The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Thursday, January 24, 2013 — 9:15 a.m. to 12:15 p.m., only

Student Name: Mr. Sibul

School Name: JMAP

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 38 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

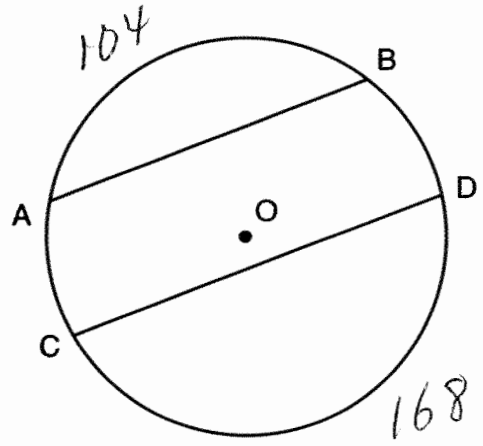
Answer all 28 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [56]

Use this space for computations.

1 If $\triangle MNP \cong \triangle VWX$ and $\overline{PM}$ is the shortest side of $\triangle MNP$, what is the shortest side of $\triangle VWX$?

(1) $\overline{XV}$
(2) $\overline{WX}$
(3) $\overline{VW}$
(4) $\overline{NP}$

2 In circle O shown in the diagram below, chords $\overline{AB}$ and $\overline{CD}$ are parallel.



360 − (104 + 168) = 88

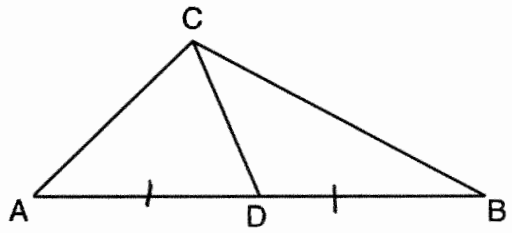
$\frac{88}{2} = 44$

If $m\overset{\frown}{AB} = 104$ and $m\overset{\frown}{CD} = 168$, what is $m\overset{\frown}{BD}$?

(1) 38
(2) 44
(3) 88
(4) 96

Use this space for computations.

3 As shown in the diagram below, $\overline{CD}$ is a median of $\triangle ABC$.



Which statement is *always* true?

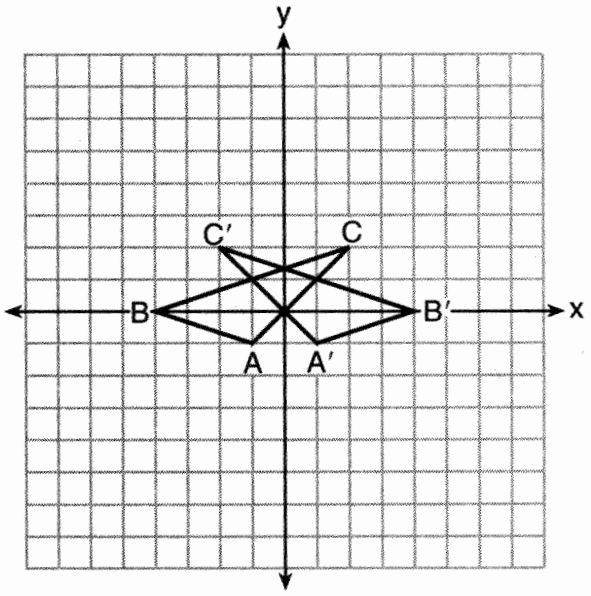
(1) $\overline{AD} \cong \overline{DB}$

(2) $\overline{AC} \cong \overline{AD}$

(3) $\angle ACD \cong \angle CDB$

(4) $\angle BCD \cong \angle ACD$

4 In the diagram below, under which transformation is $\triangle A'B'C'$ the image of $\triangle ABC$?



(1) D_2

(2) $r_{x\text{-axis}}$

(3) $r_{y\text{-axis}}$

(4) $(x,y) \rightarrow (x-2,y)$

Use this space for computations.

5 Line segment AB is a diameter of circle O whose center has coordinates (6,8). What are the coordinates of point B if the coordinates of point A are (4,2)?

(1) (1,3)
(3) (8,14)
(2) (5,5)
(4) (10,10)

$\frac{x+4}{2} = 6$

$x+4 = 12$

$x = 8$

$\frac{y+2}{2} = 8$

$y+2 = 16$

$y = 14$

6 Plane $\mathcal{A}$ and plane $\mathcal{B}$ are two distinct planes that are both perpendicular to line ℓ. Which statement about planes $\mathcal{A}$ and $\mathcal{B}$ is true?

(1) Planes $\mathcal{A}$ and $\mathcal{B}$ have a common edge, which forms a line.

(2) Planes $\mathcal{A}$ and $\mathcal{B}$ are perpendicular to each other.

(3) Planes $\mathcal{A}$ and $\mathcal{B}$ intersect each other at exactly one point.

(4) Planes $\mathcal{A}$ and $\mathcal{B}$ are parallel to each other.

7 Triangle ABC is similar to triangle DEF. The lengths of the sides of $\triangle ABC$ are 5, 8, and 11. What is the length of the shortest side of $\triangle DEF$ if its perimeter is 60?

(1) 10
(3) 20
(2) 12.5
(4) 27.5

perimeter $\triangle DEF = 5+8+11 = 24$

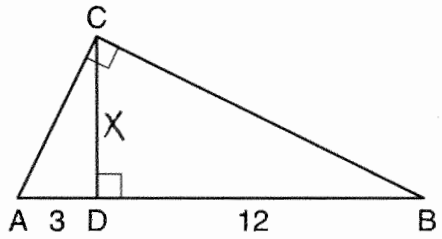
$\frac{5}{24} = \frac{x}{60}$

$24x = 300$

$x = 12.5$

Use this space for computations.

8 In the diagram below of right triangle ABC, altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.



$x^2 = 3 \cdot 12$

$x = 6$

If $AD = 3$ and $DB = 12$, what is the length of altitude $\overline{CD}$?

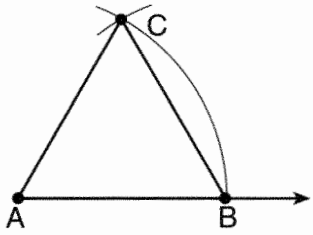
(1) 6

(2) $6\sqrt{5}$

(3) 3

(4) $3\sqrt{5}$

9 The diagram below shows the construction of an equilateral triangle.



Which statement justifies this construction?

(1) $\angle A + \angle B + \angle C = 180$

(2) $m\angle A = m\angle B = m\angle C$

(3) $AB = AC = BC$

(4) $AB + BC > AC$

10 What is the slope of the line perpendicular to the line represented by the equation $2x + 4y = 12$?

(1) -2

(2) 2

(3) $-\frac{1}{2}$

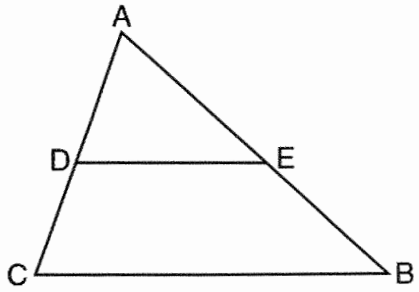
(4) $\frac{1}{2}$

$m = -\frac{A}{B} = -\frac{2}{4} = -\frac{1}{2}$

$m_\perp = 2$

Use this space for computations.

11 Triangle ABC is shown in the diagram below.



If $\overline{DE}$ joins the midpoints of $\overline{ADC}$ and $\overline{AEB}$, which statement is *not* true?

(1) $DE = \frac{1}{2}CB$

(2) $\overline{DE} \parallel \overline{CB}$

(3) $\frac{AD}{DC} = \frac{DE}{CB}$

(4) $\triangle ABC \sim \triangle AED$

12 The equations $x^2 + y^2 = 25$ and $y = 5$ are graphed on a set of axes. What is the solution of this system?

(1) (0,0)

(2) (5,0)

(3) (0,5)

(4) (5,5)

$x^2 + 5^2 = 25$

$x = 0$

13 Square $ABCD$ has vertices $A(-2,-3)$, $B(4,-1)$, $C(2,5)$, and $D(-4,3)$. What is the length of a side of the square?

(1) $2\sqrt{5}$

(2) $2\sqrt{10}$

(3) $4\sqrt{5}$

(4) $10\sqrt{2}$

$\sqrt{(-2-4)^2 + (-3-(-1))^2}$

$\sqrt{36+4}$

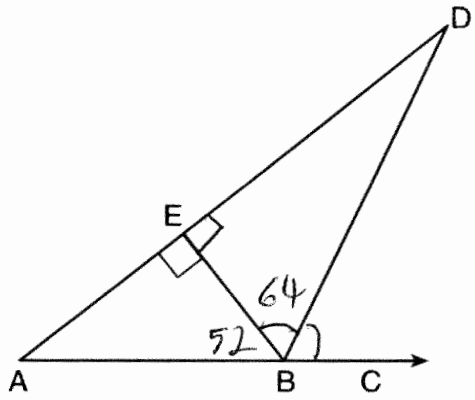
$\sqrt{40}$

$\sqrt{4}\sqrt{10}$

$2\sqrt{10}$

Use this space for computations.

14 The diagram below shows $\triangle ABD$, with $\overrightarrow{ABC}$, $\overline{BE} \perp \overline{AD}$, and $\angle EBD \cong \angle CBD$.



$180 - 52 = \frac{128}{2} = 64$

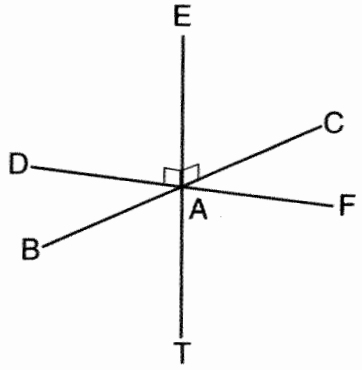
$180 - (90 + 64) = 26$

If $m\angle ABE = 52$, what is $m\angle D$?

(1) 26 (3) 52

(2) 38 (4) 64

15 As shown in the diagram below, $\overline{FD}$ and $\overline{CB}$ intersect at point A and $\overline{ET}$ is perpendicular to both $\overline{FD}$ and $\overline{CB}$ at A.



Which statement is *not* true?

(1) $\overline{ET}$ is perpendicular to plane BAD.

(2) $\overline{ET}$ is perpendicular to plane FAB.

(3) $\overline{ET}$ is perpendicular to plane CAD.

(4) $\overline{ET}$ is perpendicular to plane BAT.

Use this space for computations.

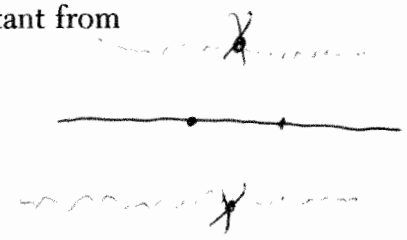
16 Which set of numbers could *not* represent the lengths of the sides of a right triangle?

(1) $\{1,3,\sqrt{10}\}$
(2) $\{2,3,4\}$
(3) $\{3,4,5\}$
(4) $\{8,15,17\}$

$2^2+3^2 \neq 4^2$

17 How many points are 5 units from a line and also equidistant from two points on the line?

(1) 1
(2) 2
(3) 3
(4) 0



18 The equation of a circle is $(x-2)^2+(y+5)^2=32$. What are the coordinates of the center of this circle and the length of its radius?

(1) $(-2,5)$ and 16
(2) $(2,-5)$ and 16
(3) $(-2,5)$ and $4\sqrt{2}$
(4) $(2,-5)$ and $4\sqrt{2}$

$\sqrt{32} = \sqrt{16}\sqrt{2} = 4\sqrt{2}$

19 The equation of a line is $y=\frac{2}{3}x+5$. What is an equation of the line that is perpendicular to the given line and that passes through the point (4,2)?

(1) $y=\frac{2}{3}x-\frac{2}{3}$
(2) $y=\frac{3}{2}x-4$
(3) $y=-\frac{3}{2}x+7$
(4) $y=-\frac{3}{2}x+8$

$m = \frac{2}{3}$

$m_\perp = -\frac{3}{2}$

$y = mx+b$
$2 = -\frac{3}{2}(4)+b$
$2 = -6+b$
$8 = b$

Use this space for computations.

20 Consider the relationship between the two statements below.

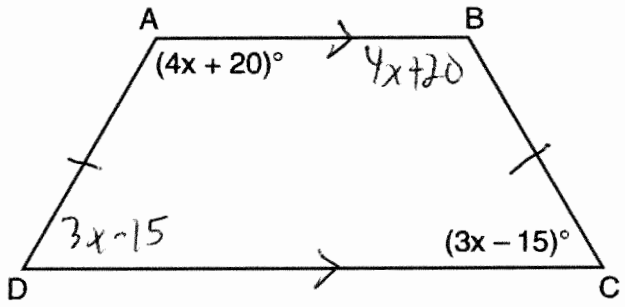
If $\sqrt{16 + 9} \neq 4 + 3$, then $5 \neq 4 + 3$.

If $\sqrt{16 + 9} = 4 + 3$, then $5 = 4 + 3$.

These statements are

(1) inverses
(2) converses
(3) contrapositives
(4) biconditionals

21 In the diagram of trapezoid $ABCD$ below, $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \cong \overline{BC}$, $m\angle A = 4x + 20$, and $m\angle C = 3x - 15$.



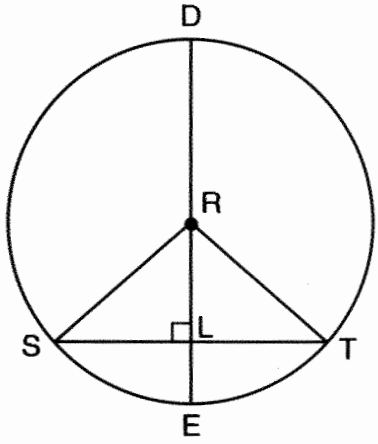
What is $m\angle D$?

(1) 25
(2) 35
(3) 60
(4) 90

$2(4x+20) + 2(3x-15) = 360$
$8x + 40 + 6x - 30 = 360$
$14x + 10 = 360$
$14x = 350$
$x = 25$
$\angle D = 3(25) - 15 = 60$

Use this space for computations.

22 In circle R shown below, diameter $\overline{DE}$ is perpendicular to chord $\overline{ST}$ at point L.



Which statement is *not* always true?

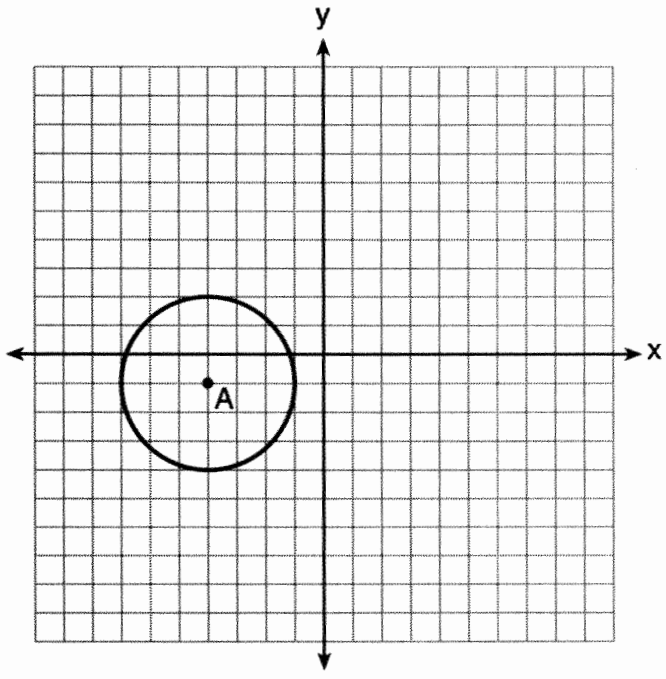
(1) $\overline{SL} \cong \overline{TL}$

(2) $RS = DR$

(3) $\overline{RL} \cong \overline{LE}$

(4) $(DL)(LE) = (SL)(LT)$

23 Which equation represents circle A shown in the diagram below?



(1) $(x - 4)^2 + (y - 1)^2 = 3$

(2) $(x + 4)^2 + (y + 1)^2 = 3$

(3) $(x - 4)^2 + (y - 1)^2 = 9$

(4) $(x + 4)^2 + (y + 1)^2 = 9$

Use this space for computations.

24 Which equation represents a line that is parallel to the line whose equation is $3x - 2y = 7$?

(1) $y = -\frac{3}{2}x + 5$

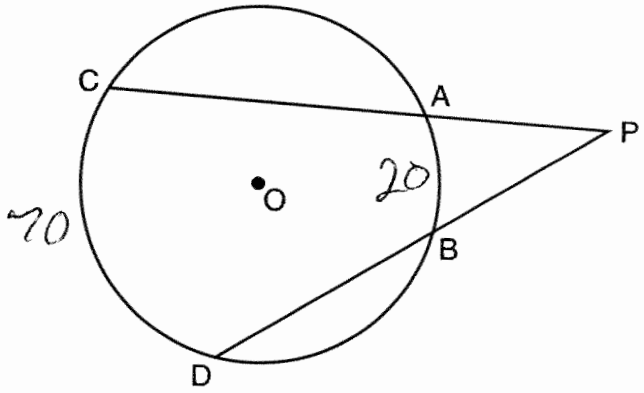
(3) $y = \frac{3}{2}x - 5$

(2) $y = -\frac{2}{3}x + 4$

(4) $y = \frac{2}{3}x - 4$

$m = -\frac{A}{B} = \frac{-3}{-2} = \frac{3}{2}$

25 In the diagram below of circle O, $\overline{PAC}$ and $\overline{PBD}$ are secants.



$\frac{70 - 20}{2} = 25$

If $m\overset{\frown}{CD} = 70$ and $m\overset{\frown}{AB} = 20$, what is the degree measure of $\angle P$?

(1) 25

(3) 45

(2) 35

(4) 50

26 The measure of an interior angle of a regular polygon is 120°. How many sides does the polygon have?

(1) 5

(3) 3

(2) 6

(4) 4

$\frac{(n-2)180}{n} = 120$

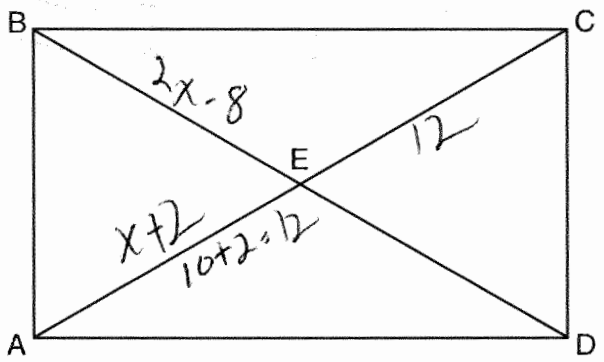
$180n - 360 = 120n$

$60n = 360$

$n = 6$

Use this space for computations.

27 As shown in the diagram of rectangle $ABCD$ below, diagonals $\overline{AC}$ and $\overline{BD}$ intersect at E.



$2x - 8 = x + 2$
$x = 10$

If $AE = x + 2$ and $BD = 4x - 16$, then the length of $\overline{AC}$ is

(1) 6
(2) 10
(3) 12
(4) 24

28 If the vertices of $\triangle ABC$ are $A(-2,4)$, $B(-2,8)$, and $C(-5,6)$, then $\triangle ABC$ is classified as

(1) right
(2) scalene
(3) isosceles
(4) equilateral

$\overline{AB} = 4$

$\overline{BC} = \sqrt{(-2-(-5))^2+(8-6)^2} = \sqrt{13}$

$AC = \sqrt{(-2-(-5))^2+(4-6)^2} = \sqrt{13}$

Part II

Answer all 6 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

29 After the transformation $r_{y=x}$, the image of $\triangle ABC$ is $\triangle A'B'C'$. If $AB = 2x + 13$ and $A'B' = 9x - 8$, find the value of x.

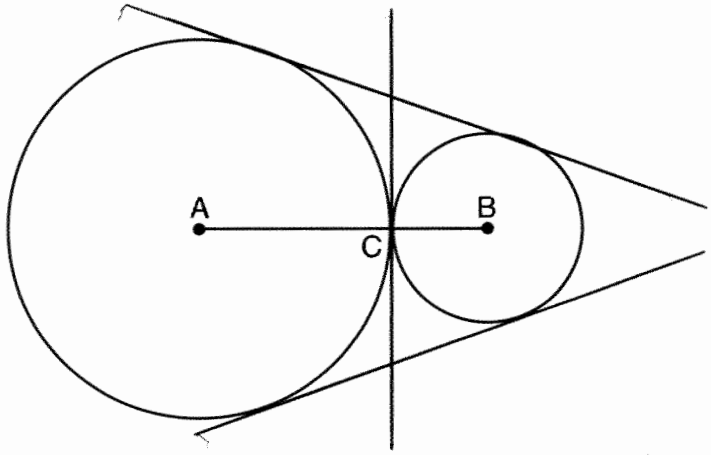
Distance is preserved

$2x + 13 = 9x - 8$

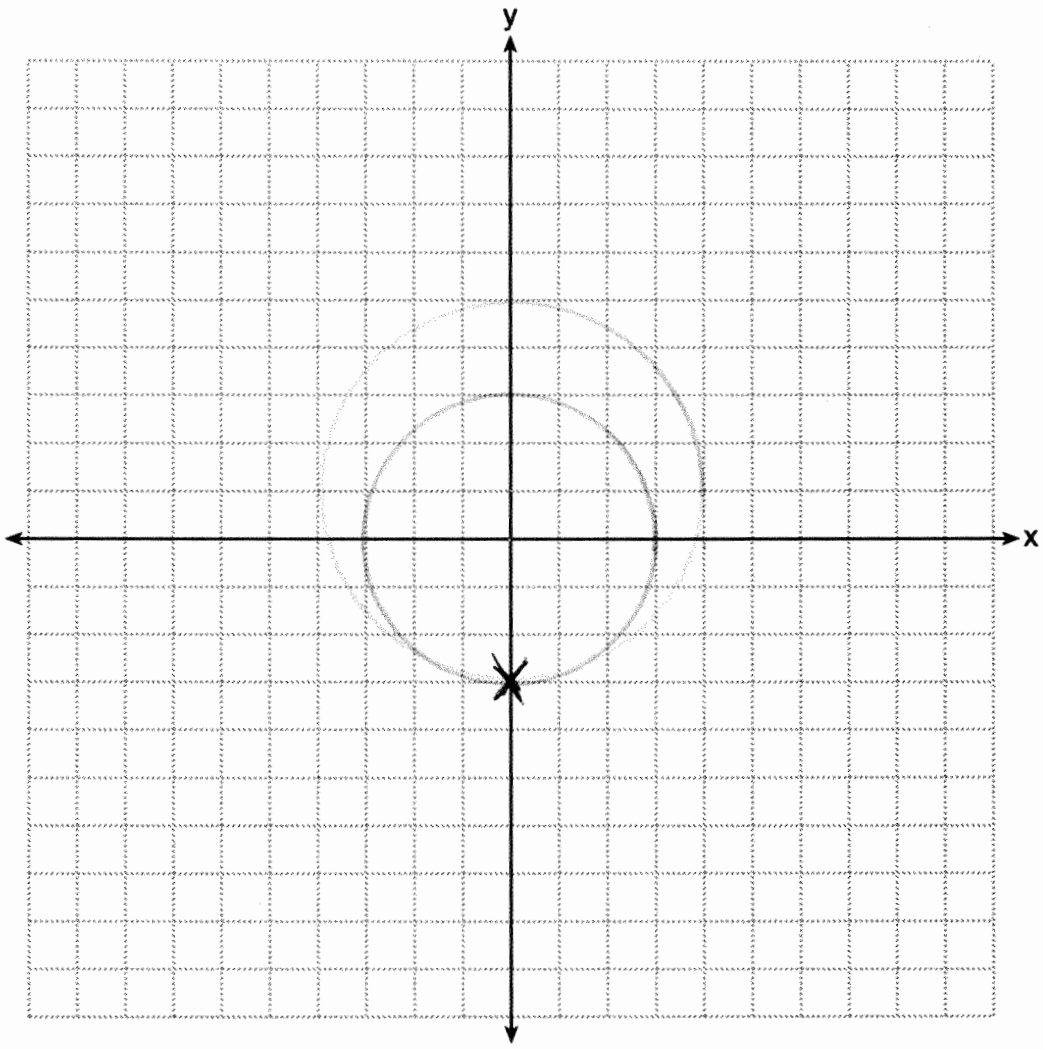
$21 = 7x$

$3 = x$

30 In the diagram below, circles A and B are tangent at point C and $\overline{AB}$ is drawn. Sketch all common tangent lines.



31 On the set of axes below, graph the locus of points 4 units from (0,1) and the locus of points 3 units from the origin. Label with an **X** *any* points that satisfy *both* conditions.



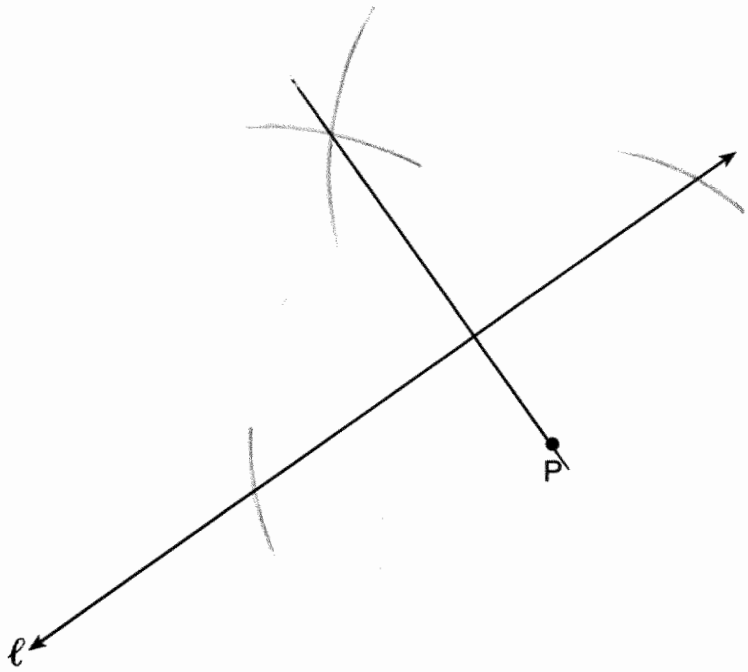
32 Write an equation of a circle whose center is $(-3,2)$ and whose diameter is 10.

$r = 5$

$r^2 = 25$

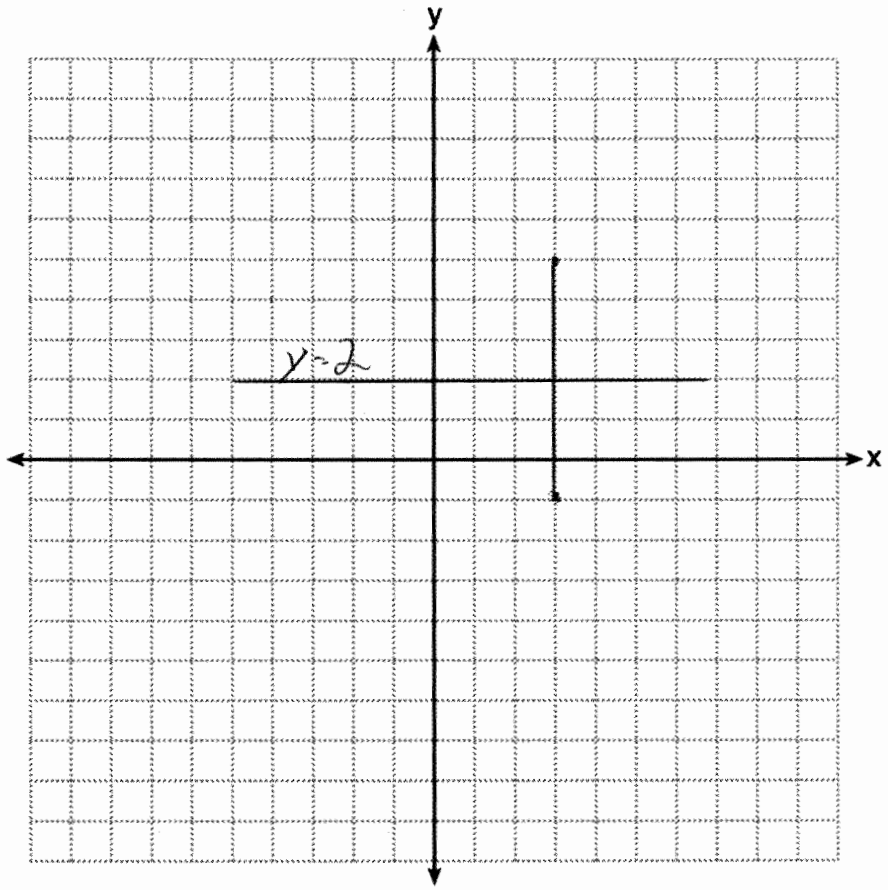
$(x+3)^2 + (y-2)^2 = 25$

33 Using a compass and straightedge, construct a line perpendicular to line ℓ through point P. [Leave all construction marks.]



34 Write an equation of the line that is the perpendicular bisector of the line segment having endpoints (3,−1) and (3,5). [The use of the grid below is optional.]

$M = \left(\frac{3+3}{2}, \frac{-1+5}{2}\right) = (3,2)$



Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

35 A right circular cylinder with a height of 5 cm has a base with a diameter of 6 cm. Find the lateral area of the cylinder to the *nearest hundredth of a square centimeter*.

$L = 2\pi rh = 2\pi(3)(5) \cong 94.25$

Find the volume of the cylinder to the *nearest hundredth of a cubic centimeter*.

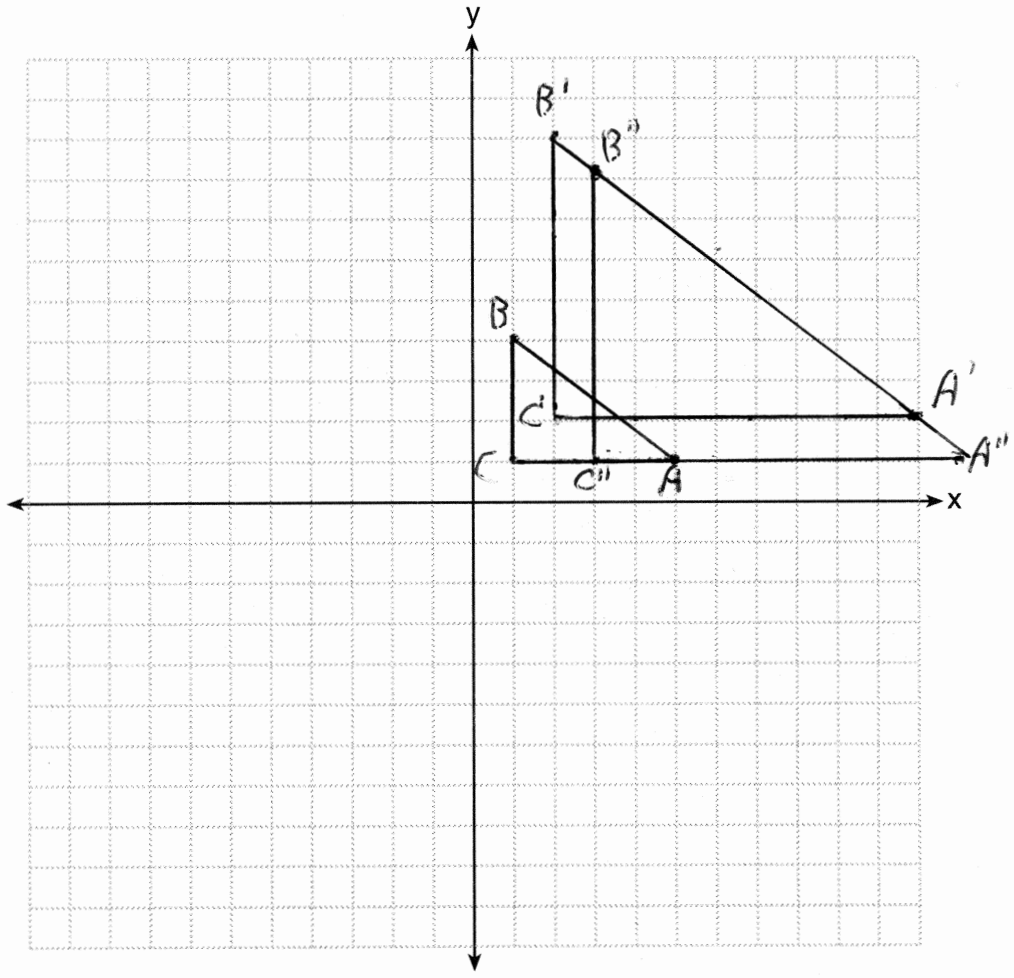
$V = \pi r^2 h = \pi(3)^2(5) \cong 141.37$

36 Triangle ABC has vertices $A(5,1)$, $B(1,4)$, and $C(1,1)$.

State and label the coordinates of the vertices of $\triangle A''B''C''$, the image of $\triangle ABC$, following the composite transformation $T_{1,-1} \circ D_2$. [The use of the set of axes below is optional.]

A'(10,2), B'(2,8), C'(2,2)

A''(11,1), B''(3,7), C''(3,1)



37 In $\triangle ABC$, $m\angle A = x^2 + 12$, $m\angle B = 11x + 5$, and $m\angle C = 13x - 17$. Determine the longest side of $\triangle ABC$.

$x^2+12+11x+5+13x-17=180$

$x^2+24x-180=0$

$(x+30)(x-6)=0$

$x=6$

$m\angle A = 6^2+12=48$

$m\angle B = 11(6)+5=71$

$m\angle C = 13(6)-17=61$

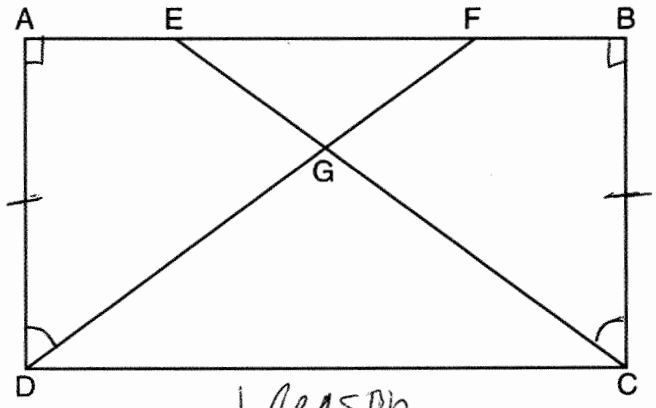
$\angle B$ is largest angle, so AC is longest side.

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

38 The diagram below shows rectangle $ABCD$ with points E and F on side $\overline{AB}$. Segments CE and DF intersect at G, and $\angle ADG \cong \angle BCG$.

Prove that $\overline{AE} \cong \overline{BF}$.



Statement	Reason
1) Rectangle ABCD with points E & F on side $\overline{AB}$; $\overline{CE}$ & $\overline{DF}$ intersect at G; $\angle ADG \cong \angle BCG$	1) Given
2) $\overline{AD} \cong \overline{BC}$	2) Opposite sides of a rectangle are congruent
3) $\angle A$ & $\angle B$ are right angles	3) All angles of a rectangle are right angles
4) $\angle A \cong \angle B$	4) All right angles are congruent
5) $\triangle ADF \cong \triangle BCE$	5) ASA
6) $\overline{AF} \cong \overline{BE}$	6) CPCTC
7) $\overline{EF} \cong \overline{FE}$	7) Reflexive Property
8) $\overline{AF} - \overline{EF} \cong \overline{BE} - \overline{FE}$	8) Subtraction Property of Segments
9) $\overline{AE} \cong \overline{BF}$	9) Definition of Segments