The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Thursday, January 27, 2011 — 9:15 a.m. to 12:15 p.m., only

Student Name: Mr. Sibol

School Name: HSCR

Print your name and the name of your school on the lines above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

This examination has four parts, with a total of 38 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

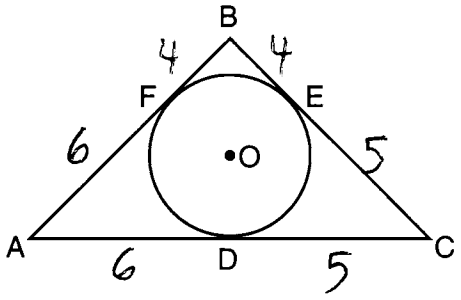
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 28 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [56]

Use this space for computations.

1 In the diagram below, $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ are tangents to circle O at points F, E, and D, respectively, $AF = 6$, $CD = 5$, and $BE = 4$.



What is the perimeter of $\triangle ABC$?

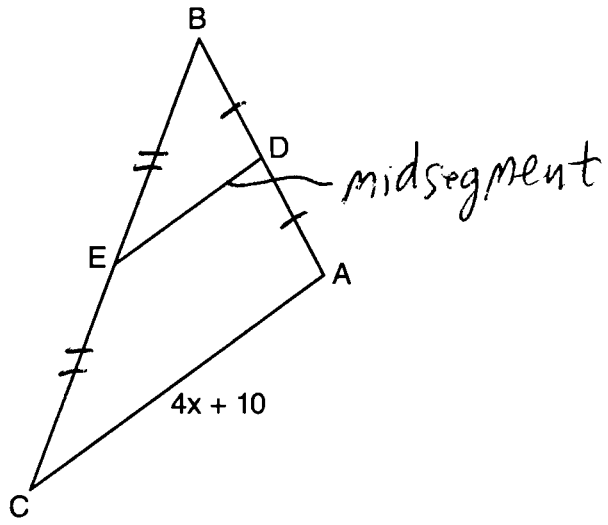
(1) 15
(2) 25
(3) 30
(4) 60

2 Quadrilateral $MNOP$ is a trapezoid with $\overline{MN} \parallel \overline{OP}$. If $M'N'O'P'$ is the image of $MNOP$ after a reflection over the x-axis, which two sides of quadrilateral $M'N'O'P'$ are parallel?

(1) $\overline{M'N'}$ and $\overline{O'P'}$
(2) $\overline{M'N'}$ and $\overline{N'O'}$
(3) $\overline{P'M'}$ and $\overline{O'P'}$
(4) $\overline{P'M'}$ and $\overline{N'O'}$

Use this space for computations.

3 In the diagram below of $\triangle ABC$, D is the midpoint of $\overline{AB}$, and E is the midpoint of $\overline{BC}$.



If $AC = 4x + 10$, which expression represents DE?

$\frac{4x+10}{2} = 2x+5$

(1) $x + 2.5$

(3) $2x + 10$

(2) $2x + 5$

(4) $8x + 20$

4 Which statement is true about every parallelogram?

(1) All four sides are congruent.

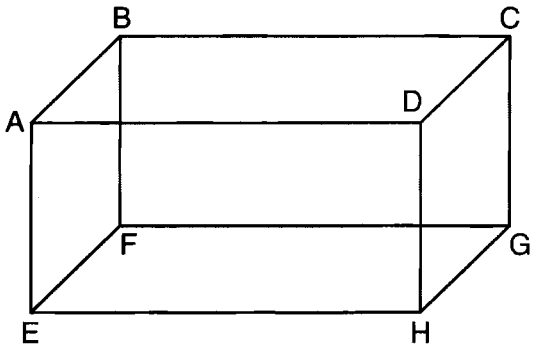
(2) The interior angles are all congruent.

(3) Two pairs of opposite sides are congruent.

(4) The diagonals are perpendicular to each other.

Use this space for computations.

5 The diagram below shows a rectangular prism.



Which pair of edges are segments of lines that are coplanar?

(1) $\overline{AB}$ and $\overline{DH}$

(3) $\overline{BC}$ and $\overline{EH}$

(2) $\overline{AE}$ and $\overline{DC}$

(4) $\overline{CG}$ and $\overline{EF}$

6 A line segment has endpoints $A(7,-1)$ and $B(-3,3)$. What are the coordinates of the midpoint of $\overline{AB}$?

(1) (1,2)

(3) (−5,2)

(2) (2,1)

(4) (5,−2)

$\left(\frac{7+-3}{2}, \frac{-1+3}{2}\right)$

$(2,1)$

Use this space for computations.

7 What is the image of the point $(-5,2)$ under the translation $T_{3,-4}$?

(1) $(-9,5)$
(2) $(-8,6)$
(3) $(-2,-2)$
(4) $(-15,-8)$

8 When writing a geometric proof, which angle relationship could be used alone to justify that two angles are congruent?

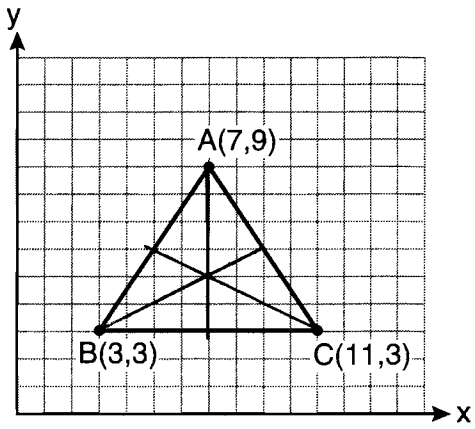
(1) supplementary angles
(2) linear pair of angles
(3) adjacent angles
(4) vertical angles

9 Plane $\mathcal{R}$ is perpendicular to line k and plane $\mathcal{D}$ is perpendicular to line k. Which statement is correct?

(1) Plane $\mathcal{R}$ is perpendicular to plane $\mathcal{D}$.
(2) Plane $\mathcal{R}$ is parallel to plane $\mathcal{D}$.
(3) Plane $\mathcal{R}$ intersects plane $\mathcal{D}$.
(4) Plane $\mathcal{R}$ bisects plane $\mathcal{D}$.

Use this space for computations.

10 The vertices of the triangle in the diagram below are $A(7,9)$, $B(3,3)$, and $C(11,3)$.



What are the coordinates of the centroid of $\triangle ABC$?

(1) (5,6)
(3) (7,5)
(2) (7,3)
(4) (9,6)

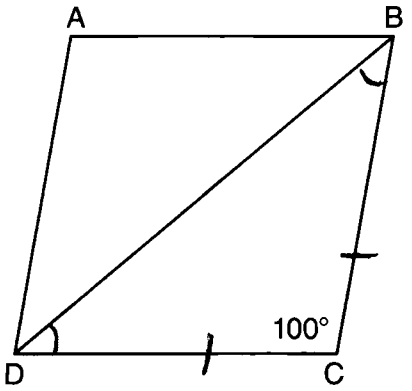
11 Which set of numbers does *not* represent the sides of a right triangle?

(1) {6, 8, 10}
(3) {8, 24, 25}
(2) {8, 15, 17}
(4) {15, 36, 39}

$8^2+24^2 \neq 25^2$

Use this space for computations.

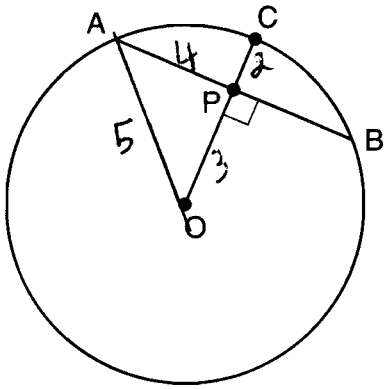
12 In the diagram below of rhombus $ABCD$, $m\angle C = 100$.



What is $m\angle DBC$?

(1) 40
(2) 45
(3) 50
(4) 80

13 In the diagram below of circle O, radius $\overline{OC}$ is 5 cm. Chord $\overline{AB}$ is 8 cm and is perpendicular to $\overline{OC}$ at point P.



What is the length of $\overline{OP}$, in centimeters?

(1) 8
(2) 2
(3) 3
(4) 4

Use this space for computations.

14 What is an equation of the line that passes through the point $(-2,3)$ and is parallel to the line whose equation is $y = \frac{3}{2}x - 4$?

$y = mx + b$

$3 = (\frac{3}{2})(-2) + b$

$3 = -3 + b$

$6 = b$

(1) $y = \frac{-2}{3}x$

(2) $y = \frac{-2}{3}x + \frac{5}{3}$

(3) $y = \frac{3}{2}x$

(4) $y = \frac{3}{2}x + 6$

15 In scalene triangle ABC, $m\angle B = 45$ and $m\angle C = 55$. What is the order of the sides in length, from longest to shortest?

$m\angle A = 80$

(1) $\overline{AB}, \overline{BC}, \overline{AC}$

(2) $\overline{BC}, \overline{AC}, \overline{AB}$

(3) $\overline{AC}, \overline{BC}, \overline{AB}$

(4) $\overline{BC}, \overline{AB}, \overline{AC}$

16 What is an equation of a circle with center $(7,-3)$ and radius 4?

(1) $(x - 7)^2 + (y + 3)^2 = 4$

(2) $(x + 7)^2 + (y - 3)^2 = 4$

(3) $(x - 7)^2 + (y + 3)^2 = 16$

(4) $(x + 7)^2 + (y - 3)^2 = 16$

Use this space for computations.

17 What is the volume, in cubic centimeters, of a cylinder that has a height of 15 cm and a diameter of 12 cm?

(1) 180π
(3) 675π
(2) 540π
(4) $2{,}160\pi$

$V = \pi r^2 h$
$= \pi \cdot 6^2 \cdot 15$
$= 540\pi$

18 Which compound statement is true?

(1) A triangle has three sides and a quadrilateral has five sides.
(2) A triangle has three sides if and only if a quadrilateral has five sides.
(3) If a triangle has three sides, then a quadrilateral has five sides.
(4) A triangle has three sides or a quadrilateral has five sides.

19 The two lines represented by the equations below are graphed on a coordinate plane.

$$x + 6y = 12$$
$$3(x - 2) = -y - 4$$

Which statement best describes the two lines?

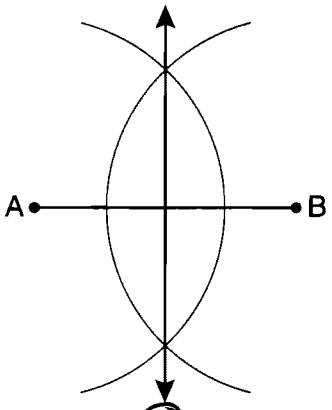
(1) The lines are parallel.
(2) The lines are the same line.
(3) The lines are perpendicular.
(4) The lines intersect at an angle other than 90°.

$x + 6y = 12$
$6y = -x + 12$
$y = -\frac{1}{6}x + 2$
$m = -\frac{1}{6}$

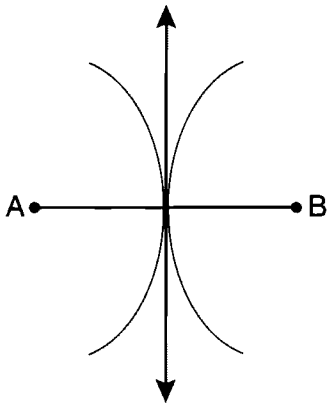
$3x - 6 = -y - 4$
$y = -3x + 6$
$m = -3$

Use this space for computations.

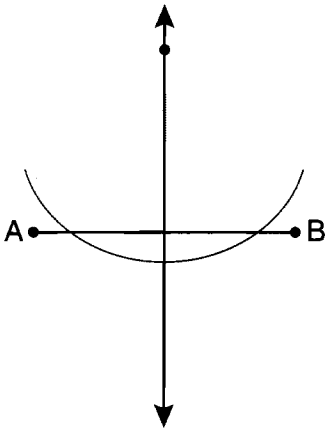
20 Which diagram shows the construction of the perpendicular bisector of $\overline{AB}$?



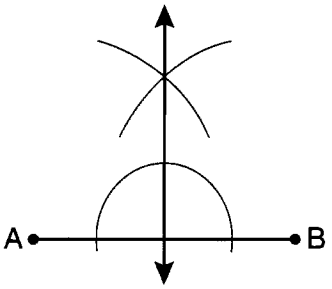
(1)



(3)



(2)



(4)

21 In circle O, a diameter has endpoints $(-5,4)$ and $(3,-6)$. What is the length of the diameter?

(1) $\sqrt{2}$

(2) $2\sqrt{2}$

(3) $\sqrt{10}$

(4) $2\sqrt{41}$

$\sqrt{(-5-3)^2+(4--6)^2}$

$\sqrt{64+100}$

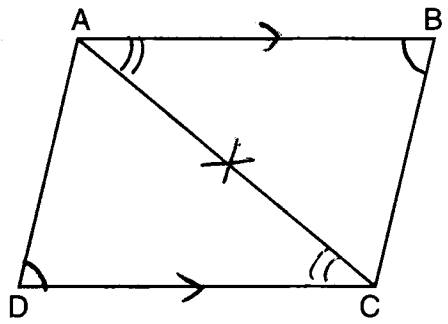
$\sqrt{164}$

$\sqrt{4}\sqrt{41}$

$2\sqrt{41}$

Use this space for computations.

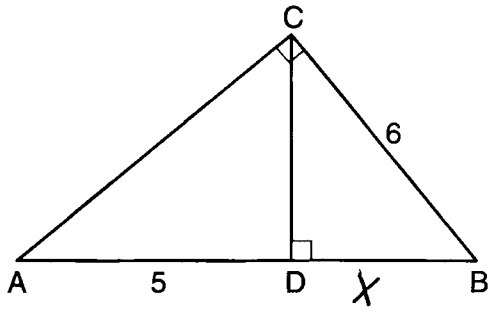
22 In the diagram below of quadrilateral $ABCD$, $\overline{AB} \parallel \overline{CD}$, $\angle ABC \cong \angle CDA$, and diagonal $\overline{AC}$ is drawn.



Which method can be used to prove that $\triangle ABC$ is congruent to $\triangle CDA$?

(1) AAS
(2) SSA
(3) SAS
(4) SSS

23 In the diagram below of right triangle ABC, $\overline{CD}$ is the altitude to hypotenuse $\overline{AB}$, $CB = 6$, and $AD = 5$.



What is the length of $\overline{BD}$?

(1) 5
(2) 9
(3) 3
(4) 4

$6^2 = x(x+5)$

$36 = x^2 + 5x$

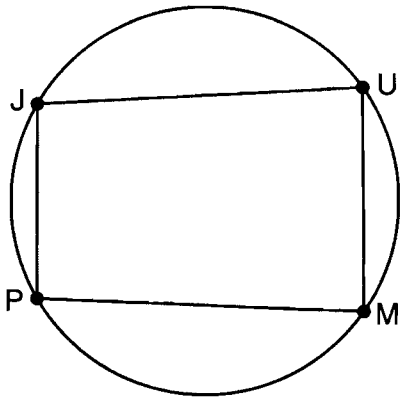
$0 = x^2 + 5x - 36$

$0 = (x+9)(x-4)$

$-9 \quad 4$

Use this space for computations.

24 In the diagram below, quadrilateral *JUMP* is inscribed in a circle.



Opposite angles *J* and *M* must be

(1) right

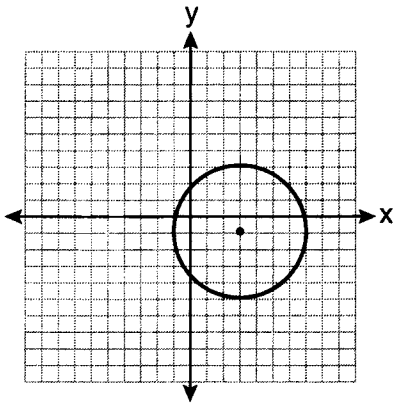
(2) complementary

(3) congruent

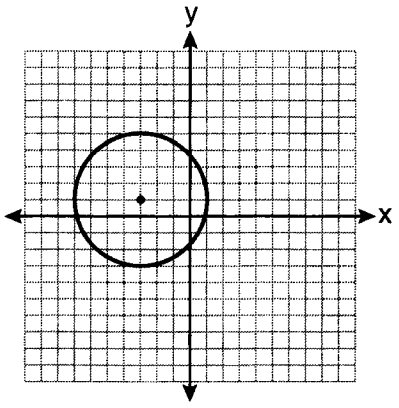
(4) supplementary

Use this space for computations.

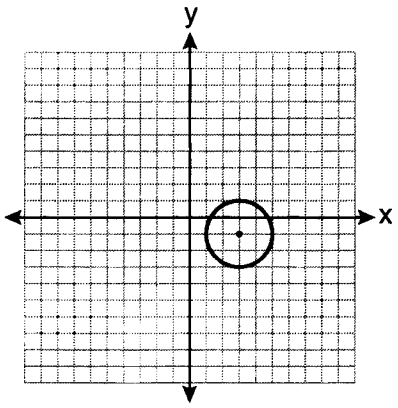
25 Which graph represents a circle with the equation $(x - 3)^2 + (y + 1)^2 = 4$?



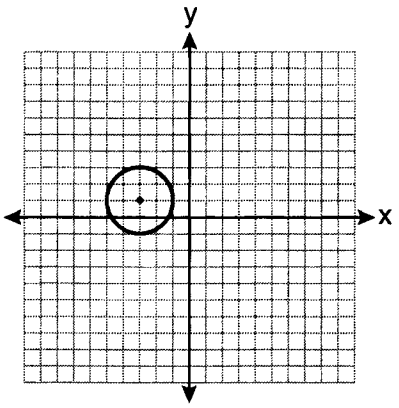
(1)



(3)



(2)



(4)

$(3,-2) \rightarrow (2,3) \rightarrow (8,12)$

26 The point $(3,-2)$ is rotated 90° about the origin and then dilated by a scale factor of 4. What are the coordinates of the resulting image?

(1) $(-12,8)$

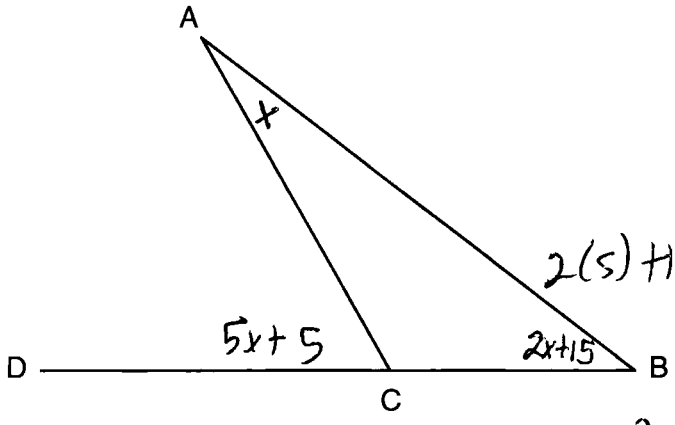
(2) $(12,-8)$

(3) $(8,12)$

(4) $(-8,-12)$

Use this space for computations.

27 In the diagram below of $\triangle ABC$, side $\overline{BC}$ is extended to point D, $m\angle A = x$, $m\angle B = 2x + 15$, and $m\angle ACD = 5x + 5$.



2(5)+15 = 25

$x + 2x + 15 = 5x + 5$

$3x + 15 = 5x + 5$

$10 = 2x$

$5 = x$

What is $m\angle B$?

(1) 5

(2) 20

(3) 25

(4) 55

28 Point P lies on line m. Point P is also included in distinct planes Q, R, S, and T. At most, how many of these planes could be perpendicular to line m?

(1) 1

(2) 2

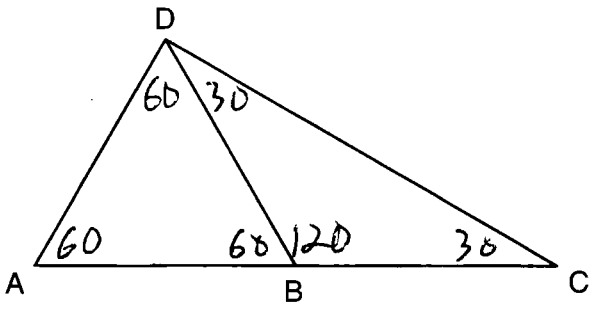
(3) 3

(4) 4

Part II

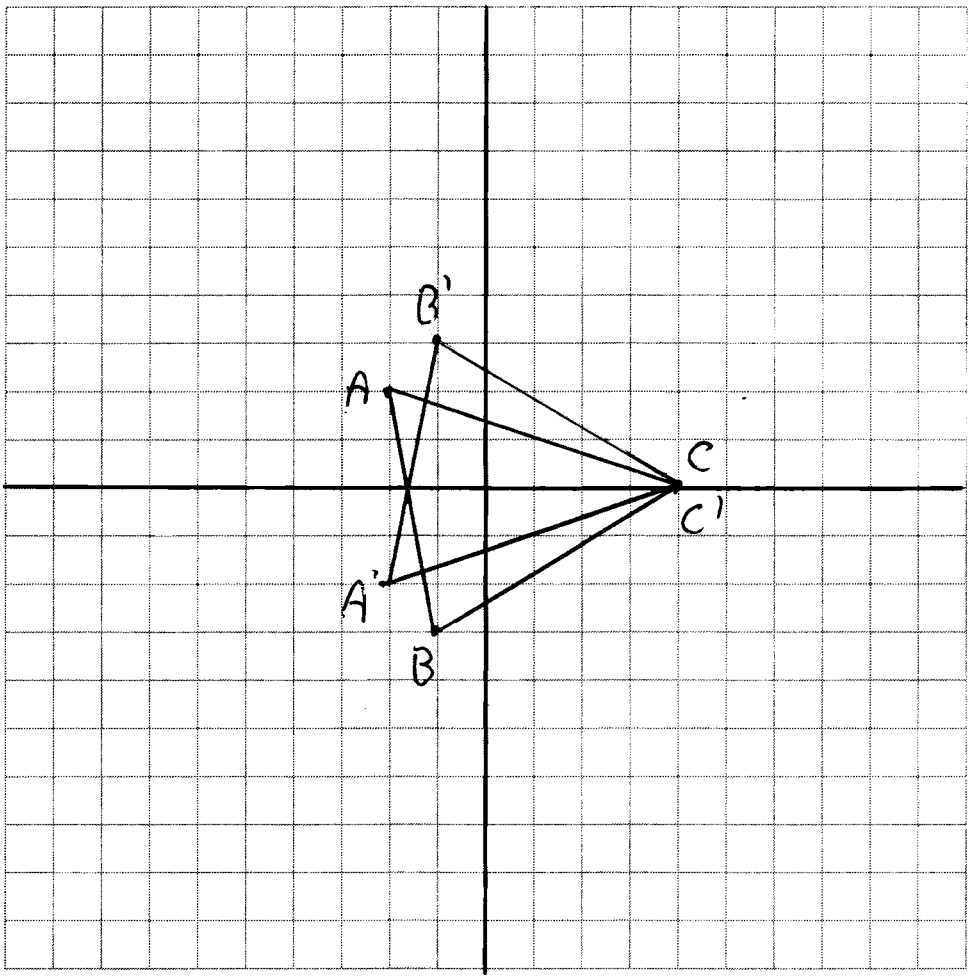
Answer all 6 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

29 In the diagram below of $\triangle ACD$, B is a point on $\overline{AC}$ such that $\triangle ADB$ is an equilateral triangle, and $\triangle DBC$ is an isosceles triangle with $\overline{DB} \cong \overline{BC}$. Find $m\angle C$.



30 Triangle ABC has vertices $A(-2,2)$, $B(-1,-3)$, and $C(4,0)$. Find the coordinates of the vertices of $\triangle A'B'C'$, the image of $\triangle ABC$ after the transformation $r_{x\text{-axis}}$.
[The use of the grid below is optional.]

$A'(-2,-2), B'(-1,3), C'(4,0)$

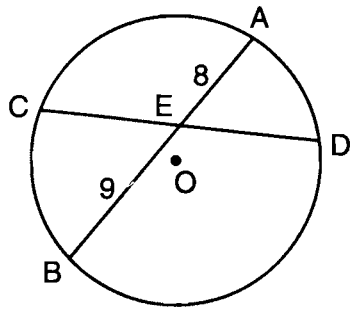


31 Find, in degrees, the measures of both an interior angle and an exterior angle of a regular pentagon.

$(n-2)180 = (5-2)(180) = \frac{540}{5} = 108$ interior

$180 - 108 = 72$ exterior

32 In the diagram below of circle O, chord $\overline{AB}$ bisects chord $\overline{CD}$ at E. If $AE = 8$ and $BE = 9$, find the length of $\overline{CE}$ in simplest radical form.



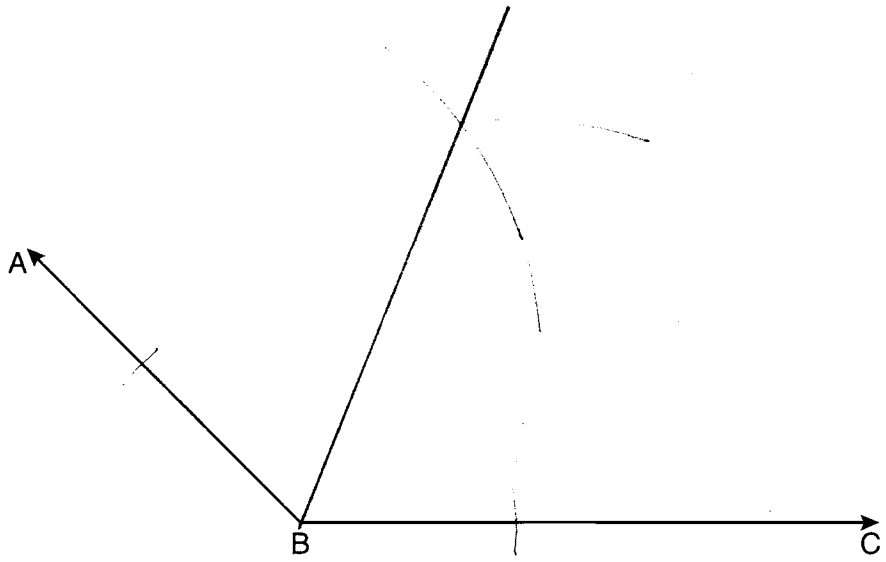
$x^2 = 9 \cdot 8$

$x = \sqrt{72}$

$x = \sqrt{36}\sqrt{2}$

$x = 6\sqrt{2}$

33 On the diagram below, use a compass and straightedge to construct the bisector of $\angle ABC$. [Leave all construction marks.]



34 Find the slope of a line perpendicular to the line whose equation is $2y - 6x = 4$.

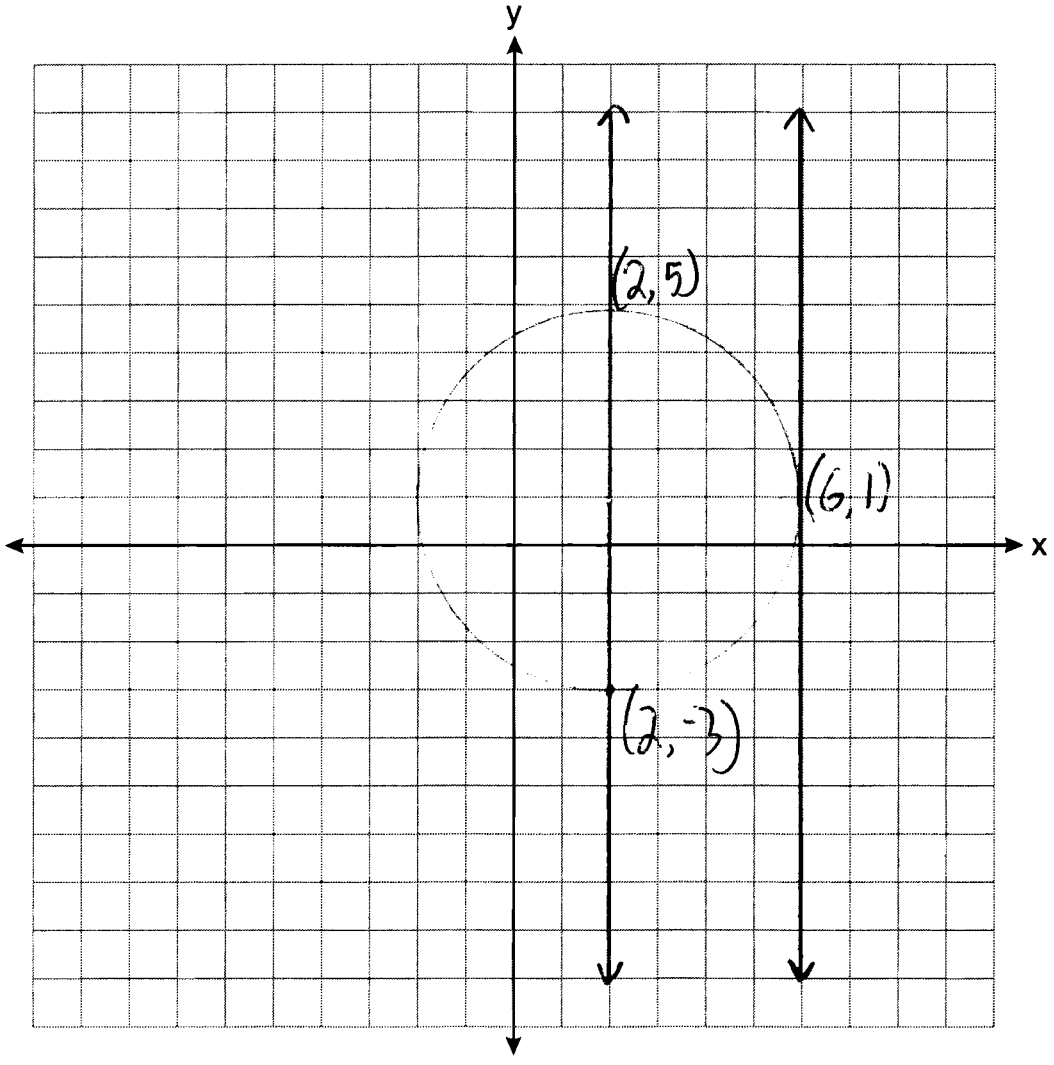
$$m = \frac{-A}{B} = \frac{6}{2} = 3$$

$$m_{\perp} = -\frac{1}{3}$$

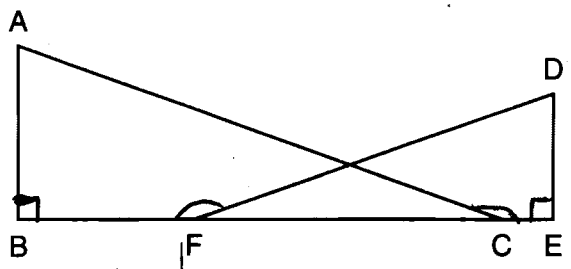
Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

35 On the set of axes below, graph the locus of points that are four units from the point (2,1). On the same set of axes, graph the locus of points that are two units from the line $x = 4$. State the coordinates of all points that satisfy both conditions.

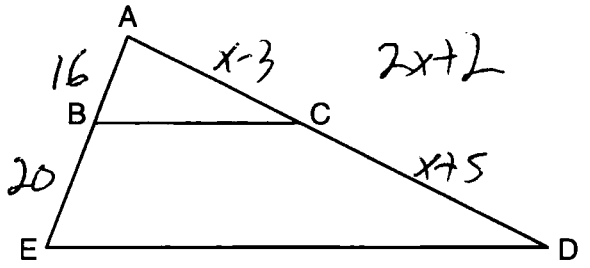


36 In the diagram below, $\overline{BFCE}$, $\overline{AB} \perp \overline{BE}$, $\overline{DE} \perp \overline{BE}$, and $\angle BFD \cong \angle ECA$. Prove that $\triangle ABC \sim \triangle DEF$.



	STATEMENT	REASON
1	$\overline{BFCE}$, $\overline{AB} \perp \overline{BE}$, $\overline{DE} \perp \overline{BE}$, $\angle BFD \cong \angle ECA$	1 Given
2	$\angle B$ & $\angle E$ are right angles	2 Definition of perpendicular
3	$\angle B \cong \angle E$	3 All right angles are congruent
4	$\angle BFD$ & $\angle DFE$ are supplementary. $\angle ECA$ & $\angle ACB$ are supplementary	4 Definition of supplementary angles
5	$\angle DFE \cong \angle ACB$	5 Angles supplementary to congruent angles are congruent
6	$\triangle ABC \sim \triangle DEF$	6 AA

37 In the diagram below of $\triangle ADE$, B is a point on $\overline{AE}$ and C is a point on $\overline{AD}$ such that $\overline{BC} \parallel \overline{ED}$, $AC = x - 3$, $BE = 20$, $AB = 16$, and $AD = 2x + 2$. Find the length of $\overline{AC}$.



$(2x+2) - (x-3) = x+5$

$\frac{16}{20} = \frac{x-3}{x+5}$

$16x + 80 = 20x - 60$

$140 = 4x$

$35 = x$

$\overline{AC} = x - 3$

$35 - 3 = 32$

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen. [6]

38 Quadrilateral *MATH* has coordinates $M(1,1)$, $A(-2,5)$, $T(3,5)$, and $H(6,1)$. Prove that quadrilateral *MATH* is a rhombus and prove that it is *not* a square.
[The use of the grid on the next page is optional.]

The length of each side of quadrilateral MATH is 5. Since each side is congruent, quadrilateral MATH is a rhombus.

The slope of $\overline{MH}$ is 0 and the slope of $\overline{HT}$ is $\frac{-4}{3}$. Since the slopes are not negative reciprocals, the sides are not perpendicular and do not form a 90° angle. Since adjacents sides are not perpendicular, quadrilateral MATH is not a square.

Question 38 continued