

**ALGEBRA I**

Tuesday, June 4, 2024 — 9:15 a.m. to 12:15 p.m., only

Student Name Steve WatsonSchool Name www.jmap.org

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III, and IV** directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice ...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

- 1 A ball was launched into the air, and its height above the ground was recorded each second, as shown in the table below.

Use this space for computations.

Time (sec)	0	1	2	3	4
Height (ft)	11	59	75	59	11

Based on these data, which statement is a valid conclusion?

- (1) The ball lands on the ground at 4 seconds. *same height above ground as launch*
 (2) The ball reaches a maximum height of 11 feet. *75 feet*
 (3) The ball was launched from a height of 0 feet. *11 feet*
 (4) The ball reaches its maximum height at 2 seconds.

- 2 A tour bus can seat, at most, 48 passengers. An adult ticket costs \$18 and a child ticket costs \$12. The bus company must collect at least \$650 to make a profit. If a represents the number of adult tickets sold and c represents the number of child tickets sold, which system of inequalities models this situation if they make a profit?

To make a profit
 $18a + 12c \geq 650$
To fit in the bus
 $a + c \leq 48$

- (1) $a + c \neq 48$
 $18a + 12c \neq 650$
 (2) $a + c \leq 48$
 $18a + 12c \geq 650$
 (3) $a + c < 48$
 $18a + 12c \neq 650$
 (4) $a + c \leq 48$
 $18a + 12c \neq 650$

- 3 Which equation is always true?

- (1) $x^2 \cdot x^3 = x^5$
 (2) $3^x \cdot 3^2 = 9^{2x}$
 (3) $-z^2 = z^2$
 (4) $7^a \cdot 7^b = 7^{ab}$

$$(-2x^2 + 4x - 2) + (3x^2 + 3x - 5)$$

Use this space for computations.

4 The expression $-2(x^2 - 2x + 1) + (3x^2 + 3x - 5)$ is equivalent to

(1) $x^2 + x - 4$

(3) $x^2 + 7x - 4$

(2) $x^2 - x - 7$

(4) $x^2 + 7x - 7$

$$\begin{array}{r} -2x^2 + 4x - 2 \\ \text{add } 3x^2 + 3x - 5 \\ \hline x^2 + 7x - 7 \end{array}$$

5 Which sum is irrational?

$-4\sqrt{5} + 10 \approx 3.0717$

$2.5 + 8 = 10.5$

$-2\sqrt{12} + \sqrt{100}$

~~(3)~~ $\frac{1}{2}\sqrt{25} + \sqrt{64}$

$-2 + 10 = 8$

$7 + 33 = 40$

~~(2)~~ $-\sqrt{4} + \frac{1}{3}\sqrt{900}$

~~(4)~~ $\sqrt{49} + 3\sqrt{121}$

6 The solution to $\frac{4(x-5)}{3} + 2 = 14$ is

(1) 15

(3) 6

(2) 14

(4) 4

S(2) $\frac{4x-20}{3} + 2 = 14$

M(3) $\frac{4x-20}{3} = 12$

A(20) $4x-20 = 36$

D(4) $4x = 56$

$x = 14$

7 On an island, a rare breed of rabbit doubled its population each month for two years. Which type of function best models the increase in population at the end of two years?

(1) linear growth

(3) exponential growth

(2) linear decay

(4) exponential decay

Month	# Rabbits
1	2
2	2 ²
3	2 ³
4	2 ⁴
5	2 ⁵

8 What is the degree of the polynomial $2x - x^2 + 4x^3$?

(1) 1

(3) 3

(2) 2

(4) 4

degree

$$4x^3 - x^2 + 2x$$

leading coefficient

- 9 The zeros of the function $f(x) = x(x - 5)(3x + 6)$ are
 (1) 0, ~~5~~, and ~~2~~ (3) ~~5~~ and ~~2~~, only
 (2) 0, 5, and -2 (4) 5 and -2, only

Use this space for computations.

factors	Set = to zero	Solve for zeros
(x)	$x = 0$	0
$(x-5)$	$x-5=0$	5
$(3x+6)$	$3x+6=0$	-2

- 10 What is the y -intercept of the line that passes through the points $(-1, 5)$ and $(2, -1)$?

- (1) -1 (2) -2 (3) 3 (4) 5

$$y = mx + b$$

$$x = -1$$

$$y = 5$$

$$m = -2$$

$$y = mx + b$$

$$5 = -2(-1) + b$$

$$5 = 2 + b$$

$$\boxed{3 = b}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 5}{2 - (-1)} = \frac{-6}{3} = -2$$

- 11 Nancy has just been hired for her first job. Her company gives her four choices for how she can collect her annual salary over the first eight years of employment.

Each function below represents the four choices she has for her annual salary in thousands of dollars, where t represents the number of years after she is hired.

$$a(t) = 2^t + 25$$

$$b(t) = 10t + 75$$

$$c(t) = \sqrt{400t} + 80$$

$$d(t) = 2(t + 1)^2 - 10t + 50$$

Input all for plans in graphing calculator.

Which pay plan should Nancy choose in order to have the highest salary in her eighth year?

- (1) $a(t)$ 281 (3) $c(t)$ 136.57
 (2) $b(t)$ 105 (4) $d(t)$ 132

- 12 The third term in a sequence is 25 and the fifth term is 625. Which number could be the common ratio of the sequence?

check

- (1) $\frac{1}{5}$ (3) $\frac{1}{25}$
 (2) 5 (4) 25

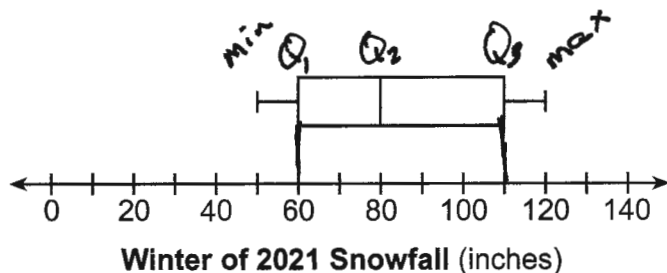
$$\frac{Q_5}{Q_3} = \frac{625}{25} = 25 = r^2$$

$$5 = r$$

Given	Q_1	?	1	$\times 5$
	Q_2	?	5	$\times 5$
	Q_3	25	25	$\times 5$
	Q_4	?	125	$\times 5$
[4] Given	Q_5	625	625	

Use this space for computations.

13 The box plot below summarizes the data for the amount of snowfall, in inches, during the winter of 2021 for 12 locations in western New York.



$$IQR = Q_3 - Q_1$$

$$Q_1 = 60$$

$$Q_3 = 110$$

$$IQR = 110 - 60$$

$$\boxed{IQR = 50}$$

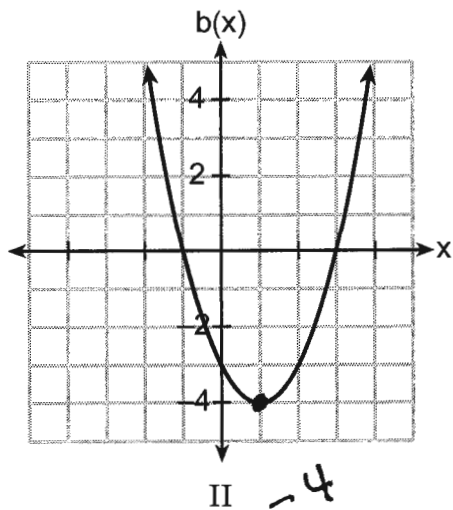
What is the interquartile range?

- (1) 30 (3) 80
 (2) 50 (4) 110

14 Four quadratic functions are represented below.

$a(x) = (x - 3)^2 - 7$ from graphing calculator
 I -7

$c(x) = x^2 + 6x + 3$ from graphing calculator
 III -6



x	d(x)
-4	-1
-3	-4
-2	-5
-1	-4
0	-1

-5

IV

Which function has the smallest minimum value?

- (1) I -7 (3) III -6
 (2) II -4 (4) IV -5

common difference is -3

Use this space for computations.

15 The equation that represents the sequence $-2, -5, -8, -11, -14, \dots$ is

(1) $a_n = -3 + (-2)(n-1)$ (3) $a_n = 3 + (-2)(n-1)$

(2) $a_n = -2 + (-3)(n-1)$ (4) $a_n = -2 + (3)(n-1)$

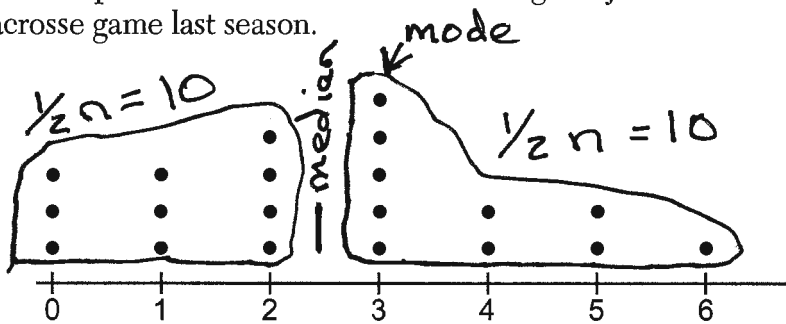
↑ common difference

from formula page

$$a_n = a_1 + d(n-1)$$

$$d = -3$$

16 The dot plot below shows the number of goals Jessica scored in each lacrosse game last season.



$$n = 20$$

$$\text{mode} = 3$$

$$\text{median} = 2.5$$

$$\text{mean} = \frac{50 \text{ goals}}{20 \text{ games}} = 2.5$$

Goals Scored per Game

$$0 + 3 + 8 + 15 + 8 + 10 + 6 = 50 \text{ goals total}$$

Which statement about the dot plot is correct?

~~(1)~~ mean $>$ mode

~~(3)~~ mode = median

mean = median

~~(4)~~ median $>$ mean

17 The students in Mrs. Smith's algebra class were asked to describe the graph of $g(x) = 2(x-3)^2$ compared to the graph of $f(x) = x^2$.

Which student response is correct?

(1) Ashley said that the graph of $g(x)$ is wider and shifted left 3 units.

(2) Beth said that the graph of $g(x)$ is narrower and shifted left 3 units.

(3) Carl said that the graph of $g(x)$ is wider and shifted right 3 units.

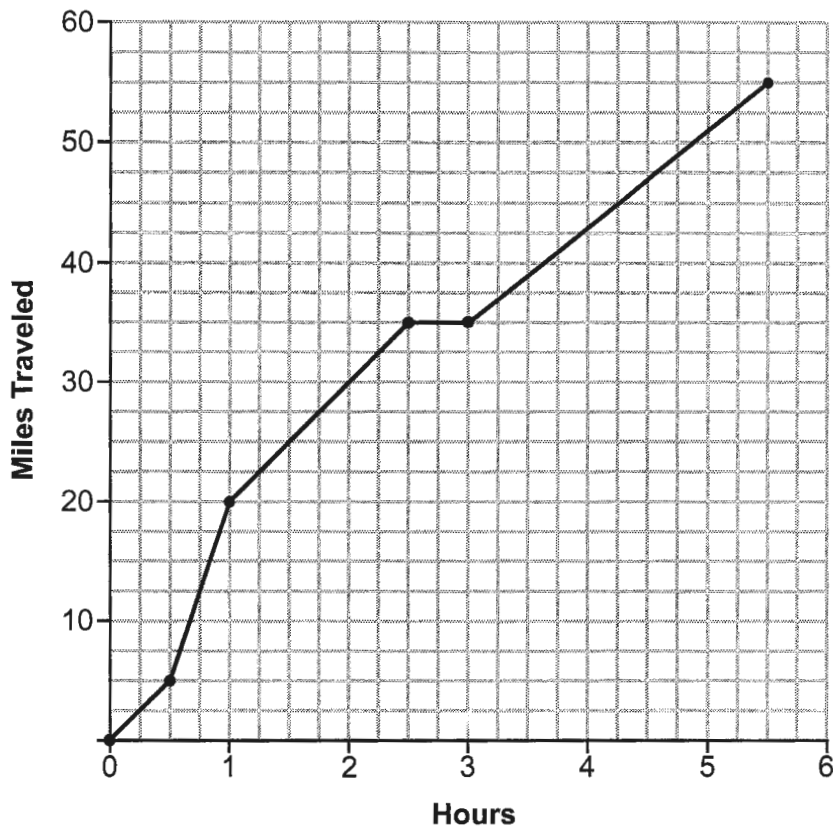
Don said that the graph of $g(x)$ is narrower and shifted right 3 units.

Caution!

Input $f(x)$ as equation #1 in graphing calculator.
Input $g(x)$ as equation #2.

Use this space for
computations.

18 One Saturday, Dave took a long bike ride. The graph below models his trip.



Average rate of
change is slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Start (0, 0)

End (5.5, 55)

$$m = \frac{55 - 0}{5.5 - 0} = 10$$

What was Dave's average rate of change, in miles per hour, on this trip?

- (1) 10
 (2) 11
 (3) 11.6
 (4) 14.5

19 Which expression is equivalent to $(x - 5)(2x + 7) - (x + 5)$?

- (1) $2x^2 - 2x - 30$
 (2) $2x^2 - 2x - 40$
 (3) $2x^2 - 4x - 30$
 (4) $2x^2 - 4x - 40$

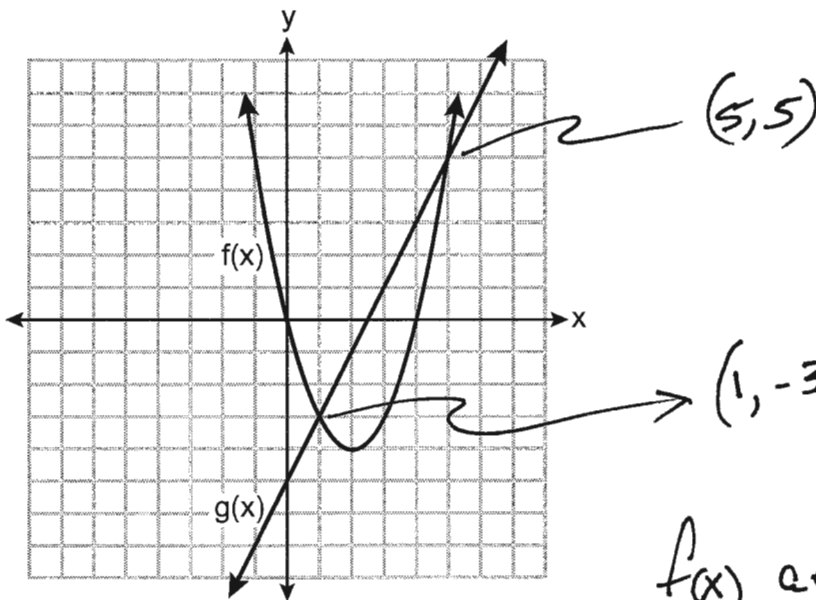
$$\begin{array}{r}
 (x-5) 2x+7 \\
 2x^2 + 7x - 10x - 35 \\
 \downarrow \\
 2x^2 - 3x - 35 \\
 x + 5
 \end{array}$$

subtract

$$2x^2 - 4x - 40$$

Use this space for computations.

20 The functions $f(x)$ and $g(x)$ are graphed on the set of axes below.



$f(x)$ and $g(x)$ intersect when $x = 1$ and $x = 5$

What is the solution to the equation $f(x) = g(x)$?

- (1) 1 and 5
- (2) -5 and 0
- (3) -3 and 5
- (4) 0 and 4

21 When babysitting, Nicole charges an hourly rate and an additional charge for gas. She uses the function $C(h) = 6h + 5$ to determine how much to charge for babysitting. The constant term of this function represents

- (1) the additional charge for gas
- (2) the hourly rate Nicole charges
- (3) the number of hours Nicole babysits
- (4) the total Nicole earns from babysitting

$C(h) = 6h + 5$
 ← variable
 ← constant
 # of hours additional charge for gas.

22 When solved for x in terms of a , the solution to the equation $3x - 7 = ax + 5$ is

- (1) $\frac{12}{3a}$
- (2) $\frac{12}{3-a}$
- (3) $\frac{3a}{12}$
- (4) $\frac{3-a}{12}$

$$\begin{aligned}
 3x - 7 &= ax + 5 \\
 3x &= ax + 12 \\
 3x - ax &= 12 \\
 x(3-a) &= 12 \\
 \boxed{x = \frac{12}{3-a}}
 \end{aligned}$$

Use this space for computations.

23 Wayde van Niekerk, a runner from South Africa, ran 400 meters in 43.03 seconds to set a world record. Which calculation would determine his average speed, in miles per hour?

~~(1)~~ $\frac{400 \text{ m}}{43.03 \text{ sec}} \cdot \frac{1000 \text{ m}}{0.62 \text{ mi}} \cdot \frac{1 \text{ hr}}{3600 \text{ sec}} \rightarrow \frac{\text{m}^2 \cdot \text{hr}}{\text{sec}^2 \cdot \text{mi}}$
~~(2)~~ $\frac{400 \text{ m}}{43.03 \text{ sec}} \cdot \frac{0.62 \text{ mi}}{1000 \text{ m}} \cdot \frac{1 \text{ hr}}{3600 \text{ sec}} \rightarrow \frac{\text{mi} \cdot \text{hr}}{\text{sec}^2}$
 $\frac{400 \text{ m}}{43.03 \text{ sec}} \cdot \frac{0.62 \text{ mi}}{1000 \text{ m}} \cdot \frac{3600 \text{ sec}}{1 \text{ hr}} \rightarrow \frac{\text{mi}}{\text{hr}}$
~~(4)~~ $\frac{400 \text{ m}}{43.03 \text{ sec}} \cdot \frac{1000 \text{ m}}{0.62 \text{ mi}} \cdot \frac{3600 \text{ sec}}{1 \text{ hr}} \rightarrow \frac{\text{m}^2 \cdot \text{sec}}{\text{sec} \cdot \text{mi} \cdot \text{hr}} \rightarrow \frac{\text{m}^2}{\text{mi} \cdot \text{hr}}$

24 Which function has a domain of all real numbers and a range greater than or equal to three?

~~(1)~~ $f(x) = -x + 3$ ~~(3)~~ $h(x) = 3^x$
 $g(x) = x^2 + 3$ ~~(4)~~ $m(x) = |x + 3|$

domain of x
range of y

The domain for all four answer choices is the set of all real numbers.

~~f(x)~~ The range for $f(x)$ is all real numbers.

~~h(x)~~ The range for $h(x)$ is $y > 0$

~~m(x)~~ The range for $m(x)$ is $y \geq 0$

Check using graphing calculator.

Part II

Answer all 6 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

25 Solve $5(x - 2) \leq 3x + 20$ algebraically.

$$5(x-2) \leq 3x + 20$$

$$5x - 10 \leq 3x + 20$$

$$2x \leq 30$$

$$x \leq 15$$

26 Given $g(x) = x^3 + 2x^2 - x$, evaluate $g(-3)$.

$$X^3 + 2X^2 - X$$

$$(-3)^3 + 2(-3)^2 - (-3)$$

$$-27 + 2(9) + 3$$

$$-27 + 18 + 3$$

$$-27 + 21$$

$$-6$$

$$g(-3) = -6$$

27 Given the relation $R = \{(-1,1), (0,3), (-2,-4), (x,5)\}$.

State a value for x that will make this relation a function.

8

Explain why your answer makes this a function.

A function can have one and only one value of y for each value of x , so any number other than -1 , 0 , or -2 will make R a function.

100 play video

28 A survey of 150 students was taken. It was determined that $\frac{2}{3}$ of the students play video games.

Of the students that play video games, 85 also use social media.

Of the students that do not play video games, 20% do not use social media.

50

$$20\% \cdot 50 = 10$$

Complete the two-way frequency table.

	Play Video Games	Do Not Play Video Games	Total
Social Media	85	40	125
No Social Media	15	10	25
Total	100	50	150

29 Use the method of completing the square to determine the exact values of x for the equation $x^2 + 10x - 30 = 0$.

$$x^2 + 10x - 30 = 0$$

$$x^2 + 10x = 30$$

$$x^2 + 10x + \left(\frac{10}{2}\right)^2 = 30 + \left(\frac{10}{2}\right)^2$$

$$x^2 + 10x + 5^2 = 30 + 5^2$$

$$(x+5)^2 = 30 + 25$$

$$(x+5)^2 = 55$$

$$x+5 = \pm \sqrt{55}$$

$$x = -5 \pm \sqrt{55}$$

30 Factor $20x^3 - 45x$ completely.

$$20x^3 - 45x$$

$$(5x) \underbrace{(4x^2 - 9)}$$

This is a difference
of perfect squares

$$a^2 - b^2 = (a+b)(a-b)$$

$$5x(2x+3)(2x-3)$$

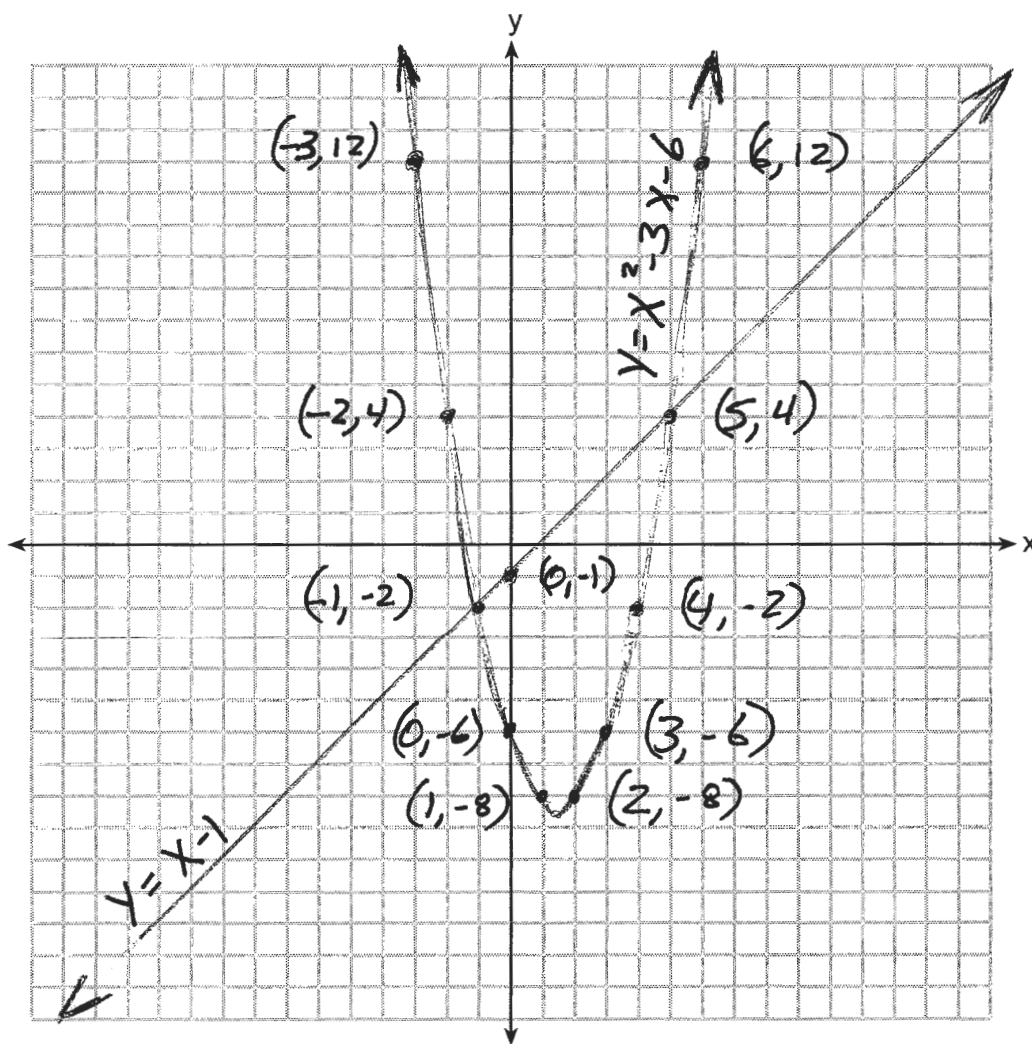
Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

31 Graph the following system of equations on the set of axes below.

$$y = x^2 - 3x - 6$$

$$y = x - 1$$



State the coordinates of all solutions.

$(-1, -2)$ and $(5, 4)$

- 32 The table below shows the amount of money a popular movie earned, in millions of dollars, during its first six weeks in theaters.

Week (x)	1	2	3	4	5	6
Dollars Earned, in Millions (y)	185	150	90	50	25	5

Write the linear regression equation for this data set, rounding all values to the nearest hundredth.

Use graphing calculator.
Turn diagnostics on.

$$Y = -37.57x + 215.67$$

State the correlation coefficient to the nearest hundredth.

$$-.98$$

State what this correlation coefficient indicates about the linear fit of the data.

There is a strong correlation between dollars earned and weeks in theater

33 Use the quadratic formula to solve the equation $3x^2 - 10x + 5 = 0$. Express the answer in simplest radical form.

$$3x^2 - 10x + 5 = 0$$

$$ax^2 + bx + c = 0$$

$$a=3 \quad b=-10 \quad c=5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(3)(5)}}{2(3)}$$

$$x = \frac{10 \pm \sqrt{100 - 60}}{6}$$

$$x = \frac{10 \pm \sqrt{40}}{6}$$

$$x = \frac{10 \pm \sqrt{4} \sqrt{10}}{6}$$

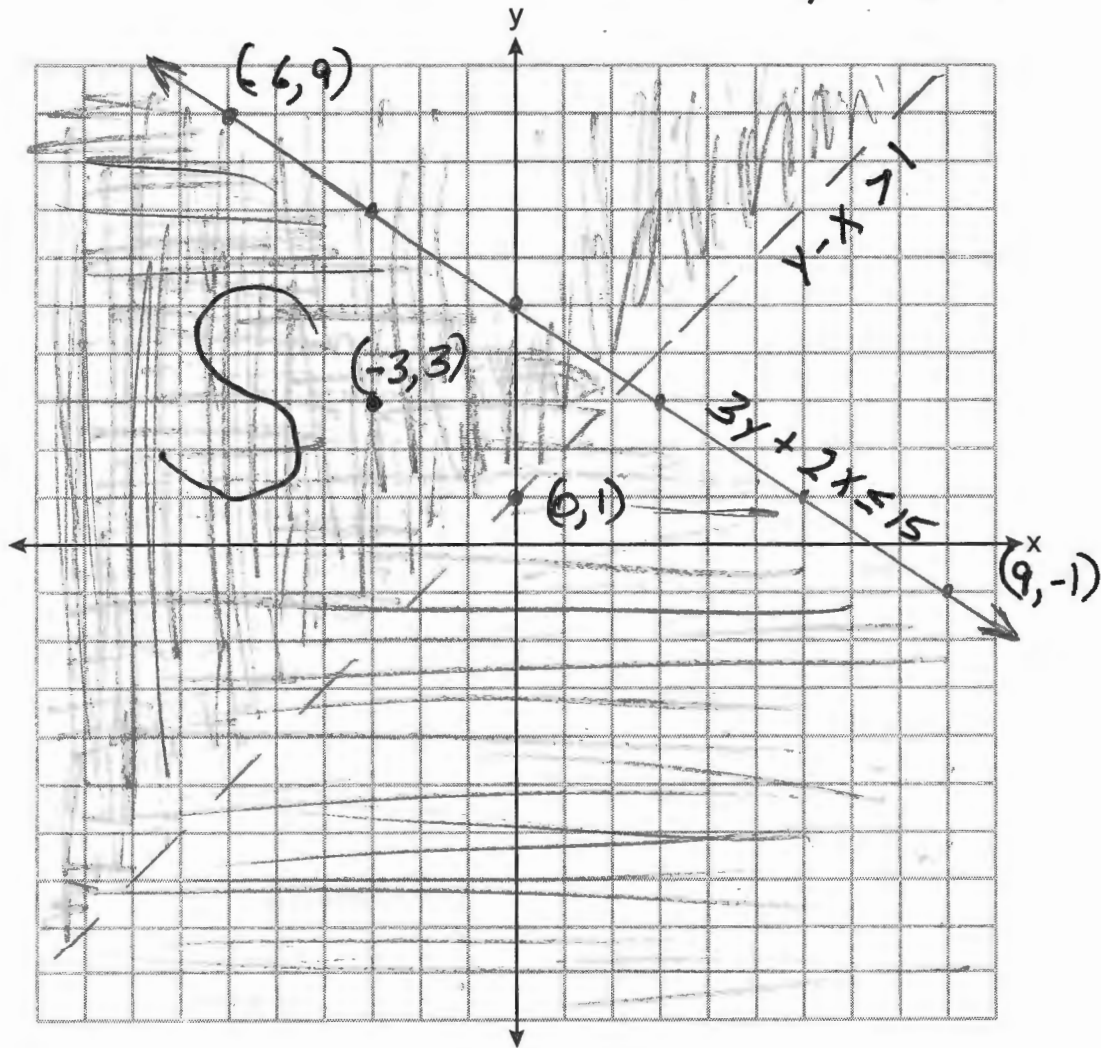
$$x = \frac{10 \pm 2\sqrt{10}}{6}$$

$$x = \frac{5 \pm \sqrt{10}}{3}$$

34 Graph the system of inequalities on the set of axes below.

$$3y + 2x \leq 15 \quad \rightarrow \quad y \leq \frac{-2x + 15}{3}$$

$$y - x > 1 \quad \rightarrow \quad y > x + 1$$



State the coordinates of a point in the solution to this system. Justify your answer.

$-3, 3$ This point satisfies both inequalities.

$$3y + 2x \leq 15 \quad \begin{array}{r} 3(3) + 2(-3) \leq 15 \\ 9 - 6 \leq 15 \\ 3 \leq 15 \quad \checkmark \end{array}$$

$$y - x > 1 \quad \begin{array}{r} 3 - (-3) > 1 \\ 6 > 1 \quad \checkmark \end{array}$$

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

35 Courtney went to a coffee shop to purchase x lattes and y donuts for her friends. One day she spent a total of \$15.50 on four lattes and two donuts. The next day she spent a total of \$18.10 on three lattes and five donuts. All prices included tax.

If x represents the cost of one latte and y represents the cost of one donut, write a system of equations that can be used to model this situation.

$E_1 \neq 1$

$$4x + 2y = 15.50$$

$E_2 \neq 2$

$$3x + 5y = 18.10$$

Courtney thinks that one latte costs \$2.75 and one donut costs \$2.25.

Is Courtney correct? Justify your answer.

No Courtney's numbers satisfy the first equation, but not the second equation.

Use your equations to determine algebraically the exact cost of one latte and the exact cost of one donut.

$$\begin{array}{r} (4x + 2y = 15.50) \quad (3) \\ (3x + 5y = 18.10) \quad (4) \end{array} \begin{array}{l} \text{Times} \\ = \\ = \end{array} \begin{array}{l} 12x + 6y = 46.50 \\ 12x + 20y = 72.40 \end{array}$$

$$-14y = -25.90$$

donut $y = 1.85$

$$4x + 2(1.85) = 15.50$$

$$4x + 3.70 = 15.50$$

$$4x = 11.80$$

latte $x = 2.95$

Check

$$4(2.95) + 2(1.85) = 15.50$$

✓

$$3(2.95) + 5(1.85) = 18.10$$

✓