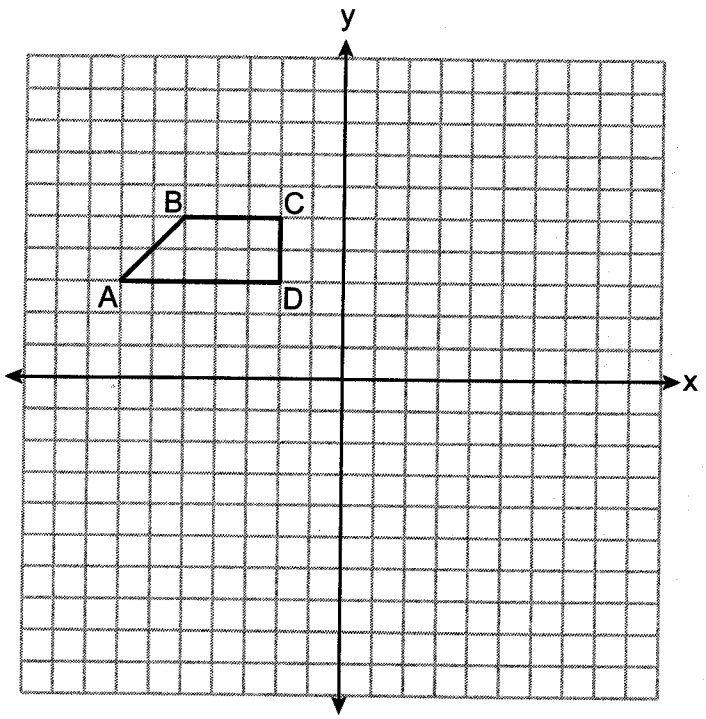
(4) $\overline{IJ}$

$A(-4,3) \rightarrow A(-2,4) \rightarrow A(-4,8) \rightarrow A(-6,7) \rightarrow E$

$B(2,1) \rightarrow B(4,2) \rightarrow A(8,4) \rightarrow A(6,3) \rightarrow F$

Use this space for computations.

13 Trapezoid $ABCD$ is graphed on the set of axes below.



Which transformation would map point A onto $A'(3,-7)$?

(1) reflection over $y = x$ (-7, 3)

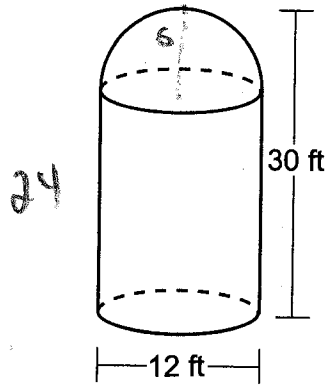
(2) reflection over the y-axis

(3) rotation of 180° about (0,0)

(4) rotation of 90° counterclockwise about (0,0)

Use this space for computations.

14 A storage building is modeled below by a hemisphere on top of a cylinder. The diameter of both the cylinder and hemisphere is 12 feet. The total height of the storage building is 30 feet.



$\pi(6)^2 \cdot 24 + \frac{4\pi(6)^3}{3 \cdot 2}$

$864\pi + 144\pi$

1008π

To the *nearest cubic foot*, what is the volume of the storage building?

(1) 942
(2) 2488
(3) 3167
(4) 3845

15 Which regular polygon will carry onto itself after a 135° rotation about its center?

$\frac{180(n-2)}{n}$

(1) triangle 60
(2) pentagon 108
(3) hexagon 120
(4) octagon 135

16 What is the length of the radius of the circle whose equation is $x^2 + y^2 - 2x + 4y - 5 = 0$?

(1) $\sqrt{5}$
(2) $\sqrt{10}$
(3) 5
(4) 10

$x^2 - 2x + 1 + y^2 + 4y + 4 = 5 + 1 + 4$

$(x-1)^2 + (y+2)^2 = 10$

Use this space for computations.

17 The line represented by the equation $y = 4x + 15$ is dilated by a scale factor of 2 centered at the origin. Which equation represents its image?

(1) $y = 4x + 15$
(2) $y = 4x + 30$
(3) $y = 8x + 15$
(4) $y = 8x + 30$

$15 \cdot 2 = 30$

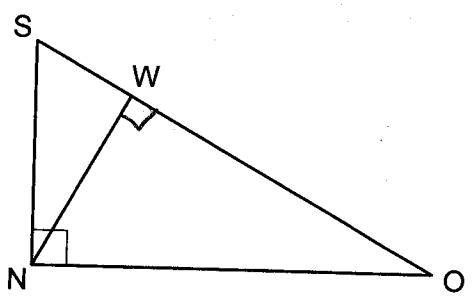
18 Line segment RH has endpoints $R(-4,4)$ and $H(2,-4)$. Which equation represents a line perpendicular to $\overline{RH}$ that passes through the point $(3,-1)$?

(1) $y + 1 = \frac{3}{4}(x - 3)$
(2) $y + 1 = -\frac{3}{4}(x - 3)$
(3) $y + 1 = \frac{4}{3}(x - 3)$
(4) $y + 1 = -\frac{4}{3}(x - 3)$

$\frac{4 - -4}{-4 - 2} = \frac{8}{-6} = -\frac{4}{3}$

$m_{\perp} = \frac{3}{4}$

19 In right triangle SNO below, altitude $\overline{NW}$ is drawn to hypotenuse $\overline{SO}$.



Which statement is *not* always true?

(1) $\frac{SO}{SN} = \frac{SN}{SW}$
(2) $\frac{SW}{NS} = \frac{NS}{OW}$
(3) $\frac{SO}{ON} = \frac{ON}{OW}$
(4) $\frac{OW}{NW} = \frac{NW}{SW}$

S

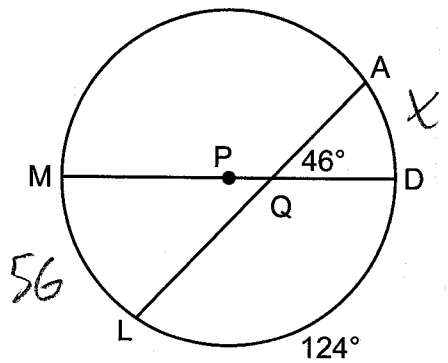
20 A rectangle has a width of 3 and a length of 4. The rectangle is dilated by a scale factor of 1.8. What is the area of its image, to the *nearest tenth*?

(1) 3.7
(2) 6.7
(3) 21.6
(4) 38.9

$(3)(4)(1.8)^2$

Use this space for computations.

21 In the diagram below of circle P, diameter $\overline{MD}$ and chord $\overline{AL}$ intersect at Q, $m\angle AQD = 46°$, and $m\overset{\frown}{LD} = 124°$.



$\frac{56+x}{2} = 46$

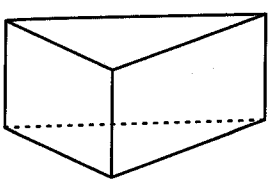
$x + 56 = 92$

$x = 36$

What is $m\overset{\frown}{AD}$?

(1) 36°
(2) 46°
(3) 51°
(4) 92°

22 The right prism with a triangular base shown below is cut by a plane perpendicular to its bases.

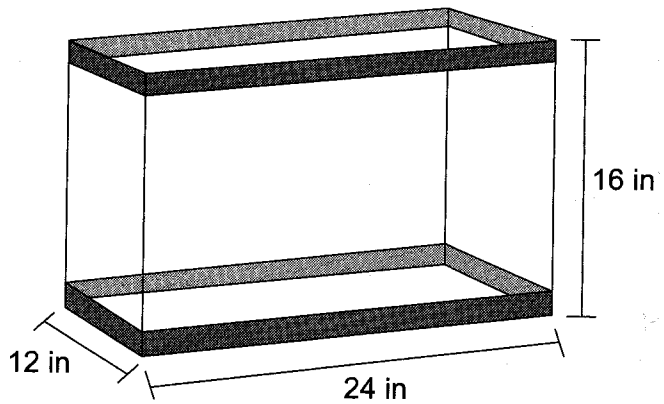


The two-dimensional shape of the cross section is always a

(1) triangle
(2) rhombus
(3) pentagon
(4) rectangle

Use this space for computations.

23 A rectangular fish tank measures 24 inches long, 12 inches wide, and 16 inches high, as modeled in the diagram below.



If the empty tank weighs 25 pounds and the fish tank is filled with water to a height of 14 inches, what is the approximate weight of the tank and water?

[27.7 $in.^3$ = 1 pound of water]

(1) 146
(2) 166
(3) 171
(4) 191

$25 + \frac{12 \cdot 24 \cdot 14}{27.7} \approx 171$

24 A circle has a radius of 4.5. What is the measure of the central angle that intercepts an arc whose length is 6.2, to the *nearest degree*?

(1) 35°
(2) 42°
(3) 64°
(4) 79°

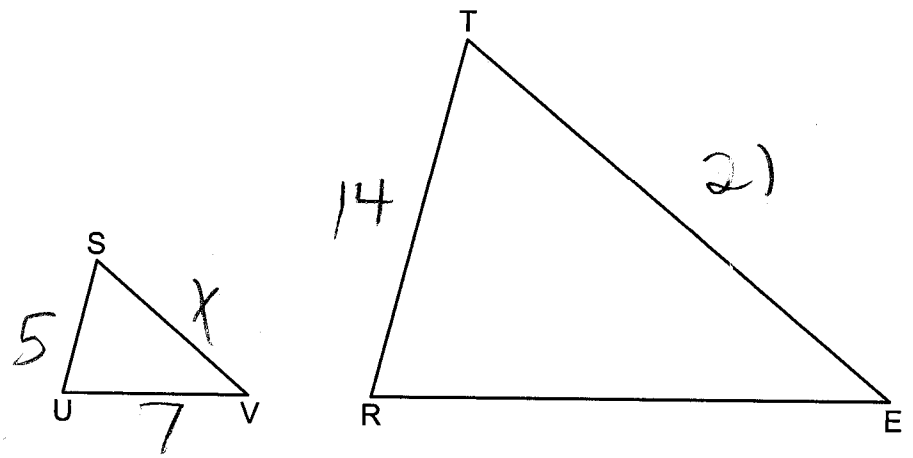
$\frac{x}{360} = \frac{6.2}{9\pi}$

$x \approx 79$

Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

25 In the diagram below, $\triangle SUV \sim \triangle TRE$.



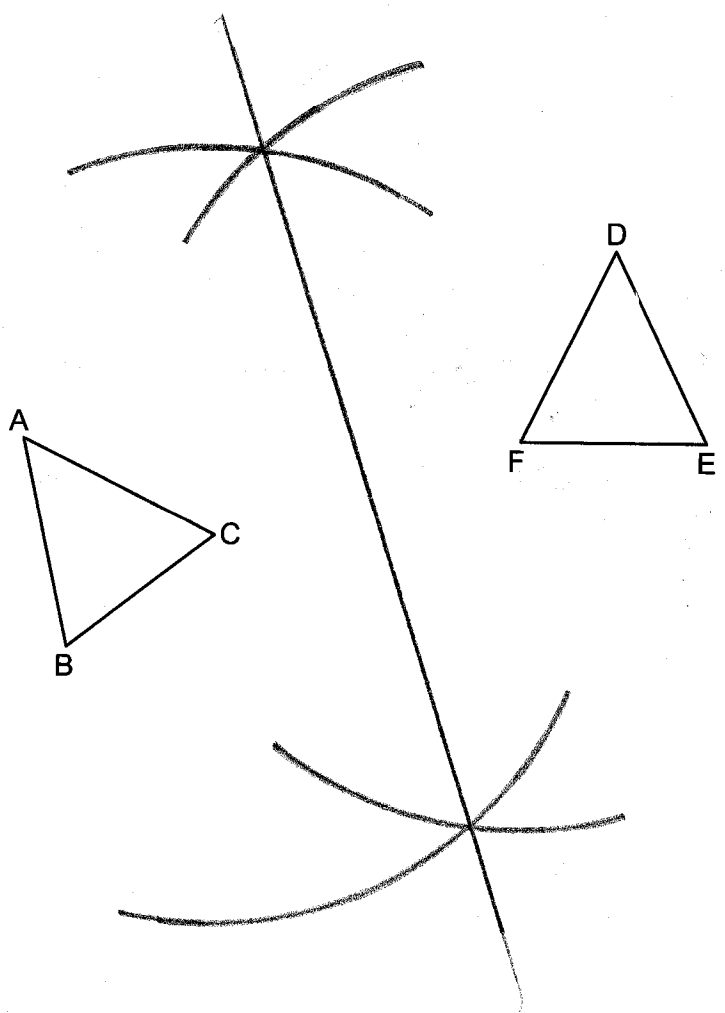
If $SU = 5$, $UV = 7$, $TR = 14$, and $TE = 21$, determine and state the length of $\overline{SV}$.

$$\frac{5}{x} = \frac{14}{21}$$

$$\frac{14x}{14} = \frac{105}{14}$$

$$x = 7.5$$

26 Using a compass and straightedge, construct the line of reflection that maps $\triangle ABC$ onto its image, $\triangle DEF$. [Leave all construction marks.]

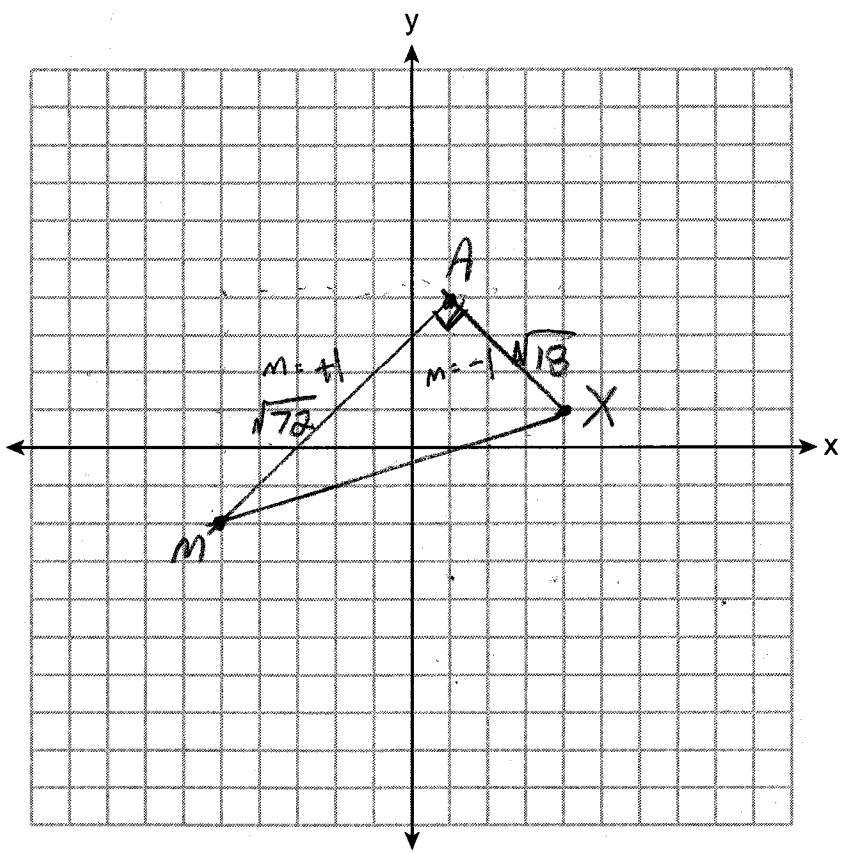


27 Triangle MAX has vertices with coordinates $M(-5,-2)$, $A(1,4)$, and $X(4,1)$.

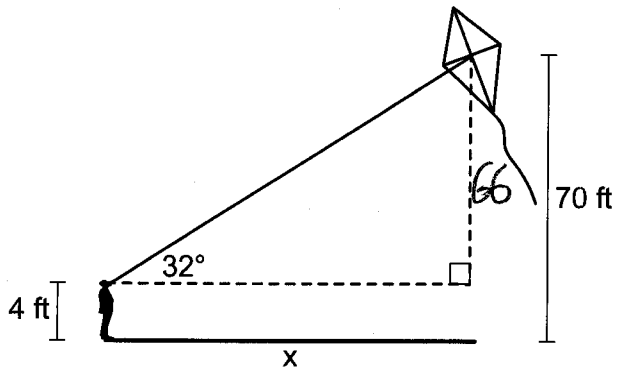
Determine and state the area of $\triangle MAX$.

[The use of the set of axes below is optional.]

$A = \frac{1}{2}\sqrt{18}\sqrt{72} = \frac{1}{2}\sqrt{9}\sqrt{2}\sqrt{9}\sqrt{8} = 18$



28 A person observes a kite at an angle of elevation of 32° from a line of sight that begins 4 feet above the ground, as modeled in the diagram below. At the moment of observation, the kite is 70 feet above the ground.

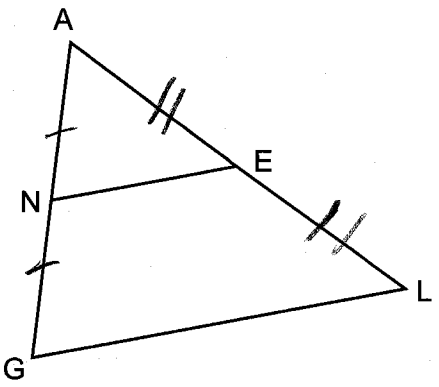


Determine and state the horizontal distance, x, between the person and the point on the ground directly below the kite, to the *nearest foot*.

$\tan 32° = \frac{66}{x}$

$x \approx 106$

29 In $\triangle AGL$ below, N and E are the midpoints of $\overline{AG}$ and $\overline{AL}$, respectively, $\overline{NE}$ is drawn.



If $NE = 15$ and $GL = 3x - 12$, determine and state the value of x.

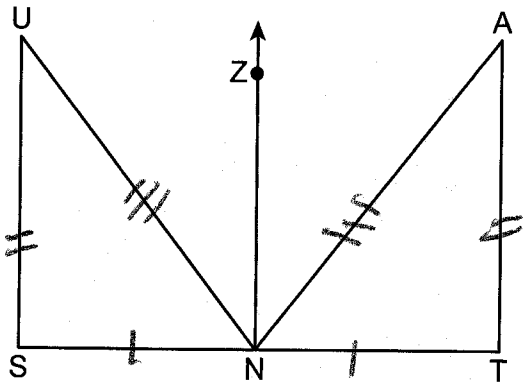
$2(15) = 3x - 12$

$30 = 3x - 12$

$42 = 3x$

$14 = x$

30 In the diagram below, $\triangle TAN$ is the image of $\triangle SUN$ after a reflection over $\overline{NZ}$.



Use the properties of rigid motions to explain why $\triangle TAN \cong \triangle SUN$.

A reflection preserves distance, so the corresponding sides are congruent.

31 A pyramid with a square base is made of solid glass. The pyramid has a base with a side length of 5.7 cm and a height of 7 cm. The density of the glass is 2.4 grams per cubic centimeter.

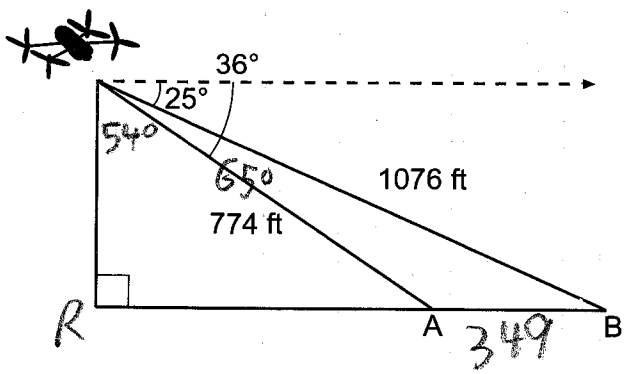
Determine and state, to the *nearest gram*, the mass of the pyramid.

$\frac{1}{3}(5.7)^2(7) \cdot 2.4 \approx 182$

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

32 A drone is used to measure the size of a brush fire on the ground. Segment AB represents the width of the fire, as shown below. The drone calculates the distance to point B to be 1076 feet at an angle of depression of 25°. At the same point, the drone calculates the distance to point A to be 774 feet at an angle of depression of 36°.



Determine and state the width of the fire, $\overline{AB}$, to the *nearest foot*.

$\sin 65 = \frac{RB}{1076}$

$RB \approx 975.2$

$\sin 54 = \frac{RA}{774}$

$RA \approx \frac{626.2}{349}$

33 Quadrilateral $ABCD$ has vertices with coordinates $A(-3,6)$, $B(6,3)$, $C(6,-2)$, and $D(-6,2)$.

Joe defines an isosceles trapezoid as a trapezoid with congruent diagonals. Use Joe's definition to prove $ABCD$ is an isosceles trapezoid.

[The use of the set of axes below is optional.]

ABCD is a trapezoid because it has one pair of parallel sides

$m_{\overline{AB}} = \frac{6-3}{-3-6} = \frac{3}{-9} = -\frac{1}{3}$

$m_{\overline{CD}} = \frac{2--2}{-6-6} = \frac{4}{-12} = -\frac{1}{3}$ } parallel sides

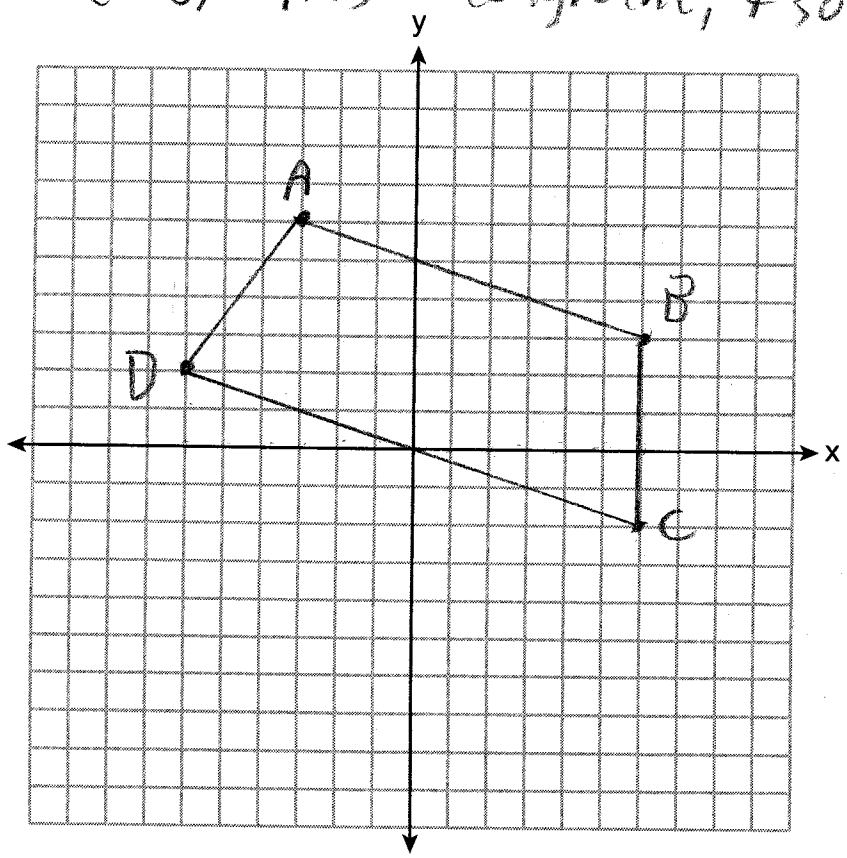
$m_{\overline{BC}} = \frac{3--2}{6-6} = \frac{5}{0}$ undefined

$m_{\overline{AD}} = \frac{6-2}{-3--6} = \frac{4}{3}$ } non-parallel sides

$\overline{BD} = \sqrt{(6--6)^2+(3-2)^2} = \sqrt{145}$

$\overline{AC} = \sqrt{(6--3)^2+(-2-6)^2} = \sqrt{145}$

ABCD's diagonals are congruent, & so isosceles.



34 Ali made six solid spherical decorations out of modeling clay. Each decoration has a radius of 2.5 inches. The weight of clay is 68 pounds per cubic foot.

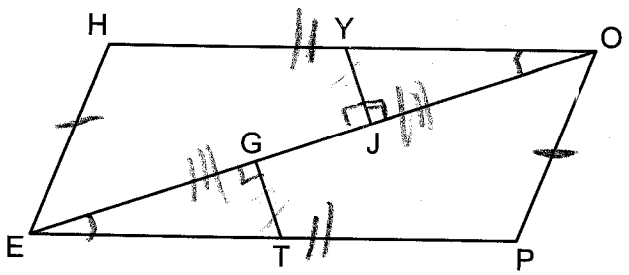
Determine and state, to the *nearest pound*, the total weight of the six decorations.

$6 \cdot \frac{4}{3}\pi\left(\frac{2.5}{12}\right)^3 \cdot 68 \approx 15$

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

35 In quadrilateral *HOPE* below, $\overline{EH} \cong \overline{OP}$, $\overline{EP} \cong \overline{OH}$, $\overline{EJ} \cong \overline{OG}$, and $\overline{TG}$ and $\overline{YJ}$ are perpendicular to diagonal $\overline{EO}$ at points G and *J*, respectively.



Prove that $\overline{TG} \cong \overline{YJ}$.

Statement	Reason
① Quad HOPE $\overline{EH} \cong \overline{OP}$, $\overline{EP} \cong \overline{OH}$, $\overline{EJ} \cong \overline{OG}$, $\overline{TJ} \perp \overline{EO}$, $\overline{YJ} \perp \overline{EO}$	① Given
② HOPE is a parallelogram	② Both pairs of opposite sides are parallel
③ $\overline{HO} \parallel \overline{PE}$	③ Opposite sides of a parallelogram are parallel
④ $\angle YOJ \cong \angle GET$	④ Parallel lines cut by a transversal form congruent alternate interior angles
⑤ $\overline{OJ} \cong \overline{OJ}$	⑤ Reflexive property
⑥ $\overline{EG} \cong \overline{OJ}$	⑥ Subtraction
⑦ $\angle EGT$ & $\angle OJY$ are right $\angle$s	⑦ Perpendicular lines form right $\angle$s
⑧ $\angle EGT \cong \angle OJY$	⑧ All right angles are congruent
⑨ $\triangle EGT \cong \triangle OJY$	⑨ ASA
⑩ $\overline{TG} \cong \overline{YJ}$	⑩ CPCTC