The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Friday, June 19, 2015 — 1:15 to 4:15 p.m., only

Student Name: Mr. Sibol

School Name: JMAP

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 38 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 28 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [56]

Use this space for computations.

1 Quadrilateral $ABCD$ undergoes a transformation, producing quadrilateral $A'B'C'D'$. For which transformation would the area of $A'B'C'D'$ *not* be equal to the area of $ABCD$?

(1) a rotation of 90° about the origin

(2) a reflection over the y-axis

(3) a dilation by a scale factor of 2

(4) a translation defined by $(x,y) \rightarrow (x + 4, y - 1)$

2 The diameter of a sphere is 12 inches. What is the volume of the sphere to the *nearest cubic inch*?

(1) 288

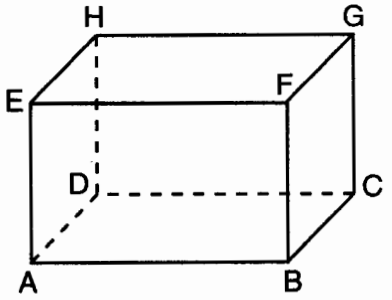
(2) 452

(3) 905

(4) 7,238

$V = \frac{4}{3}\pi\left(\frac{12}{2}\right)^3 \approx 905$

3 A right rectangular prism is shown in the diagram below.



Which line segments are coplanar?

(1) $\overline{EF}$ and $\overline{BC}$

(2) $\overline{HD}$ and $\overline{FG}$

(3) $\overline{GH}$ and $\overline{FB}$

(4) $\overline{EA}$ and $\overline{GC}$

Use this space for computations.

4 What are the coordinates of the image of point $A(2,-7)$ under the translation $(x,y) \rightarrow (x-3, y+5)$?

(1) $(-1,-2)$
(2) $(-1,2)$
(3) $(5,-12)$
(4) $(5,12)$

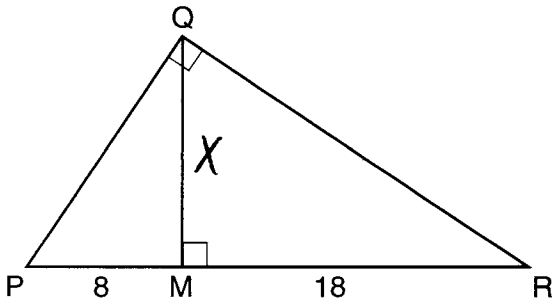
5 Point M is the midpoint of $\overline{AB}$. If the coordinates of M are (2,8) and the coordinates of A are (10,12), what are the coordinates of B?

(1) (6,10)
(2) $(-6,4)$
(3) $(-8,-4)$
(4) (18,16)

$\frac{10+x}{2} = 2$ $\quad x = -6$

$\frac{12+y}{2} = 8$ $\quad y = 4$

6 In the diagram below, $\overline{QM}$ is an altitude of right triangle PQR, $PM = 8$, and $RM = 18$.



What is the length of $\overline{QM}$?

(1) 20
(2) 16
(3) 12
(4) 10

$x^2 = 8 \cdot 18$
$x^2 = 144$
$x = 12$

7 What is an equation of the line that passes through the point (2,4) and is perpendicular to the line whose equation is $3y = 6x + 3$?

(1) $y = -\frac{1}{2}x + 5$
(2) $y = -\frac{1}{2}x + 4$
(3) $y = 2x - 6$
(4) $y = 2x$

$\frac{3y}{3} = \frac{6x}{3} + \frac{3}{3}$
$m = 2$
$m_\perp = -\frac{1}{2}$

$y = mx + b$
$4 = -\frac{1}{2}(2) + b$
$4 = -1 + b$
$5 = b$

Use this space for computations.

8 In all isosceles triangles, the exterior angle of a base angle must always be

(1) a right angle

(2) an acute angle

(3) an obtuse angle

(4) equal to the vertex angle

9 If $\triangle W'X'Y'$ is the image of $\triangle WXY$ after the transformation $R_{90°}$, which statement is *false*?

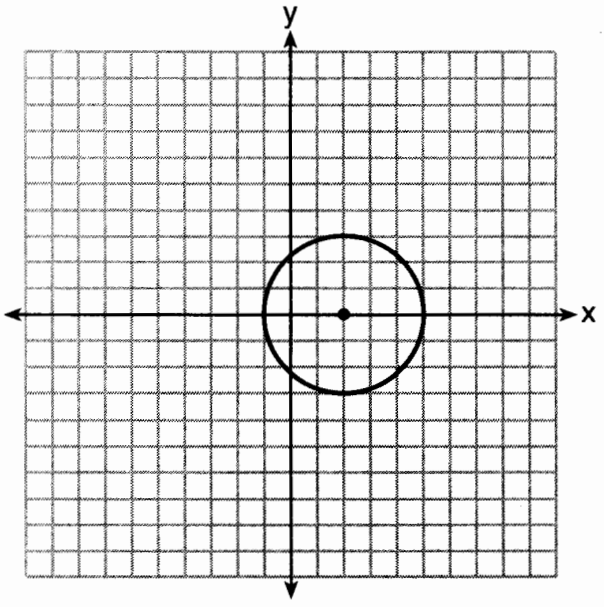
(1) $XY = X'Y'$

(2) $\overline{WX} \parallel \overline{W'X'}$

(3) $\triangle WXY \cong \triangle W'X'Y'$

(4) $m\angle XWY = m\angle X'W'Y'$

10 Which equation represents the circle shown in the graph below?



(1) $(x - 2)^2 + y^2 = 9$

(2) $(x + 2)^2 + y^2 = 9$

(3) $(x - 2)^2 + y^2 = 3$

(4) $(x + 2)^2 + y^2 = 3$

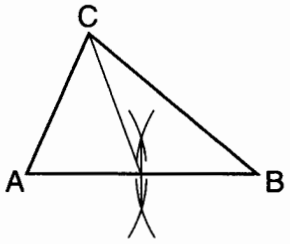
Use this space for computations.

11 In quadrilateral $ABCD$, each diagonal bisects opposite angles. If $m\angle DAB = 70$, then $ABCD$ must be a

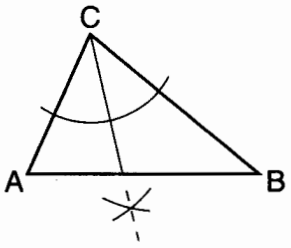
(1) rectangle } diagonals do not bisect opposite ∠s (3) rhombus

(2) trapezoid } (4) square $M\angle DAB = 90$

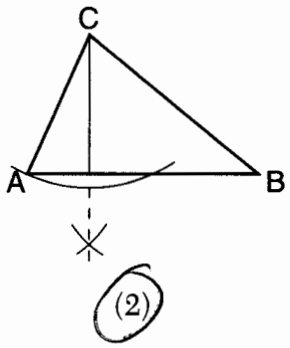
12 Which diagram illustrates a correct construction of an altitude of $\triangle ABC$?



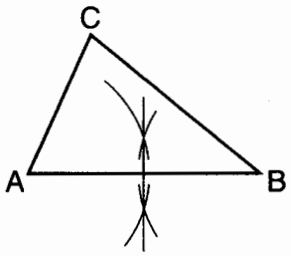
(1)



(3)



(2)



(4)

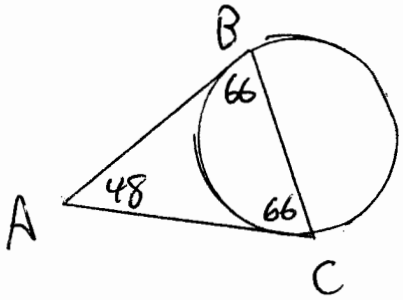
13 From external point A, two tangents to circle O are drawn. The points of tangency are B and C. Chord $\overline{BC}$ is drawn to form $\triangle ABC$. If $m\angle ABC = 66$, what is $m\angle A$?

(1) 33

(2) 48

(3) 57

(4) 66



Use this space for computations.

14 Point A lies on plane $\mathcal{P}$. How many distinct lines passing through point A are perpendicular to plane $\mathcal{P}$?

(1) 1
(2) 2
(3) 0
(4) infinite

15 Students made four statements about a circle.

A: The coordinates of its center are $(4,-3)$.
B: The coordinates of its center are $(-4,3)$.
C: The length of its radius is $5\sqrt{2}$.
D: The length of its radius is 25.

If the equation of the circle is $(x + 4)^2 + (y - 3)^2 = 50$, which statements are correct?

(1) A and C
(3) B and C
(2) A and D
(4) B and D

$r^2 = 50$
$r = \sqrt{25}\sqrt{2}$
$5\sqrt{2}$

16 Points A, B, C, and D are located on circle O, forming trapezoid $ABCD$ with $\overline{AB} \parallel \overline{DC}$. Which statement must be true?

(1) $\overline{AB} \cong \overline{DC}$
(3) $\angle A \cong \angle D$
(2) $\overset{\frown}{AD} \cong \overset{\frown}{BC}$
(4) $\overset{\frown}{AB} \cong \overset{\frown}{DC}$

17 If $\triangle ABC \sim \triangle LMN$, which statement is *not* always true?

(1) $m\angle A = m\angle N$
(3) $\dfrac{\text{area } \triangle ABC}{\text{area } \triangle LMN} = \dfrac{(AC)^2}{(LN)^2}$
(2) $m\angle B = m\angle M$
(4) $\dfrac{\text{perimeter } \triangle ABC}{\text{perimeter } \triangle LMN} = \dfrac{AB}{LM}$

Use this space for computations.

18 The equations representing lines k, m, and n are given below.

k: $3y + 6 = 2x$ $m = 2/3$

m: $3y + 2x + 6 = 0$ $m = -\frac{2}{3}$

n: $2y = 3x + 6$ $m = \frac{3}{2}$

Which statement is true?

(1) $k \parallel m$

(2) $n \parallel m$

(3) $m \perp k$

(4) $m \perp n$

19 A regular polygon with an exterior angle of 40° is a

(1) pentagon

(2) hexagon

(3) nonagon

(4) decagon

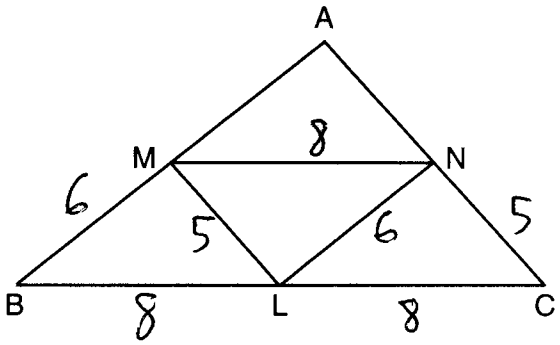
$180 - \frac{180(n-2)}{n} = 40$

$140 = \frac{180n - 360}{n}$

$360 = 40n$

$9 = n$

20 In $\triangle ABC$ shown below, L is the midpoint of $\overline{BC}$, M is the midpoint of $\overline{AB}$, and N is the midpoint of $\overline{AC}$.



If $MN = 8$, $ML = 5$, and $NL = 6$, the perimeter of trapezoid $BMNC$ is

(1) 26

(2) 28

(3) 30

(4) 35

Use this space for computations.

21 The sum of the interior angles of a regular polygon is 720°. How many sides does the polygon have?

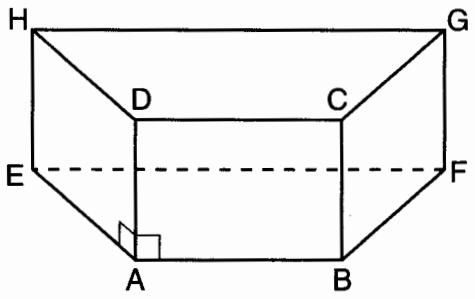
(1) 8
(2) 6
(3) 5
(4) 4

$\frac{180(n-2)}{180} = \frac{720}{180}$

$n-2=4$

$n=6$

22 In the prism shown below, $\overline{AD} \perp \overline{AE}$ and $\overline{AD} \perp \overline{AB}$.



Which plane is perpendicular to $\overline{AD}$?

(1) *HEA*
(2) *BAD*
(3) *EAB*
(4) *EHG*

23 In $\triangle ABC$, $m\angle A = 65$ and $m\angle B$ is greater than $m\angle A$. The lengths of the sides of $\triangle ABC$ in order from smallest to largest are

(1) $\overline{AB}, \overline{BC}, \overline{AC}$
(2) $\overline{BC}, \overline{AB}, \overline{AC}$
(3) $\overline{AC}, \overline{BC}, \overline{AB}$
(4) $\overline{AB}, \overline{AC}, \overline{BC}$

$\angle C < \angle A < \angle B$

24 Which equation represents a circle whose center is the origin and that passes through the point $(-4,0)$?

(1) $x^2 + y^2 = 8$
(2) $x^2 + y^2 = 16$
(3) $(x + 4)^2 + y^2 = 8$
(4) $(x + 4)^2 + y^2 = 16$

Use this space for computations.

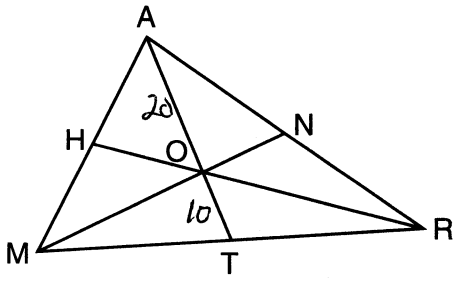
25 The lengths of two sides of a triangle are 7 and 11. Which inequality represents all possible values for x, the length of the third side of the triangle?

(1) $4 \leq x \leq 18$
(2) $4 < x \leq 18$
(3) $4 \leq x < 18$
(4) $4 < x < 18$

26 Which statement is the inverse of "If $x + 3 = 7$, then $x = 4$"?

(1) If $x = 4$, then $x + 3 = 7$.
(2) If $x \neq 4$, then $x + 3 \neq 7$.
(3) If $x + 3 \neq 7$, then $x \neq 4$.
(4) If $x + 3 = 7$, then $x \neq 4$.

27 In the diagram below of $\triangle MAR$, medians $\overline{MN}$, $\overline{AT}$, and $\overline{RH}$ intersect at O.



If $TO = 10$, what is the length of $\overline{TA}$?

(1) 30
(2) 25
(3) 20
(4) 15

28 What is an equation of the line that passes through the point (4,5) and is parallel to the line whose equation is $y = \frac{2}{3}x - 4$?

(1) $2y + 3x = 11$
(2) $2y + 3x = 22$
(3) $3y - 2x = 2$
(4) $3y - 2x = 7$

$\frac{2}{3}(x-4) = y-5$
$2x-8 = 3y-15$
$7 = 3y-2x$

Part II

Answer all 6 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

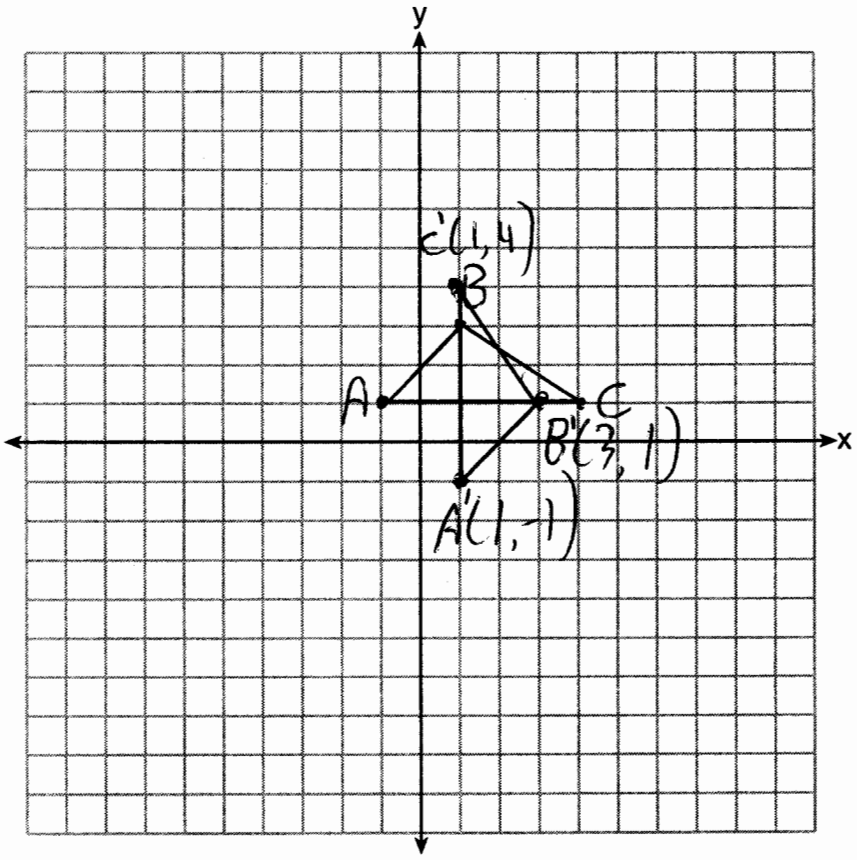
29 The measures of the angles of a triangle are in the ratio 5:6:7. Determine the measure, in degrees, of the *smallest* angle of the triangle.

18

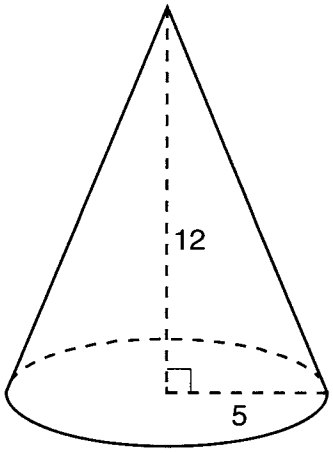
$\frac{5}{18} \times 180 = 50$

30 Triangle ABC has vertices $A(-1,1)$, $B(1,3)$, and $C(4,1)$. The image of $\triangle ABC$ after the transformation $r_{y=x}$ is $\triangle A'B'C'$. State and label the coordinates of $\triangle A'B'C'$.

[The use of the set of axes below is optional.]



31 As shown in the diagram below, a right circular cone has a height of 12 and a radius of 5.

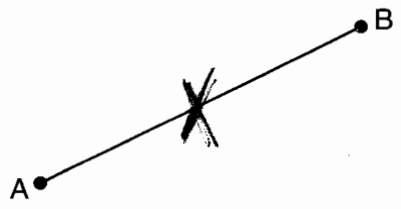


$\ell = \sqrt{5^2+12^2} = 13$

Determine, in terms of π, the lateral area of the right circular cone.

$L = \pi \cdot 5 \cdot 13 = 65\pi$

32 Using a compass and straightedge, locate the midpoint of $\overline{AB}$ by construction.
[Leave all construction marks.]



33 The coordinates of the endpoints of $\overline{CD}$ are $C(3,8)$ and $D(6,-1)$. Express the length of $\overline{CD}$ in simplest radical form.

$\sqrt{(6-3)^2+(-1-8)^2}$

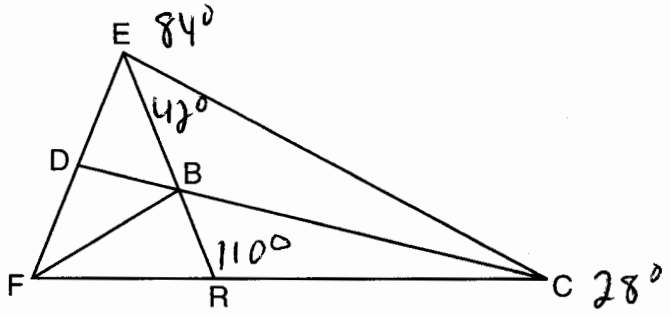
$\sqrt{9+81}$

$\sqrt{90}$

$\sqrt{9}\sqrt{10}$

$3\sqrt{10}$

34 In the diagram below, point B is the incenter of $\triangle FEC$, and $\overline{EBR}$, $\overline{CBD}$, and $\overline{FB}$ are drawn.



If $m\angle FEC = 84$ and $m\angle ECF = 28$, determine and state $m\angle BRC$.

$180-(42+28)$

$180-70$

$110°$

Part III

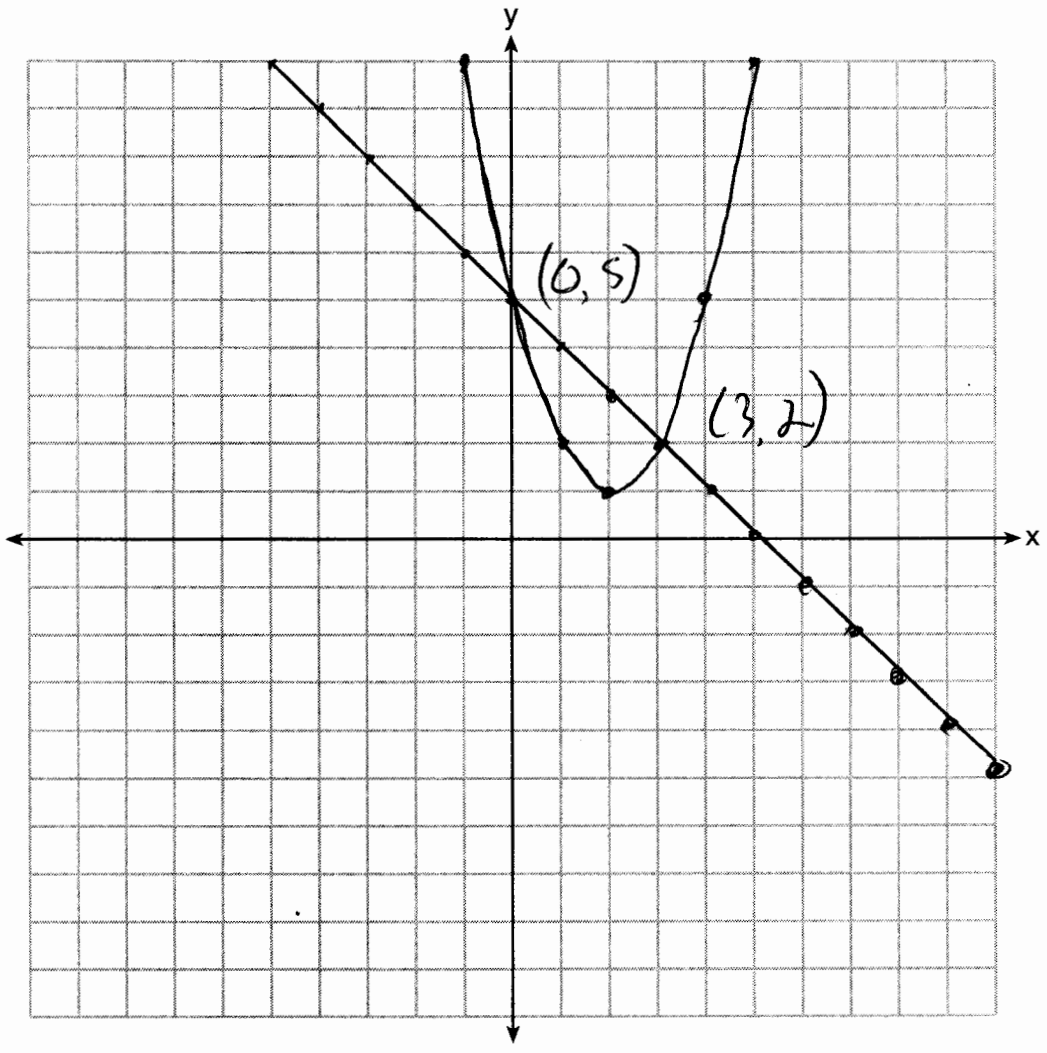
Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

35 Solve the following system of equations graphically. State the coordinates of all points in the solution.

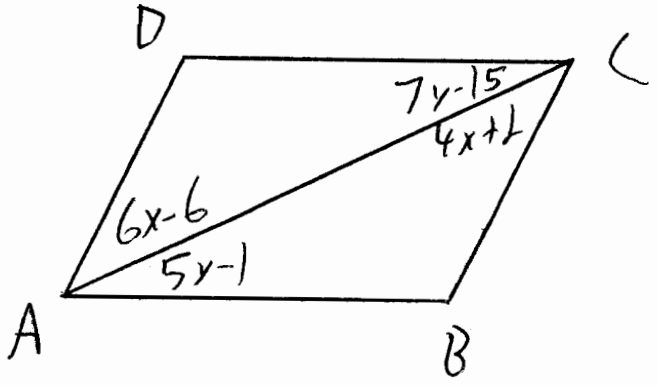
$y + 4x = x^2 + 5$ $y = x^2 - 4x + 5$

$x + y = 5$

$y = -x + 5$



36 In parallelogram $ABCD$, with diagonal $\overline{AC}$ drawn, $m\angle BCA = 4x + 2$, $m\angle DAC = 6x - 6$, $m\angle BAC = 5y - 1$, and $m\angle DCA = 7y - 15$. Determine $m\angle B$.



$6x - 6 = 4x + 2$

$2x = 8$

$x = 4$

$4(4) + 2 = 18^\circ$

$7y - 15 = 5x - 1$

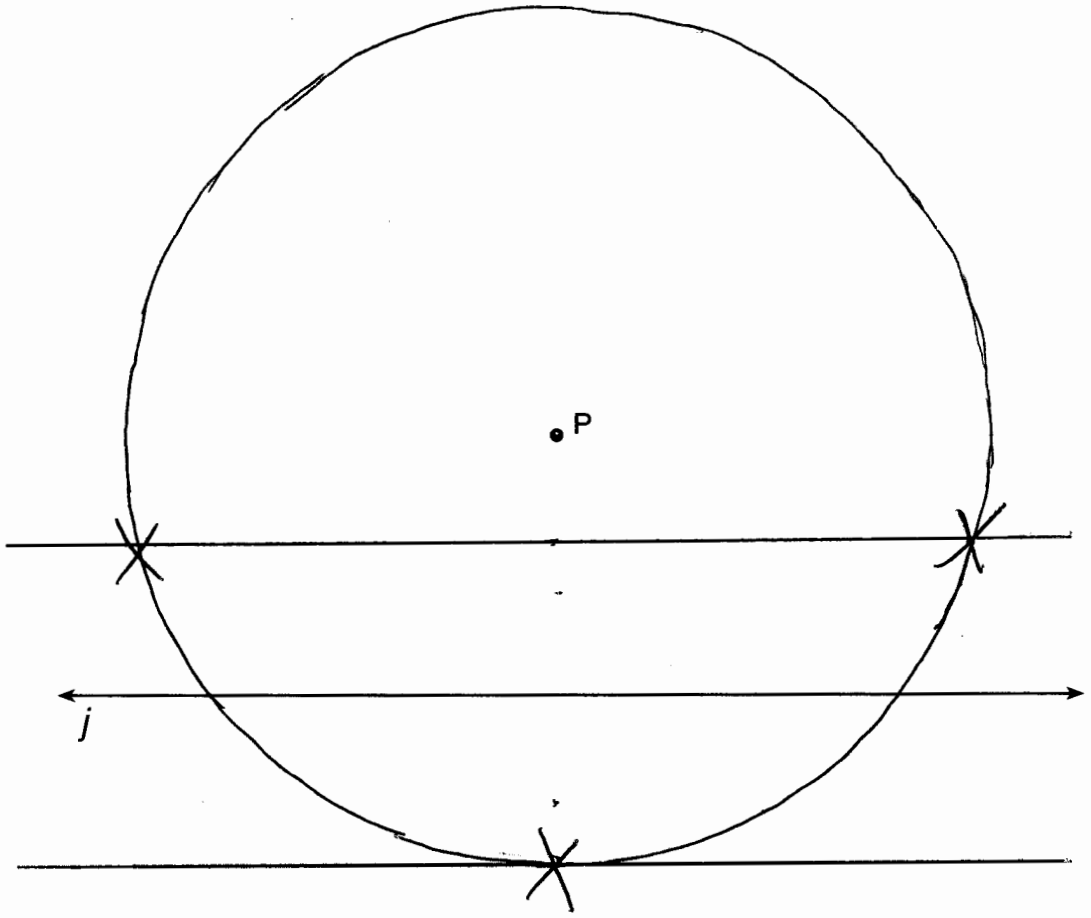
$2y = 14$

$y = 7$

$5(7) - 1 = 34^\circ$

$180 - (18 + 34) = 128$

37 Point P is 5 units from line j. Sketch the locus of points that are 3 units from line j and also sketch the locus of points that are 8 units from P. Label with an **X** all points that satisfy *both* conditions.

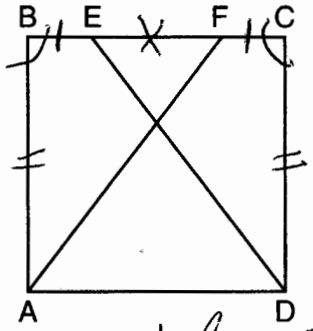


Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

38 The diagram below shows square $ABCD$ where E and F are points on $\overline{BC}$ such that $\overline{BE} \cong \overline{FC}$, and segments AF and DE are drawn.

Prove that $\overline{AF} \cong \overline{DE}$.



Statement	Reason
(1) Square ABCD; E & F are points on $\overline{BC}$ such that $\overline{BE} \cong \overline{FC}$; segments AF & DE drawn	(1) Given
(2) $\overline{AB} \cong \overline{CD}$	(2) All sides of a square are congruent
(3) $\angle ABF \cong \angle DCE$	(3) All angles of a square are equiangular
(4) $\overline{EF} \cong \overline{FE}$	(4) Reflexive property
(5) $\overline{BE} + \overline{EF} \cong \overline{FC} + \overline{FE}$	(5) Additive property of line segments
(6) $\overline{BF} \cong \overline{CE}$	(6) Angle Addition
(7) $\triangle ABF \cong \triangle DCE$	(7) SAS
(8) $\overline{AF} \cong \overline{DE}$	(8) CPCTC