The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Wednesday, August 18, 2010 — 8:30 to 11:30 a.m., only

Student Name: Mr. Sibol

School Name: HS For Civil Rights

Print your name and the name of your school on the lines above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

This examination has four parts, with a total of 38 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

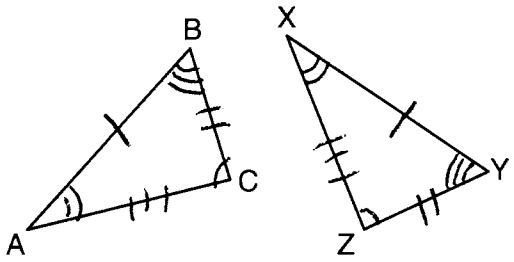
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 28 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [56]

Use this space for computations.

1 In the diagram below, $\triangle ABC \cong \triangle XYZ$.



Which two statements identify corresponding congruent parts for these triangles?

(1) $\overline{AB} \cong \overline{XY}$ and $\angle C \cong \angle Y$

(2) $\overline{AB} \cong \overline{YZ}$ and $\angle C \cong \angle X$

(3) $\overline{BC} \cong \overline{XY}$ and $\angle A \cong \angle Y$

(4) $\overline{BC} \cong \overline{YZ}$ and $\angle A \cong \angle X$

2 A support beam between the floor and ceiling of a house forms a 90° angle with the floor. The builder wants to make sure that the floor and ceiling are parallel. Which angle should the support beam form with the ceiling?

(1) 45°

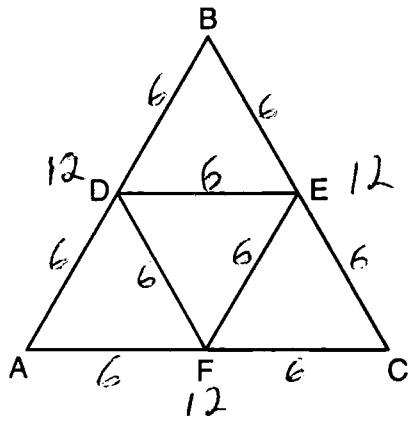
(2) 60°

(3) 90°

(4) 180°

Use this space for computations.

3 In the diagram below, the vertices of $\triangle DEF$ are the midpoints of the sides of equilateral triangle ABC, and the perimeter of $\triangle ABC$ is 36 cm.



What is the length, in centimeters, of $\overline{EF}$?

(1) 6
(2) 12
(3) 18
(4) 4

4 What is the solution of the following system of equations?

$$y = (x + 3)^2 - 4$$
$$y = 2x + 5$$

(1) (0,−4)
(2) (−4,0)
(3) (−4,−3) and (0,5)
(4) (−3,−4) and (5,0)

$(x+3)^2 - 4 = 2x + 5$

$x^2 + 6x + 9 - 4 = 2x + 5$

$x^2 + 4x = 0$

$x(x+4) = 0$

$x = 0 \quad x + 4 = 0$

$x = -4$

$y = 2x + 5$

$= 2(0) + 5$

$= 5$

$y = 2(-4) + 5$

$= -3$

Use this space for computations.

5 One step in a construction uses the endpoints of $\overline{AB}$ to create arcs with the same radii. The arcs intersect above and below the segment. What is the relationship of $\overline{AB}$ and the line connecting the points of intersection of these arcs?

(1) collinear

(2) congruent

(3) parallel

(4) perpendicular

$m\angle B = m\angle X$

6 If $\triangle ABC \sim \triangle ZXY$, $m\angle A = 50$, and $m\angle C = 30$, what is $m\angle X$?

(1) 30

(2) 50

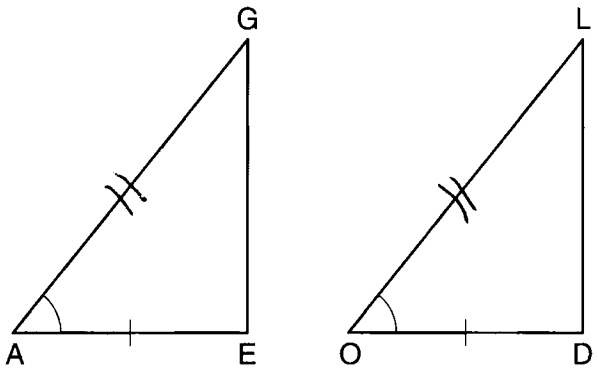
(3) 80

(4) 100

$180 - (50+30) = 100$

Use this space for computations.

7 In the diagram below of $\triangle AGE$ and $\triangle OLD$, $\angle GAE \cong \angle LOD$, and $\overline{AE} \cong \overline{OD}$.



To prove that $\triangle AGE$ and $\triangle OLD$ are congruent by SAS, what other information is needed?

(1) $\overline{GE} \cong \overline{LD}$
(2) $\overline{AG} \cong \overline{OL}$
(3) $\angle AGE \cong \angle OLD$
(4) $\angle AEG \cong \angle ODL$

8 Point A is not contained in plane $\mathcal{B}$. How many lines can be drawn through point A that will be perpendicular to plane $\mathcal{B}$?

(1) one
(2) two
(3) zero
(4) infinite

9 The equation of a circle is $x^2 + (y - 7)^2 = 16$. What are the center and radius of the circle?

(1) center = (0,7); radius = 4
(2) center = (0,7); radius = 16
(3) center = (0,−7); radius = 4
(4) center = (0,−7); radius = 16

Use this space for computations.

10 What is an equation of the line that passes through the point (7,3) and is parallel to the line $4x + 2y = 10$?

(1) $y = \frac{1}{2}x - \frac{1}{2}$

(2) $y = -\frac{1}{2}x + \frac{13}{2}$

(3) $y = 2x - 11$

(4) $y = -2x + 17$

$m = \frac{-A}{B} = \frac{-4}{2} = -2$

$y = mx + b$

$3 = -2(7) + b$

$3 = -14 + b$

$17 = b$

$\angle C \quad \angle A \quad \angle B$

11 In $\triangle ABC$, $AB = 7$, $BC = 8$, and $AC = 9$. Which list has the angles of $\triangle ABC$ in order from smallest to largest?

(1) $\angle A, \angle B, \angle C$

(2) $\angle B, \angle A, \angle C$

(3) $\angle C, \angle B, \angle A$

(4) $\angle C, \angle A, \angle B$

12 Tangents $\overline{PA}$ and $\overline{PB}$ are drawn to circle O from an external point, P, and radii $\overline{OA}$ and $\overline{OB}$ are drawn. If $m\angle APB = 40$, what is the measure of $\angle AOB$?

(1) 140°

(2) 100°

(3) 70°

(4) 50°

$180 - 40 = 140$

13 What is the length of the line segment with endpoints $A(-6,4)$ and $B(2,-5)$?

(1) $\sqrt{13}$

(2) $\sqrt{17}$

(3) $\sqrt{72}$

(4) $\sqrt{145}$

$\sqrt{(-6-2)^2 + (4-(-5))^2}$

$\sqrt{64+81}$

$\sqrt{145}$

Use this space for computations.

14 The lines represented by the equations $y + \frac{1}{2}x = 4$ and $3x + 6y = 12$ are

$y = -\frac{1}{2}x + 4$ $\quad m = -\frac{A}{B} = \frac{-3}{6} = -\frac{1}{2}$

$m = -\frac{1}{2}$

(1) the same line

(2) parallel

(3) perpendicular

(4) neither parallel nor perpendicular

15 A transformation of a polygon that always preserves both length and orientation is

(1) dilation

(2) translation

(3) line reflection

(4) glide reflection

16 In which polygon does the sum of the measures of the interior angles equal the sum of the measures of the exterior angles?

(1) triangle

(2) hexagon

(3) octagon

(4) quadrilateral

Sum of interior $\angle$s = sum of exterior $\angle$s

$(n-2)180 = n\left(180 - \frac{(n-2)180}{n}\right)$

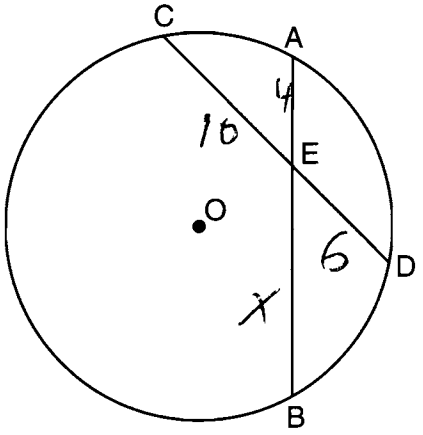
$180n - 360 = 180n - 180n + 360$

$\frac{180n}{180} = \frac{720}{180}$

$n = 4$

Use this space for computations.

17 In the diagram below of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E.



$4x = 6 \cdot 10$

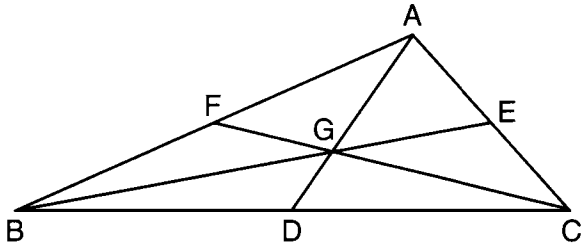
$\frac{4x}{4} = \frac{60}{4}$

$x = 15$

If $CE = 10$, $ED = 6$, and $AE = 4$, what is the length of $\overline{EB}$?

(1) 15
(2) 12
(3) 6.7
(4) 2.4

18 In the diagram below of $\triangle ABC$, medians $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ intersect at G.



$\overline{GC} = 2\overline{FG}$

$\overline{FG} + \overline{GC} = 24$

$\overline{FG} + 2\overline{FG} = 24$

$3\overline{FG} = 24$

$\overline{FG} = 8$

If $CF = 24$, what is the length of $\overline{FG}$?

(1) 8
(2) 10
(3) 12
(4) 16

Use this space for computations.

19 If a line segment has endpoints $A(3x + 5, 3y)$ and $B(x - 1, -y)$, what are the coordinates of the midpoint of $\overline{AB}$?

(1) $(x + 3, 2y)$
(2) $(2x + 2, y)$
(3) $(2x + 3, y)$
(4) $(4x + 4, 2y)$

$\left(\frac{3x+5+x-1}{2}, \frac{3y+(-y)}{2}\right)$

$\left(\frac{4x+4}{2}, \frac{2y}{2}\right)$

$(2x+2, y)$

20 If the surface area of a sphere is represented by 144π, what is the volume in terms of π?

(1) 36π
(2) 48π
(3) 216π
(4) 288π

$SA = 4\pi r^2$

$\frac{144\pi}{4\pi} = \frac{4\pi r^2}{4\pi}$

$36 = r^2$

$6 = r$

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi 6^3$

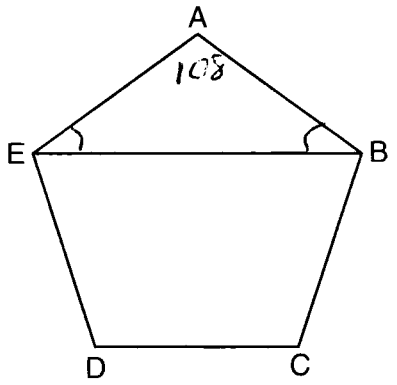
$=$

21 Which transformation of the line $x = 3$ results in an image that is perpendicular to the given line?

(1) $r_{x\text{-axis}}$
(2) $r_{y\text{-axis}}$
(3) $r_{y=x}$
(4) $r_{x=1}$

Use this space for computations.

22 In the diagram below of regular pentagon $ABCDE$, $\overline{EB}$ is drawn.



$\angle A = \frac{(n-2)180}{n}$

$= \frac{(5-2)180}{5}$

$= 108$

What is the measure of $\angle AEB$?

(1) 36°
(2) 54°
(3) 72°
(4) 108°

$\angle AEB = \frac{180-108}{2}$

$= 36$

23 $\triangle ABC$ is similar to $\triangle DEF$. The ratio of the length of $\overline{AB}$ to the length of $\overline{DE}$ is 3:1. Which ratio is also equal to 3:1?

(1) $\frac{m\angle A}{m\angle D}$
(2) $\frac{m\angle B}{m\angle F}$
(3) $\frac{\text{area of } \triangle ABC}{\text{area of } \triangle DEF}$
(4) $\frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF}$

24 What is the slope of a line perpendicular to the line whose equation is $2y = -6x + 8$?

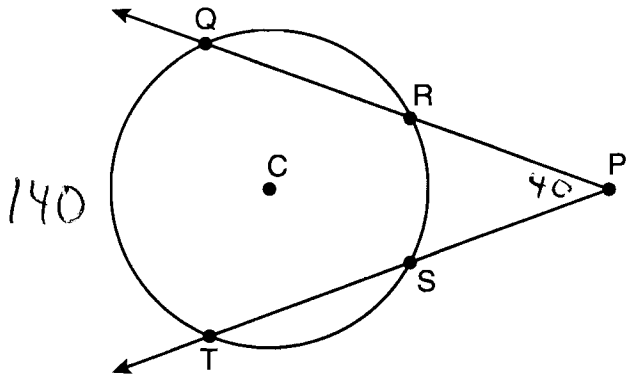
(1) -3
(2) $\frac{1}{6}$
(3) $\frac{1}{3}$
(4) -6

$y = -3x + 4$

$m = -3$ $\quad m_\perp = \frac{1}{3}$

Use this space for computations.

25 In the diagram below of circle C, $m\overset{\frown}{QT} = 140$ and $m\angle P = 40$.



$\frac{140 - \overline{RS}}{2} = 40$

$140 - \overline{RS} = 80$

$60 = \overline{RS}$

What is $m\overset{\frown}{RS}$?

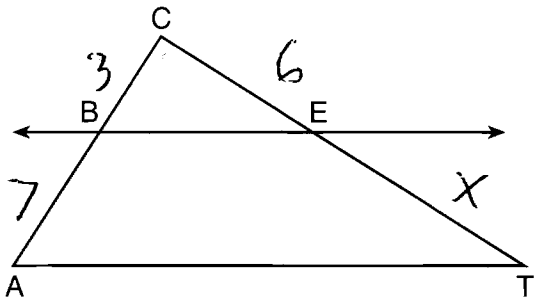
(1) 50
(2) 60
(3) 90
(4) 100

26 Which statement is logically equivalent to "If it is warm, then I go swimming"?

(1) If I go swimming, then it is warm.
(2) If it is warm, then I do not go swimming.
(3) If I do not go swimming, then it is not warm.
(4) If it is not warm, then I do not go swimming.

Use this space for computations.

27 In the diagram below of $\triangle ACT$, $\overleftrightarrow{BE} \parallel \overline{AT}$.



$\frac{3}{7} = \frac{6}{x}$

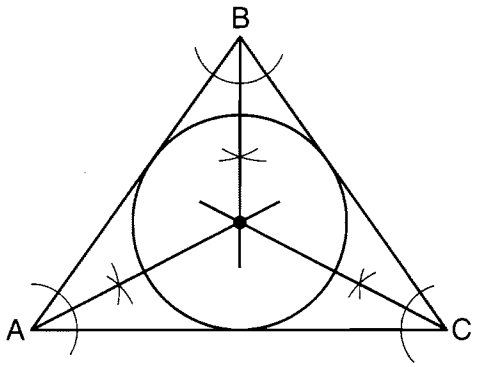
$\frac{3x}{3} = \frac{42}{3}$

$x = 14$

If $CB = 3$, $CA = 10$, and $CE = 6$, what is the length of $\overline{ET}$?

(1) 5
(3) 20
(2) 14
(4) 26

28 Which geometric principle is used in the construction shown below?



(1) The intersection of the angle bisectors of a triangle is the center of the inscribed circle.

(2) The intersection of the angle bisectors of a triangle is the center of the circumscribed circle.

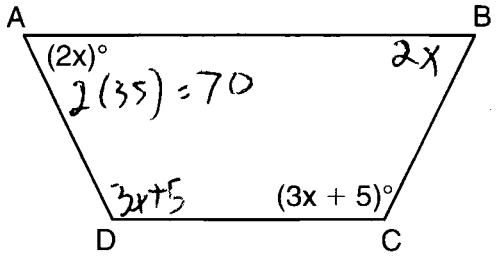
(3) The intersection of the perpendicular bisectors of the sides of a triangle is the center of the inscribed circle.

(4) The intersection of the perpendicular bisectors of the sides of a triangle is the center of the circumscribed circle.

Part II

Answer all 6 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

29 The diagram below shows isosceles trapezoid $ABCD$ with $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \cong \overline{BC}$. If $m\angle BAD = 2x$ and $m\angle BCD = 3x + 5$, find $m\angle BAD$.



$3x+5+3x+5+2x+2x=360$

$10x+10=360$

$\frac{10x}{10}=\frac{350}{10}$

$x=35$

$2x=70$

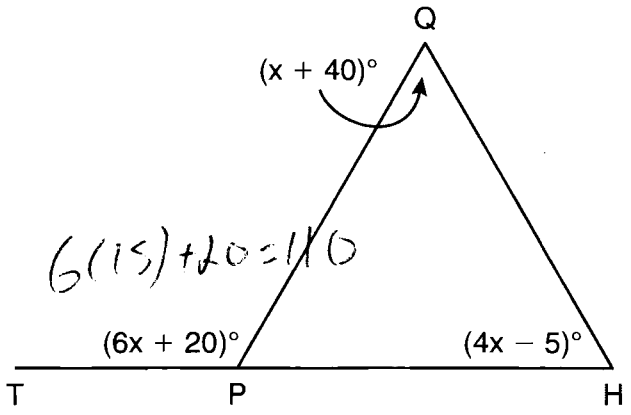
30 A right circular cone has a base with a radius of 15 cm, a vertical height of 20 cm, and a slant height of 25 cm. Find, in terms of π, the number of square centimeters in the lateral area of the cone.

$L = \pi r \ell$

$= \pi(15)(25)$

$= 375\pi$

31 In the diagram below of $\triangle HQP$, side $\overline{HP}$ is extended through P to T, $m\angle QPT = 6x + 20$, $m\angle HQP = x + 40$, and $m\angle PHQ = 4x - 5$. Find $m\angle QPT$.



(Not drawn to scale)

$6(15)+20=110$

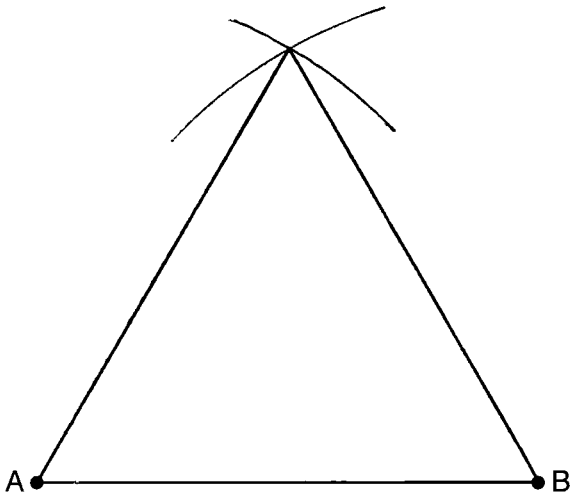
$6x+20 = x+40+4x-5$

$6x+20 = 5x+35$

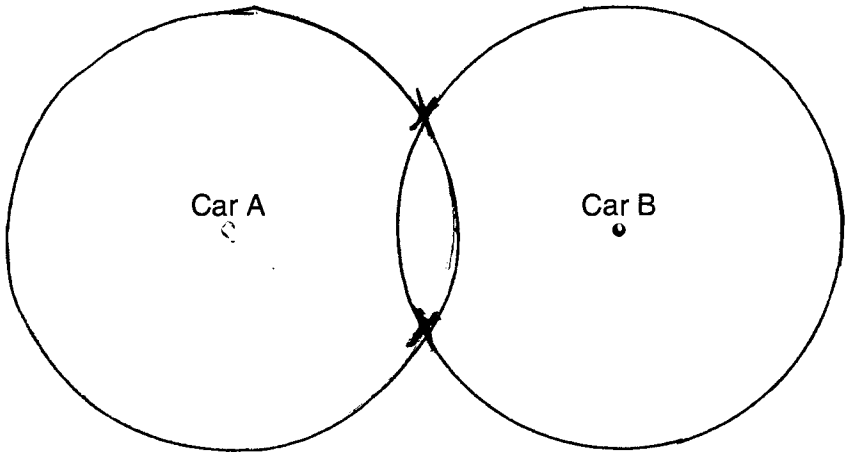
$x = 15$

$6(15)+20 = 110$

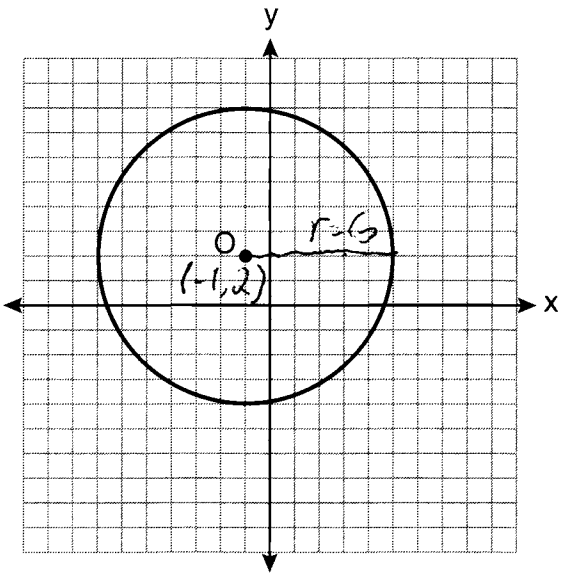
32 On the line segment below, use a compass and straightedge to construct equilateral triangle *ABC*. [Leave all construction marks.]



33 In the diagram below, car A is parked 7 miles from car B. Sketch the points that are 4 miles from car A and sketch the points that are 4 miles from car B. Label with an **X** all points that satisfy both conditions.



34 Write an equation for circle O shown on the graph below.



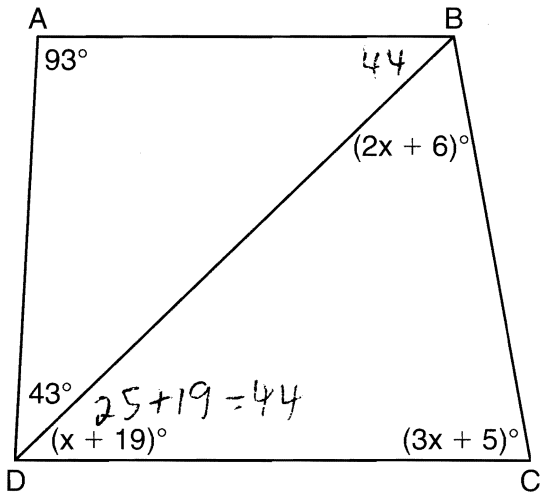
$(x+1)^2+(y-2)^2=36$

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

35 In the diagram below of quadrilateral $ABCD$ with diagonal $\overline{BD}$, $m\angle A = 93$, $m\angle ADB = 43$, $m\angle C = 3x + 5$, $m\angle BDC = x + 19$, and $m\angle DBC = 2x + 6$. Determine if $\overline{AB}$ is parallel to $\overline{DC}$. Explain your reasoning.

35



$x+19+2x+6+3x+5=180$

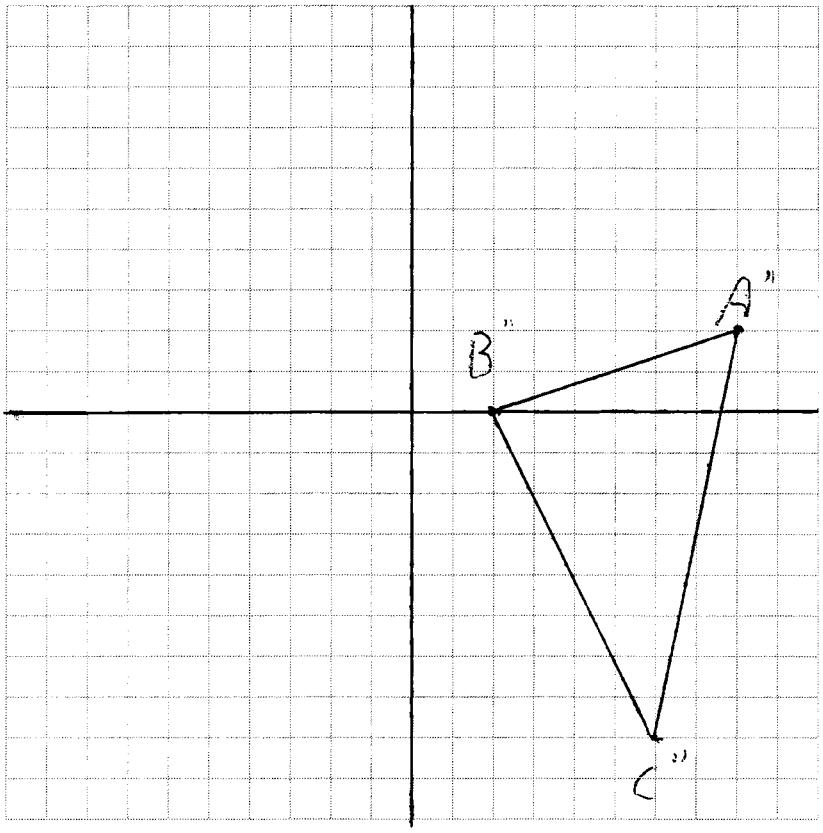
$6x+30=180$

$\frac{6x}{6} = \frac{150}{6}$

$x=25$

Because alternate interior angles $\angle ABD$ and $\angle CDB$ are equal, $\overline{AB}$ is parallel to $\overline{DC}$

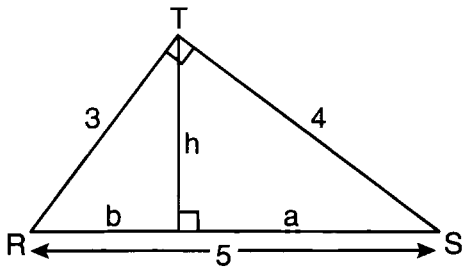
36 The coordinates of the vertices of $\triangle ABC$ are $A(1,3)$, $B(-2,2)$, and $C(0,-2)$. On the grid below, graph and label $\triangle A''B''C''$, the result of the composite transformation $D_2 \circ T_{3,-2}$. State the coordinates of A'', B'', and C''.



A'(4,1) B'(1,0) C'(3,-4)

A''(8,2) B''(2,0) C''(6,-8)

37 In the diagram below, $\triangle RST$ is a 3-4-5 right triangle. The altitude, h, to the hypotenuse has been drawn. Determine the length of h.



$5a = 4^2$ $\quad 5b = 3^2$

$a = 3.2$ $\quad b = 1.8$

$h^2 = 3.2 \cdot 1.8$

$h = \sqrt{5.76}$

$h = 2.4$

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen. [6]

38 Given: Quadrilateral $ABCD$ has vertices $A(-5,6)$, $B(6,6)$, $C(8,-3)$, and $D(-3,-3)$.

Prove: Quadrilateral $ABCD$ is a parallelogram but is neither a rhombus nor a rectangle.

[The use of the grid below is optional.]

$\overline{AB} \parallel \overline{CD}$ because their slopes are equal.
$\overline{AD} \parallel \overline{CB}$ because their slopes are equal.
ABCD is a parallelogram because opposite sides are parallel.
$\overline{AB} \neq \overline{BC}$. ABCD is not a rhombus because all sides are not equal.
$\overline{AB}$ is not perpendicular to $\overline{BC}$ because their slopes are not opposite reciprocals.
ABCD is not a rectangle because $\angle ABC$ is not a right angle