ALGEBRA

II

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Monday, June 24, 2024 — 9:15 a.m. to 12:15 p.m., only

Student Name Mr. Sibol

School Name JMAP

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice ...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 Which expression is equivalent to $(x + 3)^2 + 4(x + 3) - 5$?

(1) $(x + 5)(x - 1)$

(2) $(x + 8)(x + 2)$

(3) $(x - 2)(x + 4)$

(4) $x^2 + 4x + 16$

$u = x+3$

u^2+4u-5

$(u+5)(u-1)$

$(x+3+5)(x+3-1)$

2 Which value, to the *nearest tenth*, is an approximate solution for the equation $f(x) = g(x)$, if $f(x) = \frac{5}{x-3}$ and $g(x) = 2(1.3)^x$?

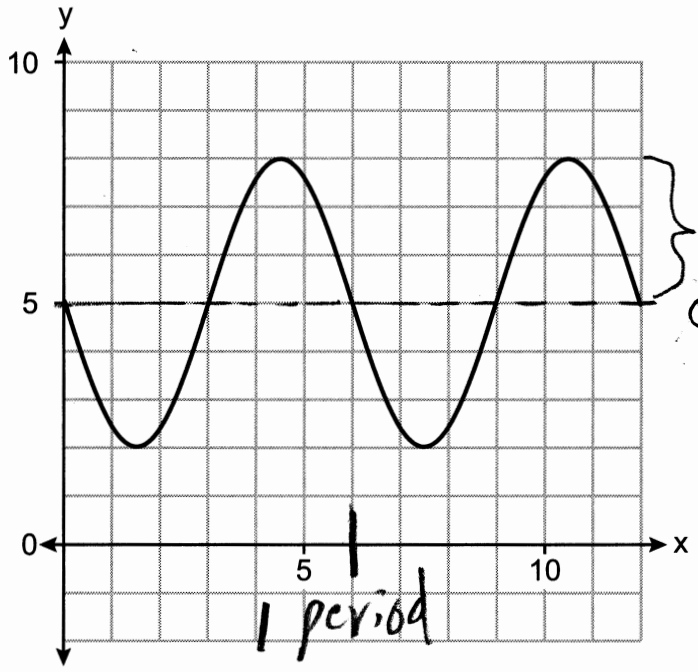
(1) 3.2

(2) 3.9

(3) 4.0

(4) 5.6

3 Which equation is graphed in the diagram below?



$a = -3$

$c = 5$

$b = \frac{2\pi}{6} = \frac{\pi}{3}$

(1) $y = -3\sin\left(\frac{\pi}{3}x\right) + 5$

(2) $y = -3\cos\left(\frac{\pi}{3}x\right) + 5$

(3) $y = -5\sin\left(\frac{\pi}{3}x\right) + 3$

(4) $y = -5\cos\left(\frac{\pi}{3}x\right) + 3$

Use this space for computations.

4 Which point is in Quadrant III and is a solution to the system below?

$$y = x^2 - 24$$
$$y = x - 12$$

(1) $(4,-8)$
(2) $(-3,-15)$
(3) $(-4,-16)$
(4) $(-3,-33)$

$x^2 - 24 = x - 12$
$x^2 - x - 12 = 0$
$(x-4)(x+3) = 0$
$x = 4, -3$
$y = -3 - 12 = -15$

5 For which equations will the value $s = 4$ make the statement an identity?

I. $(2x - 3)^2 = 4x^2 - 3sx + 9$
II. $(x - 2)^3 = (x - 2)(x^2 + sx + s)$

(1) I, only
(2) II, only
(3) I and II
(4) neither I nor II

$(2x-3)^2 = 4x^2 - 12x + 9$
$s = 4$
$(x-2)(x-2)^2 = (x-2)(x^2 - 4x + 4)$
$s = -4 \quad s = 4$

6 The number of bacteria in a sample, which can be modeled by an exponential regression, is shown in the table below.

Time Since Observation Began (hours)	0	1	2	3.5	4
Number of Bacteria	40	48	57	75	82

$y = 40(1.2)^8$
≈ 168

Assuming this trend continues, approximately how many bacteria would be present 8 hours after the observation began?

(1) 123
(2) 127
(3) 168
(4) 180

Use this space for computations.

7 Factored completely, $x^4 + 4x^3 - 9x^2 - 36x$ is equivalent to

(1) $x(x + 9)(x - 9)(x + 4)$ (3) $(x^3 - 9x)(x + 4)$

(2) $x(x + 3)(x - 3)(x + 4)$ (4) $x(x^2 - 9)(x + 4)(x + 4)$

$x(x^3+4x^2-9x-36)$
$x(x^2(x+4)-9(x+4))$
$x(x^2-9)(x+4)$

8 The solution set for the equation $x + 1 = \sqrt{4x + 25}$ is

(1) $\{\}$ (3) $\{6,-4\}$

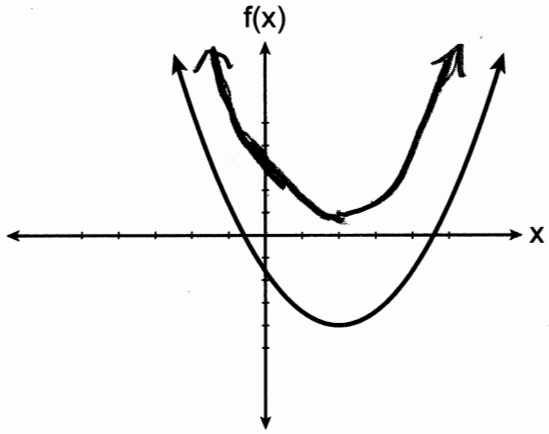
(2) $\{6\}$ (4) $\{-4\}$

$x^2+2x+1=4x+25$

$x^2-2x-24=0$
$(x-6)(x+4)=0$
$x=6, -4$

$-4+1<0$

9 If $f(x)$ is represented by the graph below, which translation of $f(x)$ would have imaginary roots?



(1) $f(x + 5)$ (3) $f(x) + 5$

(2) $f(x - 5)$ (4) $f(x) - 5$

Use this space for computations.

10 The solutions to the equation $3x^2 - 4x + 2 = 2x - 3$ are

(1) $\frac{2}{3} \pm \frac{\sqrt{2}}{3}i$

(3) $1 \pm \frac{\sqrt{12}}{3}$

(2) $1 \pm \frac{\sqrt{6}}{3}i$

(4) $1 \pm 2\sqrt{6}i$

$3x^2 - 6x + 5 = 0$

$x = \frac{6 \pm \sqrt{(-6)^2 - 4(3)(5)}}{2(3)}$

$\frac{6 \pm \sqrt{-24}}{6} = \frac{6 \pm 2i\sqrt{6}}{6}$

11 A culture of 1000 bacteria triples every 10 hours. Which expression models the number of bacteria in the sample after t hours?

(1) $1000e^{3t}$

(3) $1000(3)^{10t}$

(2) $1000(3)^{t}$

(4) $1000(3)^{\frac{t}{10}}$

12 An initial investment of \$5000 in an account earns 3.5% annual interest. Which function correctly represents a recursive model of the investment after n years?

(1) $A = 5000(0.035)^n$

(3) $A = 5000(1.035)^n$

(2) $a_0 = 5000$
$a_n = a_{n-1}(0.035)$

(4) $a_0 = 5000$
$a_n = a_{n-1}(1.035)$

Use this space for computations.

13 Which expression is equivalent to $2xy^2\sqrt[3]{x^2y}$?

(1) $2x^{\frac{5}{3}}y^{\frac{7}{3}}$

(2) $2xy$

(3) $2x^{\frac{2}{3}}y^{\frac{2}{3}}$

(4) $2x^7y^4$

$2x^{\frac{3}{3}}y^{\frac{6}{3}}x^{\frac{2}{3}}y^{\frac{1}{3}}$

14 Which statements must be true about the polynomial function $k(x) = -2x^3 - 11x^2 - 12x + 9$?

I. $(x - 3)$ is a factor of $k(x)$

✓ II. $k(0) = 9$

III. $\frac{k(x)}{x+2}$ has a remainder of 5

(1) II, only

(2) I and II

(3) II and III

(4) I, II, and III

3 | -2 -11 -12 9
-6 -51 -189
-2 -17 -63 -180

$x-3$ not a factor

-2 | -2 -11 -12 9
4 14 -4
-2 -7 2 5

15 To prepare for lacrosse tryouts, Kole is increasing the amount of time he spends at the gym. This week he is spending 150 minutes there and he plans to increase this amount by 2% each week. The amount of time, in minutes, that he plans to spend at the gym t weeks from now is given by the function $A(t) = 150(1.02)^t$.
In terms of a daily growth rate, the amount of time Kole is planning to spend at the gym can best be modeled by the function

(1) $A(t) = 150(1.14869)^{\frac{t}{7}}$

(2) $A(t) = 150(1.14869)^{7t}$

(3) $A(t) = 150(1.00283)^{\frac{t}{7}}$

(4) $A(t) = 150(1.00283)^{7t}$

$A(t) = 150(1.02^{\frac{1}{7}})^{7t}$

$\approx 150(1.00283)^{7t}$

Use this space for computations.

16 Mr. Zachary posts review assignments on the Betamath website for his students. On his last test, 49% of his students used Betamath and passed. Overall, 68% of his students used Betamath. Approximately what percentage of Mr. Zachary's students passed, given that they used Betamath?

(1) 19%
(3) 33%
(2) 32%
(4) 72%

$P(B) \cdot P(P|B) = P(P \text{ and } B)$
$.68x = .49$
$x \approx .72$

7/24/25

17 Given $\sin\theta = \frac{7}{25}$ and θ terminates in Quadrant II, what is the value of $\tan\theta$?

(1) $-\frac{7}{24}$
(3) $\frac{7}{24}$
(2) $-\frac{24}{7}$
(4) $\frac{24}{7}$

$\cos\theta = \frac{-24}{25}$
$\tan\theta = \frac{7/25}{-24/25}$

18 A family owned grocery store in New Hartford, NY employs 49 people whose ages are approximately normally distributed with a mean of 36 years and a standard deviation of 6.2 years. Ryan has been hired to work at this store. He is 30 years old. How many people who work at this store would you expect to be younger than Ryan?

(1) 17
(3) 41
(2) 7
(4) 8

$.167 \cdot 49 \approx 8$

19 For the function $d(x) = \sqrt[3]{x+2}$, the inverse function, $d^{-1}(x)$, equals

(1) $\sqrt[3]{x+2}$
(3) $-\sqrt[3]{x+2}$
(2) $x^3 + 2$
(4) $x^3 - 2$

$y = \sqrt[3]{x+2}$
$x = \sqrt[3]{y+2}$
$x^3 = y + 2$
$x^3 - 2 = y$

Use this space for computations.

20 Given $f(x) = x^4 + x^3 - 3x^2 + 9x - 108$ and $f(3) = 0$, which values satisfy $f(x) = 0$?

(1) $-4, 3$ only

(3) $\pm 3i, -4, 3$

(2) $-3, 4$ only

(4) $\pm 3i, -3, 4$

$$\begin{array}{r|rrrrr} 3 & 1 & 1 & -3 & 9 & -108 \\ & & 3 & 12 & 27 & 108 \\ \hline & 1 & 4 & 9 & 36 & 0 \end{array}$$

21 The profit function, $p(x)$, is found by subtracting the cost function, $c(x)$, from the revenue function, $r(x)$. Which function below represents the cost function given $p(x) = -15x^2 + 600x + 60$ and $r(x) = -0.4x^2 + 130x + 1200$?

(1) $c(x) = -14.6x^2 + 470x - 1140$

(2) $c(x) = -14.6x^2 + 730x - 1260$

(3) $c(x) = 14.6x^2 - 470x + 1140$

(4) $c(x) = 14.6x^2 + 730x - 1260$

$x^3 + 4x^2 + 9x + 36 = 0$

$x^2(x+4) + 9(x+4) = 0$

$(x^2+9)(x+4) = 0$

$\pm 3i \quad -4$

$p(x) = r(x) - c(x)$

$-15x^2 + 600x + 60 = -0.4x^2 + 130x + 1200 - c(x)$

$c(x) = 14.6x^2 - 470x + 1140$

22 What is the remainder when $4x^3 - 3x + 3$ is divided by $x - 2$?

(1) -23

(3) 13

(2) -7

(4) 29

$$\begin{array}{r|rrrr} 2 & 4 & 0 & -3 & 3 \\ & & 8 & 16 & 26 \\ \hline & 4 & 8 & 13 & 29 \end{array}$$

Use this space for computations.

23 The parabola with equation $12(y + 1) = (x - 4)^2$ has

(1) a vertex at ~~(4,2)~~ (4, -1)

(2) a focus at (4, ~~−1~~) (4, 2)

(3) a directrix $y = -4$ $-1-3=-4$

(4) ~~four~~ three units between the focus and vertex

$\frac{12(y+1)}{12} = \frac{(x-4)^2}{12}$ $y = \frac{1}{4(3)}(x-4)^2 - 1$

24 Jay is training for a bike race over fifteen weeks. At the end of the first week, he has ridden ten miles, and he is planning to increase his weekly distance by nine percent each week. Approximately how many miles total will he have ridden from the beginning of his training to the end of the fifteenth week?

(1) 10.989

(2) 33.417

(3) 163.5

(4) 293.609

$S_{15} = \frac{10 - 10(1.09)^{15}}{1 - 1.09} \approx 293.609$

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 Given x is a real number, write the expression in simplest $a + bi$ form:

$$(x + 2i)(3 - 2xi) + 2x^2i$$

$3x - 2x^2i + 6i - 4xi^2 + 2x^2i$

$3x + 6i + 4x$

$7x + 6i$

26 Solve $3.8e^{1.5t} = 16$ algebraically for t to the *nearest hundredth*.

$\ln e^{1.5t} = \ln \frac{16}{3.8}$

$\frac{1.5t}{1.5} = \frac{\ln \frac{16}{3.8}}{1.5}$

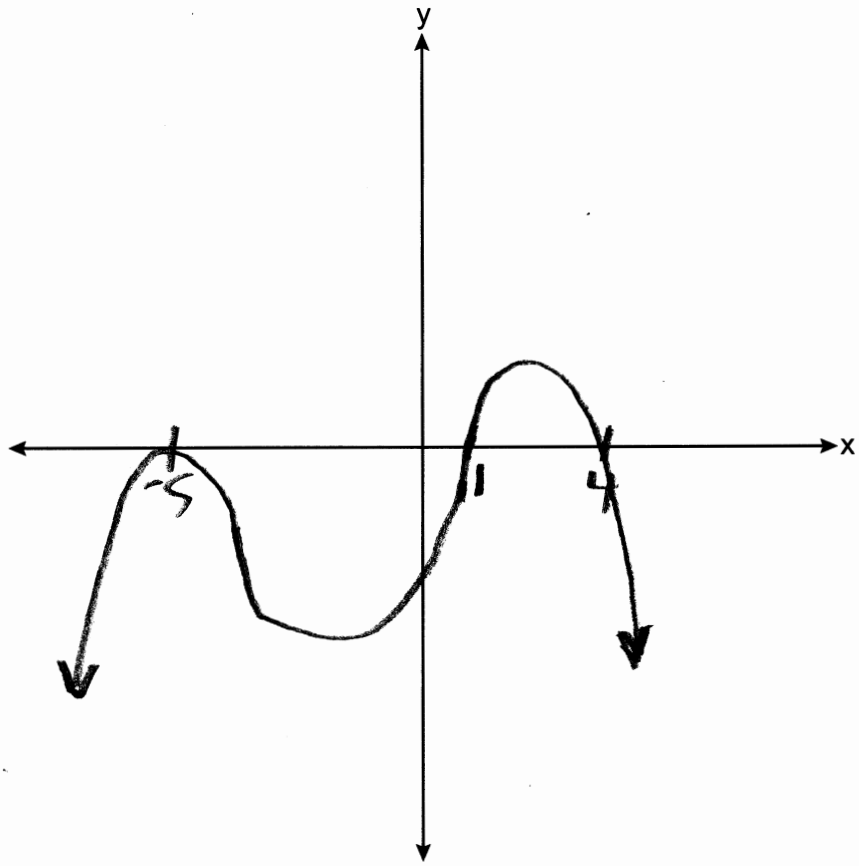
$t \approx .96$

27 In an attempt to get the student body's opinion of a new dress code, members of the statistics class surveyed the students of the first period computer science class. Explain a statistical bias in the method of data collection.

The opinion sought is that of the entire student body, but the first period computer science class may not be representative of the entire student body.

28 Sketch a graph of polynomial $P(x)$, given the criteria below:

- $P(x)$ has zeros only at -5, 1, and 4
- As $x \to \infty$, $P(x) \to -\infty$
- As $x \to -\infty$, $P(x) \to -\infty$



29 The height, above ground, of a Ferris wheel car can be modeled by the function $h(t) = -103.5\cos\left(\frac{2\pi t}{5}\right) + 108.5$ where h is measured in feet and t is measured in minutes. State the period of the function and describe what the period represents in this context.

$\frac{2\pi}{2\pi/5} = 5$ The wheel rotates every 5 minutes

30 Solve algebraically for all values of x:

$$\frac{8}{x+5} - \frac{3}{x} = 5$$

$$\frac{8x - 3(x+5)}{x(x+5)} = 5$$

$$8x - 3x - 15 = 5x^2 + 25x$$

$$0 = 5x^2 + 20x + 15$$

$$0 = x^2 + 4x + 3$$

$$0 = (x+3)(x+1)$$

$$x = -3, -1$$

31 The transportation methods used by the upperclassmen at Calhoun High School are summarized in the table below.

Upperclassmen Transportation Methods

	Drive	Take the Bus	Walk	
Junior	58	75	12	145
Senior	81	39	12	132
	139	114	24	277

Are the events "being a junior" and "driving to school" independent? Using statistical evidence, justify your answer.

$P(J) = \frac{145}{277} \approx .52$

$P(J|D) = \frac{58}{139} \approx .42$

No, because these probabilities are not equal.

32 Can $f(x) = x^3 + 7$ be classified as an odd function? Justify your answer.

No, because a 180° rotation of f about the origin does not map f onto itself.

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Solve the following system of equations algebraically for all values of x, y, and z:

$$\begin{aligned} 3x - 8y + 2z &= -60 \\ 2x - 7y - 5z &= -31 \\ -6x + 2y - 4z &= 36 \end{aligned}$$

$6x - 16y + 4z = -120$

$6x - 21y - 15z = -93$

$-6x + 2y - 4z = 36$

$-19y - 19z = -57$

$y + z = 3$

$-14y = -84$

$y = 6$

$6 + z = 3$

$z = -3$

$-6x + 2(6) - 4(-3) = 36$

$-6x + 24 = 36$

$-6x = 12$

$x = -2$

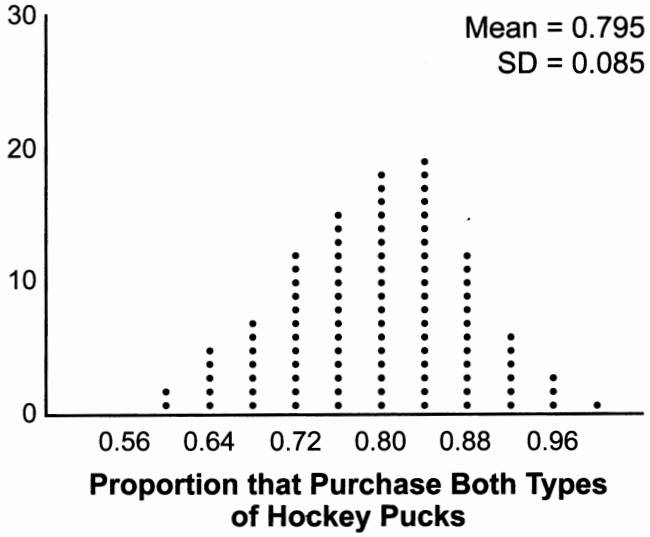
34 In the town of Skaneateles, New York, house prices since 2008 have changed based on the function $H(t) = 200{,}000(1.045)^t$, where t is the number of years since 2008 and $H(t)$ is the median house price. Determine the average rate of change for the median house price in Skaneateles from 2010 to 2018 to the *nearest dollar per year*.

$$\frac{H(10) - H(2)}{10 - 2} \approx 11{,}524$$

Explain what this rate of change means as it relates to median house prices.

From 2010-2018, the median house price increased $11524 per year on average.

35 A sporting goods manufacturer is trying to determine if they should continue to produce multiple types of hockey pucks. The company surveyed 50 randomly chosen customers and asked them if they purchased both game regulation pucks and lighter training pucks. Of those surveyed, 40 of them said that they purchase both types of pucks. A simulation that was run 100 times based on the survey results produced the approximately normal results below.



a) Determine an interval containing the middle 95% of plausible values that estimates the proportion of all customers who would purchase both types of pucks from the company.

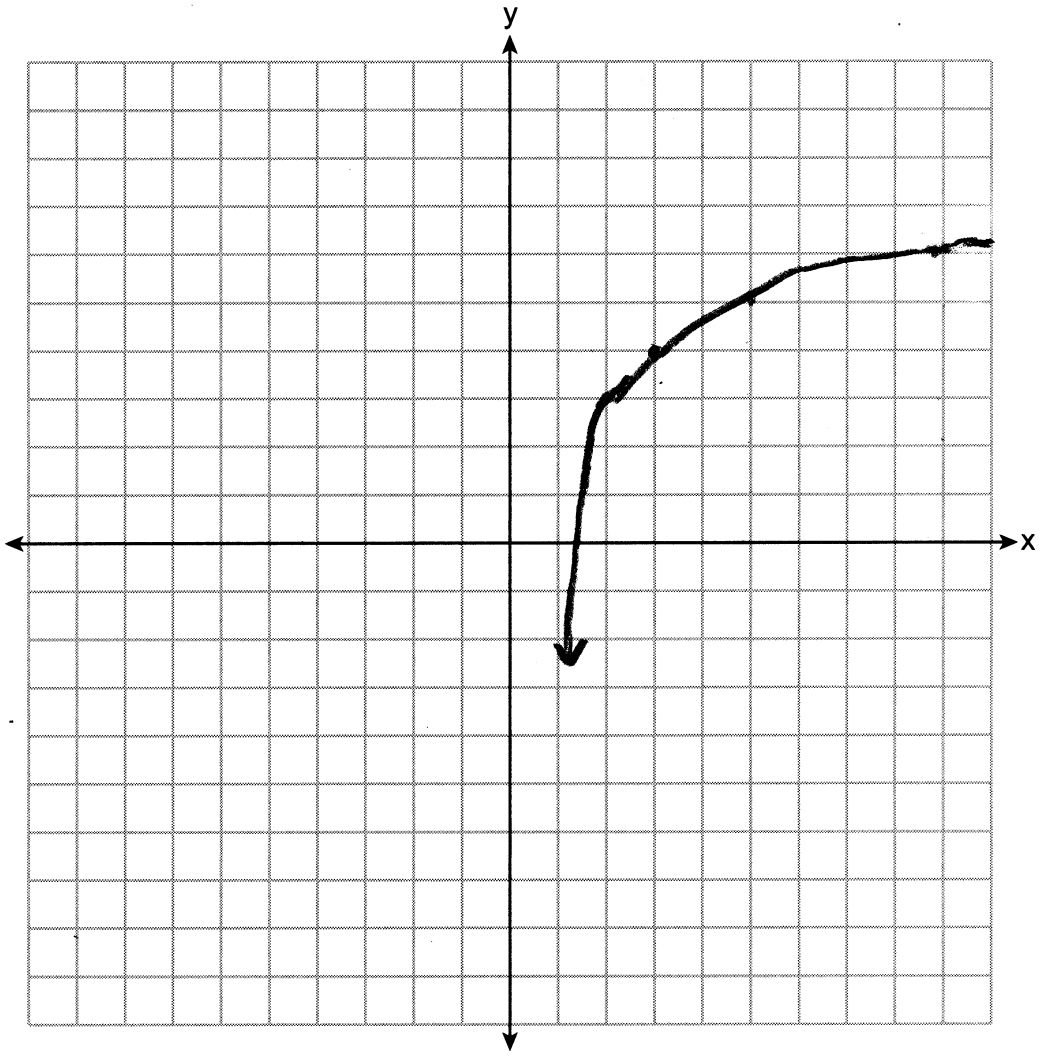
.795 ± 2(.085)

.625 - .965

b) The company will continue to manufacture both types of hockey pucks if it is reasonable to assume that the true proportion of customers who buy both types of hockey pucks is above 0.60. Using the interval from part *a*, explain whether or not the company should continue to produce both types of hockey pucks.

Yes, as it is probable at least .625 of the customer will purchase both.

36 Graph $y = f(x)$, where $f(x) = \log_2(x - 1) + 3$ on the set of axes below.



State the equation of the asymptote of $f(x)$.

$x = 1$

When $f(x)$ is reflected over the line $y = x$, a new function is formed: $g(x) = 2^{x-3} + 1$.
State the equation of the asymptote of $g(x)$.

$y = 1$

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 Megan is performing an experiment in a lab where the air temperature is a constant 73°F and the liquid is 237°F. One and a half hours later, the temperature of the liquid is 112°F. Newton's law of cooling states $T(t) = T_a + (T_0 - T_a)e^{-kt}$ where:

$T(t)$: temperature, °F, of the liquid at t hours
T_a: air temperature
T_0: initial temperature of the liquid
k: constant

Determine the value of k, to the *nearest thousandth*, for this liquid.

$112 = 73 + (237 - 73)e^{-1.5k}$

$k \approx .958$

Question 37 is continued on the next page.

Question 37 continued

Determine the temperature of the liquid using your value for k, to the *nearest degree*, after two and a half hours.

$73+(237-73)e^{-.958(2.5)} \approx 88$

Megan needs the temperature of the liquid to be 80°F to perform the next step in her experiment. Use your value for k to determine, to the *nearest tenth of an hour*, how much time she must wait since she first began the experiment.

$80 = 73+(237-73)e^{-.958t}$

$t \approx 3.3$