The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Wednesday, June 19, 2013 — 9:15 a.m. to 12:15 p.m., only

Student Name: Mr. Sibol

School Name: JMAP

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 38 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

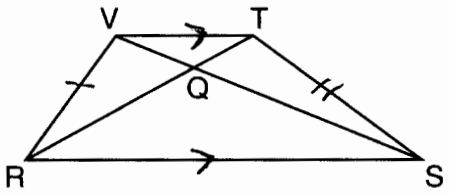
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 28 questions in this part. Each correct answer will receive 2 credits. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [56]

Use this space for computations.

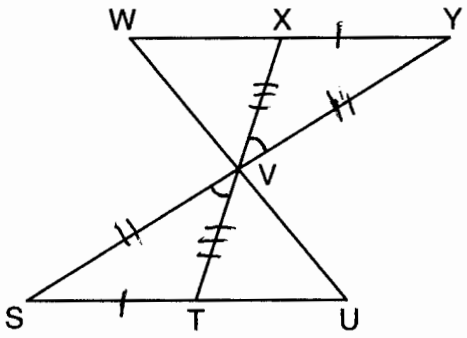
1 In trapezoid $RSTV$ below with bases $\overline{RS}$ and $\overline{VT}$, diagonals $\overline{RT}$ and $\overline{SV}$ intersect at Q.



If trapezoid $RSTV$ is *not* isosceles, which triangle is equal in area to $\triangle RSV$?

(1) $\triangle RQV$
(2) $\triangle RST$
(3) $\triangle RVT$
(4) $\triangle SVT$

2 In the diagram below, $\triangle XYV \cong \triangle TSV$.

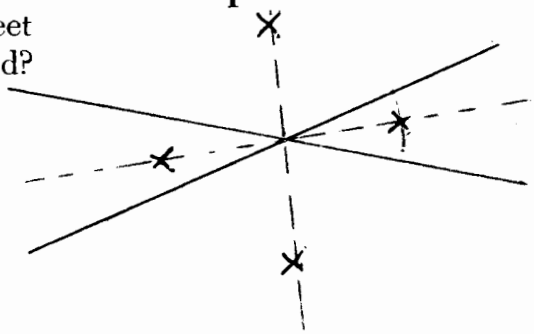


Which statement can *not* be proven?

(1) $\angle XVY \cong \angle TVS$
(2) $\angle VYX \cong \angle VUT$
(3) $\overline{XY} \cong \overline{TS}$
(4) $\overline{YV} \cong \overline{SV}$

Use this space for computations.

3 In a park, two straight paths intersect. The city wants to install lampposts that are both equidistant from each path and also 15 feet from the intersection of the paths. How many lampposts are needed?



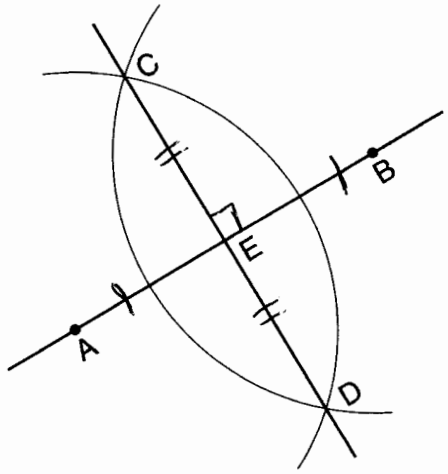
(1) 1
(2) 2
(3) 3
(4) 4

4 What are the coordinates of A', the image of point $A(-3,4)$, after a rotation of 180° about the origin?

$(x, y) \rightarrow (-x, -y)$

(1) (4,−3)
(2) (−4,−3)
(3) (3,4)
(4) (3,−4)

5 Based on the construction below, which conclusion is *not* always true?



(1) $\overline{AB} \perp \overline{CD}$
(2) $AB = CD$
(3) $AE = EB$
(4) $CE = DE$

Use this space for computations.

6 Which equation represents the circle whose center is $(-5,3)$ and that passes through the point $(-1,3)$?

$r = 4$

$r^2 = 16$

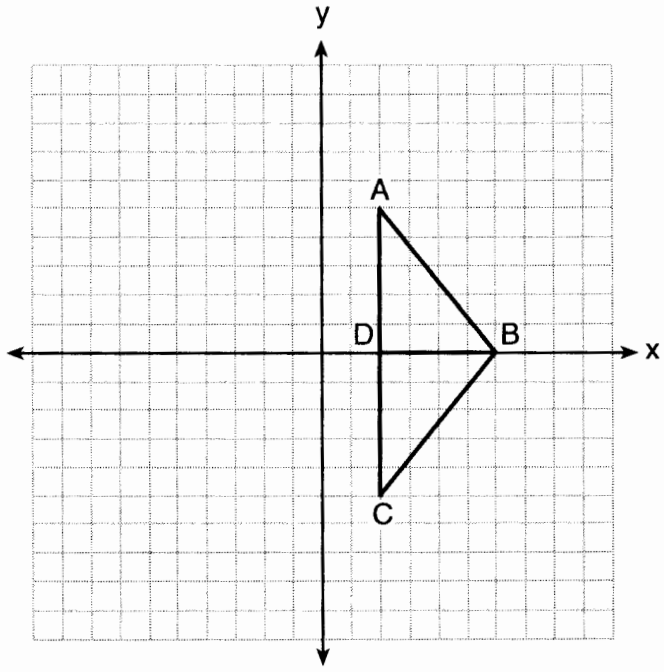
(1) $(x + 1)^2 + (y - 3)^2 = 16$

(2) $(x - 1)^2 + (y + 3)^2 = 16$

(3) $(x + 5)^2 + (y - 3)^2 = 16$

(4) $(x - 5)^2 + (y + 3)^2 = 16$

7 As shown in the diagram below, when right triangle DAB is reflected over the x-axis, its image is triangle DCB.



Which statement justifies why $\overline{AB} \cong \overline{CB}$?

(1) Distance is preserved under reflection.

(2) Orientation is preserved under reflection.

(3) Points on the line of reflection remain invariant.

(4) Right angles remain congruent under reflection.

55 55 70
3(18)+1 4(18)-17 5(18)-20

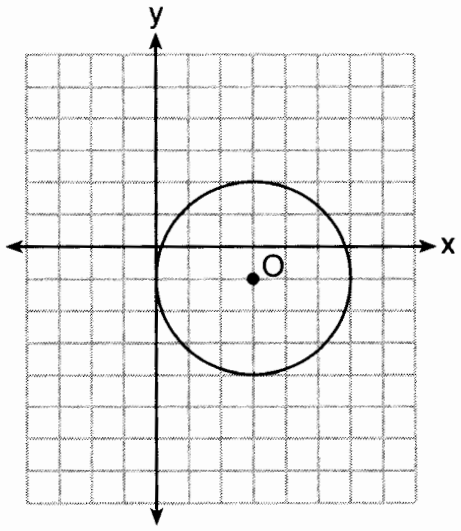
8 In $\triangle ABC$, $m\angle A = 3x + 1$, $m\angle B = 4x - 17$, and $m\angle C = 5x - 20$. Which type of triangle is $\triangle ABC$?

(1) right
(2) scalene
(3) isosceles
(4) equilateral

Use this space for computations.

$3x+1+4x-17+5x-20=180$
$12x-36=180$
$12x=216$
$x=18$

9 What is the equation for circle O shown in the graph below?



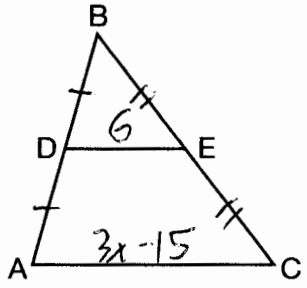
(1) $(x-3)^2+(y+1)^2=6$
(2) $(x+3)^2+(y-1)^2=6$
(3) $(x-3)^2+(y+1)^2=9$
(4) $(x+3)^2+(y-1)^2=9$

Use this space for computations.

10 Point A is on line m. How many distinct planes will be perpendicular to line m and pass through point A?

(1) one
(2) two
(3) zero
(4) infinite

11 In $\triangle ABC$, D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{BC}$. If $AC = 3x - 15$ and $DE = 6$, what is the value of x?



$3x - 15 = (2)(6)$
$3x - 15 = 12$
$3x = 27$
$x = 9$

(1) 6
(2) 7
(3) 9
(4) 12

12 What are the coordinates of the center of a circle if the endpoints of its diameter are $A(8,-4)$ and $B(-3,2)$?

(1) (2.5,1)
(2) (2.5,−1)
(3) (5.5,−3)
(4) (5.5,3)

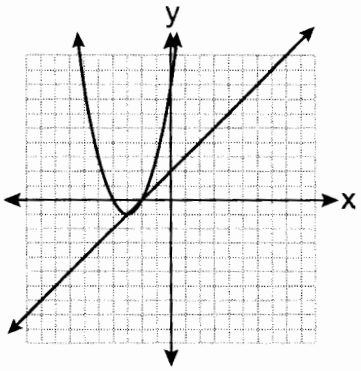
$\left(\frac{8+-3}{2}, \frac{-4+2}{2}\right)$
$\left(\frac{5}{2}, \frac{-2}{2}\right)$
$(2.5, -1)$

Use this space for computations.

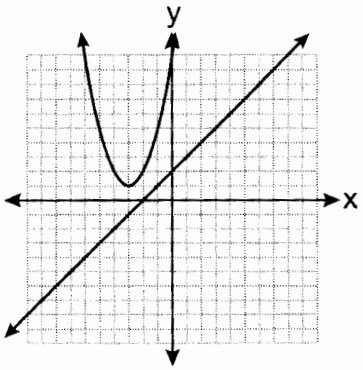
13 Which graph could be used to find the solution to the following system of equations?

$y = (x + 3)^2 - 1$ vertex $(-3, -1)$

$x + y = 2$ $y = -x + 2$



(1)



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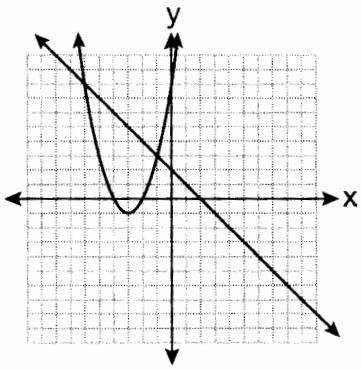

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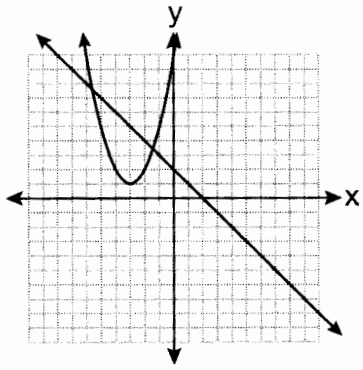


(4)

14 What is the converse of "If an angle measures 90 degrees, then it is a right angle"?

(1) If an angle is a right angle, then it measures 90 degrees.

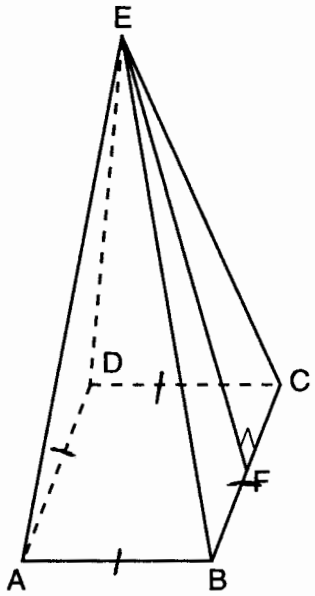
(2) An angle is a right angle if it measures 90 degrees.

(3) If an angle is not a right angle, then it does not measure 90 degrees.

(4) If an angle does not measure 90 degrees, then it is not a right angle.

Use this space for computations.

15 As shown in the diagram below, a right pyramid has a square base, $ABCD$, and $\overline{EF}$ is the slant height.



Which statement is *not* true?

(1) $\overline{EA} \cong \overline{EC}$

(2) $\overline{EB} \cong \overline{EF}$

(3) $\triangle AEB \cong \triangle BEC$

(4) $\triangle CED$ is isosceles

16 What is the equation of a line passing through the point (6,1) and parallel to the line whose equation is $3x = 2y + 4$?

(1) $y = -\frac{2}{3}x + 5$

(2) $y = -\frac{2}{3}x - 3$

(3) $y = \frac{3}{2}x - 8$

(4) $y = \frac{3}{2}x - 5$

$2y = 3x - 4$

$y = \frac{3}{2}x - 2$

$y = \frac{3}{2}(6) - 8$

$y = 9 - 8 = 1$

17 The volume of a sphere is approximately 44.6022 cubic centimeters. What is the radius of the sphere, to the *nearest tenth of a centimeter*?

(1) 2.2

(2) 3.3

(3) 4.4

(4) 4.7

$V = \frac{4}{3}\pi r^3$

$44.6022 = \frac{4\pi}{3} r^3$

$10.648 \cong r^3$

$2.2 \cong r$

Use this space for computations.

18 Points $A(5,3)$ and $B(7,6)$ lie on $\overleftrightarrow{AB}$. Points $C(6,4)$ and $D(9,0)$ lie on $\overleftrightarrow{CD}$. Which statement is true?

$m_{\overleftrightarrow{AB}} = \frac{6-3}{7-5} = \frac{3}{2}$

$m_{\overleftrightarrow{CD}} = \frac{4-0}{6-9} = \frac{4}{-3}$

(1) $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

(2) $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$

(3) $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are the same line.

(4) $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect, but are not perpendicular.

19 Which set of equations represents two circles that have the same center?

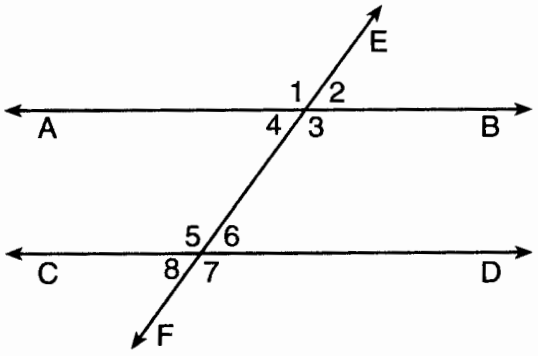
(1) $x^2 + (y + 4)^2 = 16$ and $(x + 4)^2 + y^2 = 16$

(2) $(x + 3)^2 + (y - 3)^2 = 16$ and $(x - 3)^2 + (y + 3)^2 = 25$

(3) $(x - 7)^2 + (y - 2)^2 = 16$ and $(x + 7)^2 + (y + 2)^2 = 25$

(4) $(x - 2)^2 + (y - 5)^2 = 16$ and $(x - 2)^2 + (y - 5)^2 = 25$

20 Transversal $\overleftrightarrow{EF}$ intersects $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$, as shown in the diagram below.



Which statement could always be used to prove $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$?

(1) $\angle 2 \cong \angle 4$

(2) $\angle 7 \cong \angle 8$

(3) $\angle 3$ and $\angle 6$ are supplementary

(4) $\angle 1$ and $\angle 5$ are supplementary

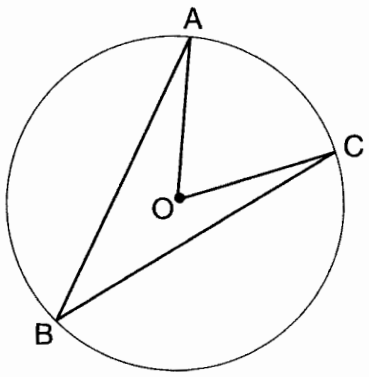
AC > BC > AB

Use this space for computations.

21 In $\triangle ABC$, $m\angle A = 60$, $m\angle B = 80$, and $m\angle C = 40$. Which inequality is true?

(1) $AB > BC$
(2) $AC > BC$
(3) $AC < BA$
(4) $BC < BA$

22 Circle O with $\angle AOC$ and $\angle ABC$ is shown in the diagram below.



What is the ratio of $m\angle AOC$ to $m\angle ABC$?

(1) 1:1
(2) 2:1
(3) 3:1
(4) 1:2

23 A rectangular prism has a base with a length of 25, a width of 9, and a height of 12. A second prism has a square base with a side of 15. If the volumes of the two prisms are equal, what is the height of the second prism?

(1) 6
(3) 12
(2) 8
(4) 15

$25 \cdot 9 \cdot 12 = 15^2 \cdot h$

$12 = \frac{2700}{15^2} = h$

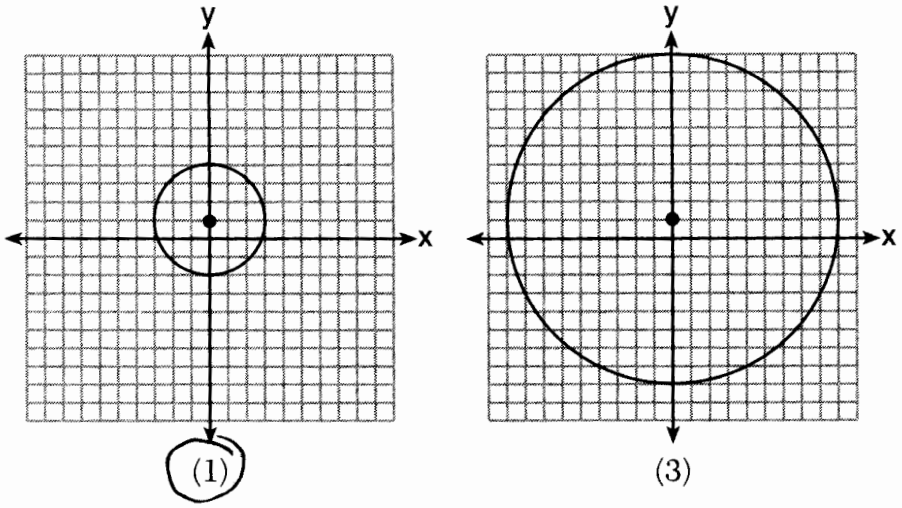
5 5

24 In triangles ABC and DEF, $AB = 4$, $AC = 5$, $DE = 8$, $DF = 10$, and $\angle A \cong \angle D$. Which method could be used to prove $\triangle ABC \sim \triangle DEF$?

(1) AA
(2) SAS
(3) SSS
(4) ASA

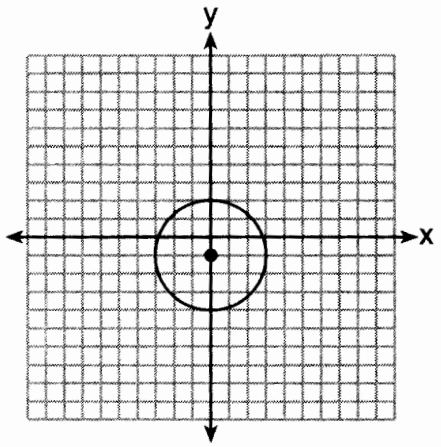
Use this space for computations.

25 Which graph represents a circle whose equation is $x^2 + (y - 1)^2 = 9$?

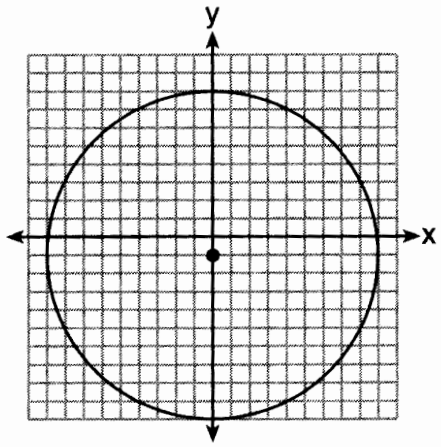


(1)

(3)



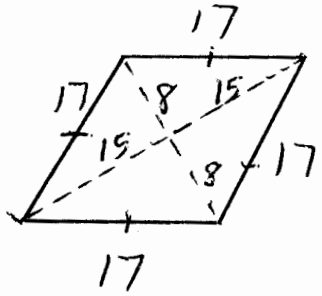
(2)



(4)

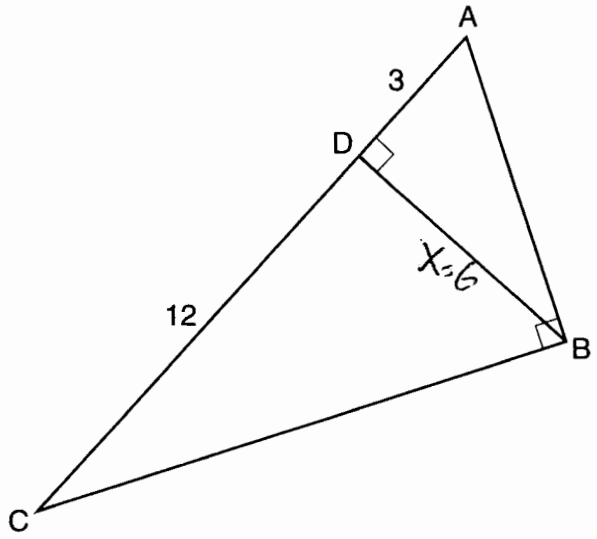
26 What is the perimeter of a rhombus whose diagonals are 16 and 30?

(1) 92
(2) 68
(3) 60
(4) 17



Use this space for computations.

27 In right triangle ABC shown in the diagram below, altitude $\overline{BD}$ is drawn to hypotenuse $\overline{AC}$, $CD = 12$, and $AD = 3$.



$12 \cdot 3 = x^2$

$36 = x^2$

$6 = x$

$\overline{AB} = \sqrt{3^2 + 6^2}$

$= \sqrt{45}$

$= 3\sqrt{5}$

What is the length of $\overline{AB}$?

(1) $5\sqrt{3}$

(3) $3\sqrt{5}$

(2) 6

(4) 9

28 Secants $\overline{JKL}$ and $\overline{JMN}$ are drawn to circle O from an external point, J. If $JK = 8$, $LK = 4$, and $JM = 6$, what is the length of $\overline{JN}$?

(1) 16

(3) 10

(2) 12

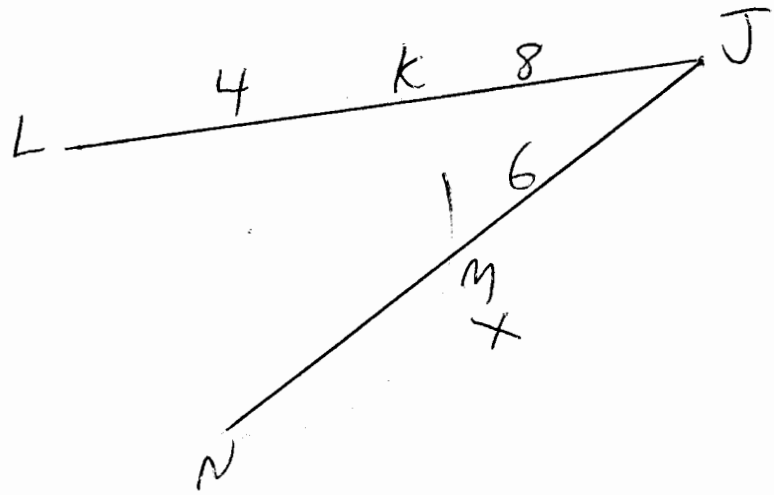
(4) 8

$WE \cdot WE$

$12(8) = x \cdot 6$

$96 = 6x$

$16 = x$



Part II

Answer all 6 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

29 A right circular cylinder has a height of 7 inches and the base has a diameter of 6 inches. Determine the lateral area, in square inches, of the cylinder in terms of π.

$L = 2\pi rh$

$= 2\pi(3)(7)$

$= 42\pi$

30 Determine, in degrees, the measure of each interior angle of a regular octagon.

$$\frac{(n-2)180}{n}$$

$$\frac{(8-2)180}{8}$$

$$135$$

31 Triangle ABC has vertices at $A(3,0)$, $B(9,-5)$, and $C(7,-8)$. Find the length of $\overline{AC}$ in simplest radical form.

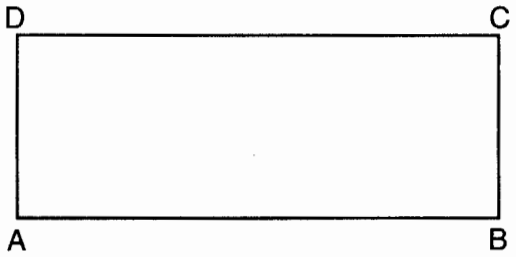
$\sqrt{(7-3)^2+(-8-0)^2}$

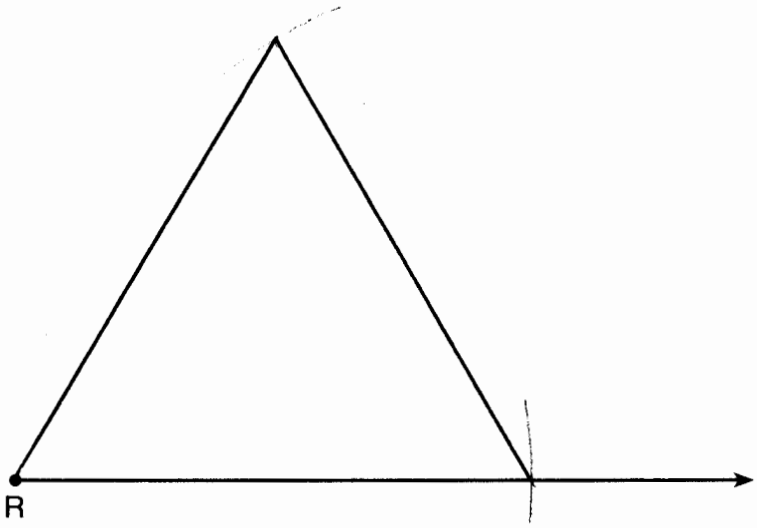
$\sqrt{16+64}$

$\sqrt{80}$

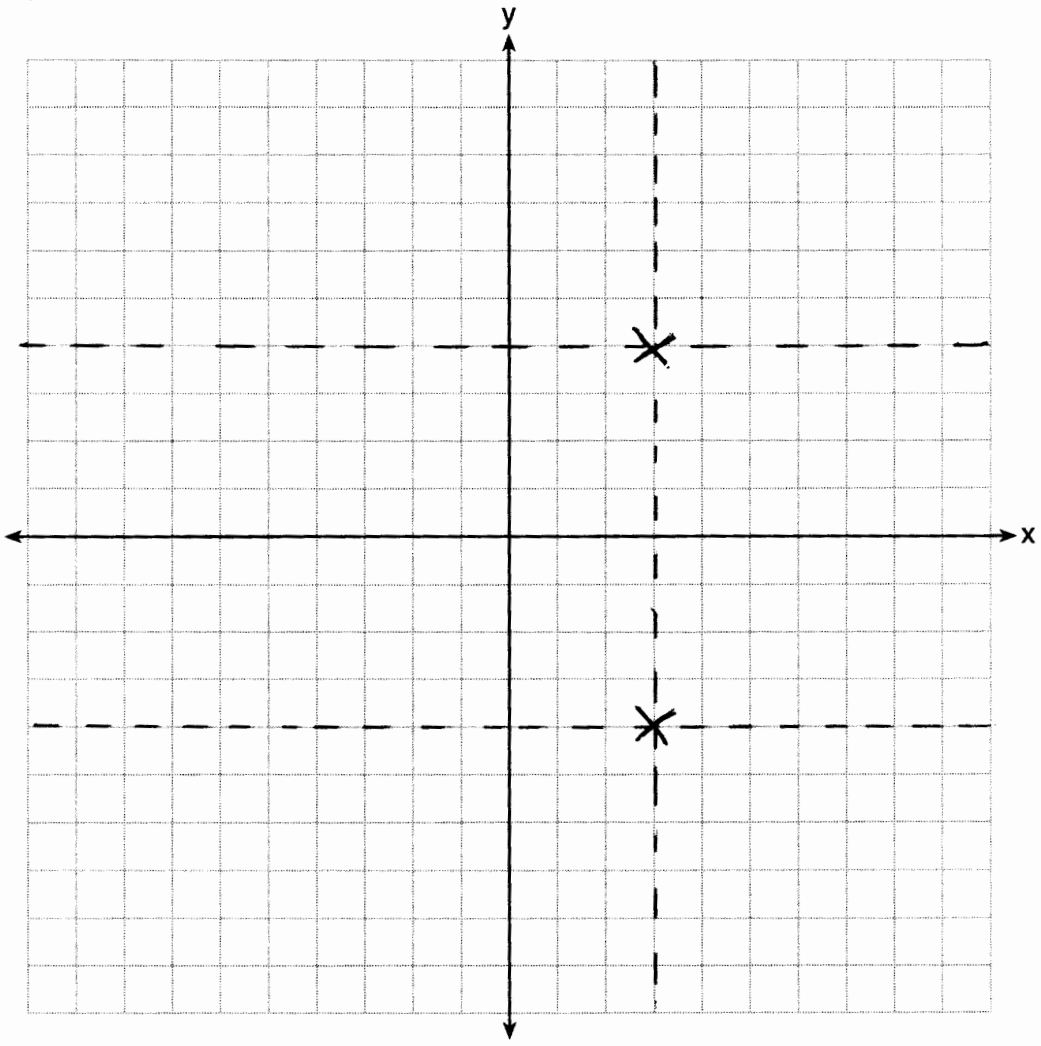
$4\sqrt{5}$

32 On the ray drawn below, using a compass and straightedge, construct an equilateral triangle with a vertex at R. The length of a side of the triangle must be equal to a length of the diagonal of rectangle $ABCD$.





33 On the set of axes below, graph the locus of points 4 units from the x-axis and equidistant from the points whose coordinates are $(-2,0)$ and $(8,0)$.
Mark with an **X** all points that satisfy *both* conditions.



34 The coordinates of two vertices of square $ABCD$ are $A(2,1)$ and $B(4,4)$. Determine the slope of side $\overline{BC}$.

$$m_{\overline{AB}} = \frac{4-1}{4-2} = \frac{3}{2}$$

$$m_{\overline{BC}} = \frac{-2}{3}$$

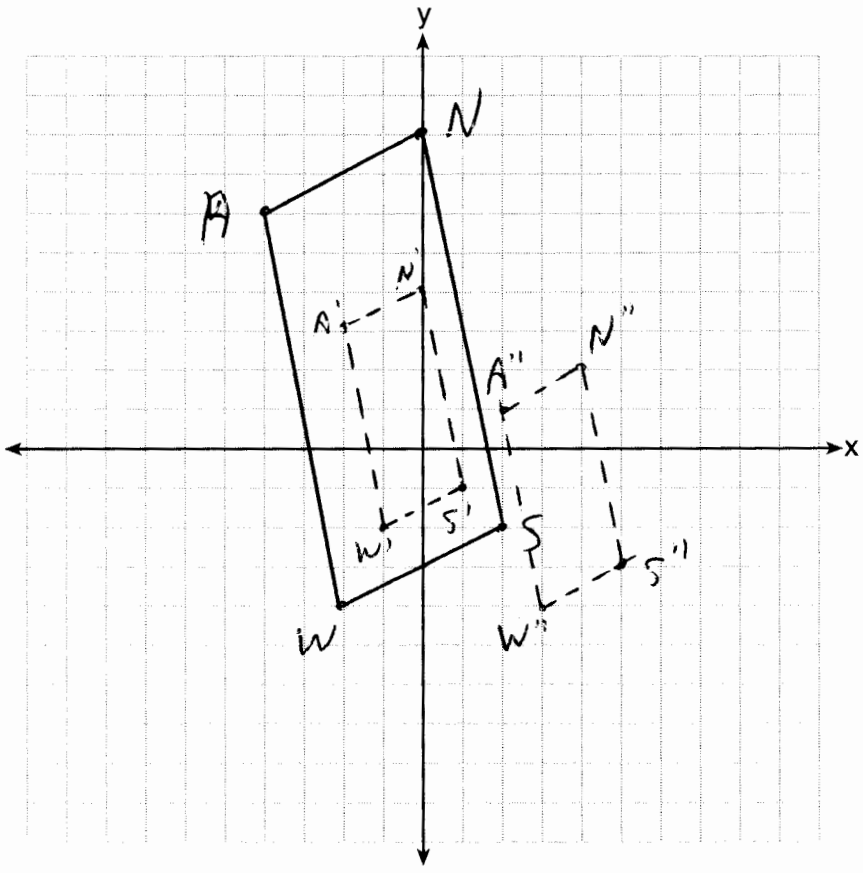
Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

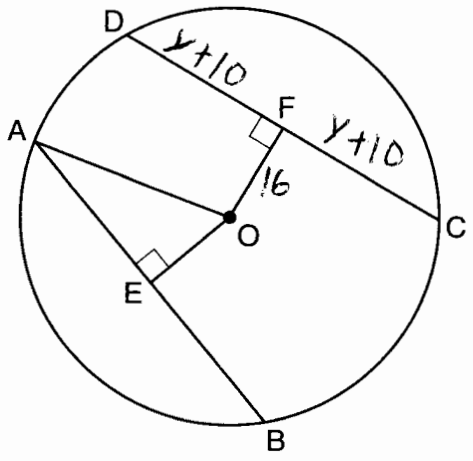
35 The coordinates of the vertices of parallelogram $SWAN$ are $S(2,-2)$, $W(-2,-4)$, $A(-4,6)$, and $N(0,8)$. State and label the coordinates of parallelogram $S''W''A''N''$, the image of $SWAN$ after the transformation $T_{4,-2} \circ D_{\frac{1}{2}}$. [The use of the set of axes below is optional.]

$S'(1,-1)$, $W'(-1,-2)$, $A'(-2,3)$, $N'(0,4)$

$S''(5,-3)$, $W''(3,-4)$, $A''(2,1)$, $N''(4,2)$



36 In circle O shown below, chords $\overline{AB}$ and $\overline{CD}$ and radius $\overline{OA}$ are drawn, such that $\overline{AB} \cong \overline{CD}$, $\overline{OE} \perp \overline{AB}$, $\overline{OF} \perp \overline{CD}$, $OF = 16$, $CF = y + 10$, and $CD = 4y - 20$.



Determine the length of $\overline{DF}$.

$2(y+10) = 4y-20$

$2y+20 = 4y-20$

$40 = 2y$

$20 = y$

$\overline{DF} = y+10$

$= 20+10$

$= 30$

Determine the length of $\overline{OA}$.

$\overline{OA} = \overline{OD} = \sqrt{16^2+30^2}$

$= 34$

37 If $\triangle RST \sim \triangle ABC$, $m\angle A = x^2 - 8x$, $m\angle C = 4x - 5$, and $m\angle R = 5x + 30$, find $m\angle C$.

[Only an algebraic solution can receive full credit.]

$x^2 - 8x = 5x + 30$

$x^2 - 13x - 30 = 0$

$(x - 15)(x + 2) = 0$

$x = 15$

$m\angle C = 4(15) - 5$

$= 60 - 5$

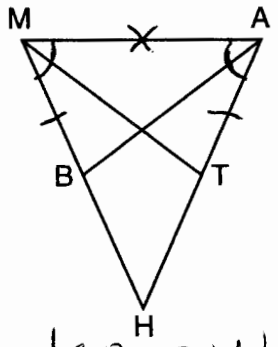
$= 55$

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

38 In the diagram of $\triangle MAH$ below, $\overline{MH} \cong \overline{AH}$ and medians $\overline{AB}$ and $\overline{MT}$ are drawn.

Prove: $\angle MBA \cong \angle ATM$



STATEMENT	REASON
(1) $\triangle MAH$, $\overline{MH} \cong \overline{AH}$, medians $\overline{AB}$ & $\overline{MT}$	(1) Given
(2) $\overline{MA} \cong \overline{AM}$	(2) Reflexive Property
(3) $\triangle MAH$ is an isosceles triangle	(3) Definition of isosceles triangle
(4) $\angle AMB \cong \angle MAT$	(4) Isosceles triangle theorem
(5) B is the midpoint of $\overline{MH}$ T is the midpoint of $\overline{AH}$	(5) Definition of median
(6) $m\overline{MB} = \frac{1}{2} m\overline{MH}$ $m\overline{AT} = \frac{1}{2} m\overline{AH}$	(6) Definition of midpoint
(7) $\overline{MB} \cong \overline{AT}$	(7) Multiplication postulate
(8) $\triangle MBA \cong \triangle ATM$	(8) SAS
(9) $\angle MBA \cong \angle ATM$	(9) CPCTC