

Lesson 12-1: Graphing Rational Functions

Part 1: Graphing Rational Functions

1. 010704b
What is the total number of points of intersection of the graphs of the equations $xy = 12$ and $y = -x^2 + 3$?

[A] 4 [B] 1 [C] 3 [D] 2

2. 080422a, P.I. A.A.15
For which value of x is the expression $\frac{x-7}{x+2}$ undefined?

[A] 7 [B] -2 [C] 0 [D] 2

3. 060319a, P.I. A.A.15
For which value of x is the expression $\frac{3x-6}{x-4}$ undefined?

[A] -4 [B] 0 [C] 4 [D] 2

4. 010607a, P.I. A.A.15
For which value of x will the fraction $\frac{3}{2x+4}$ be undefined?

[A] 2 [B] -4 [C] 0 [D] -2

5. 080610a, P.I. A.A.15
For which value of x is the expression $\frac{3}{x-2}$ undefined?

[A] -2 [B] 3 [C] 2 [D] 0

6. 010822a, P.I. A.A.15
For which value of x is the expression $\frac{6-x}{x+2}$ undefined?

[A] 2 [B] -2 [C] 0 [D] 6

7. fall0728ia, P.I. A.A.15

For which value of x is $\frac{x-3}{x^2-4}$ undefined?

[A] 4 [B] 3 [C] 0 [D] -2

8. 010716a, P.I. A.A.15

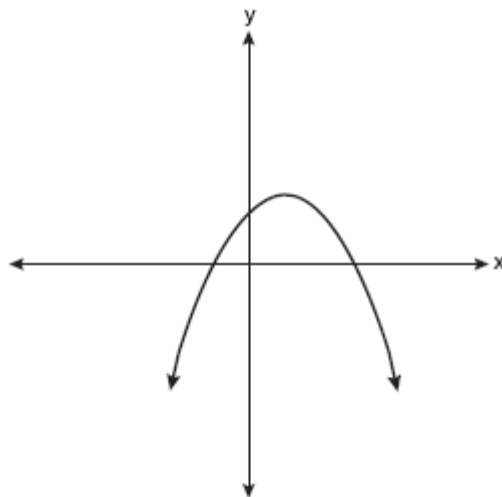
Which expression is undefined when $w = 3$?

[A] $\frac{w+1}{w^2-3w}$ [B] $\frac{3w}{3w^2}$
[C] $\frac{w-3}{w+1}$ [D] $\frac{w^2+2w}{5w}$

Part 2: Types of Functions

9. fall0717ia, P.I. A.G.4

Which type of graph is shown in the diagram below?

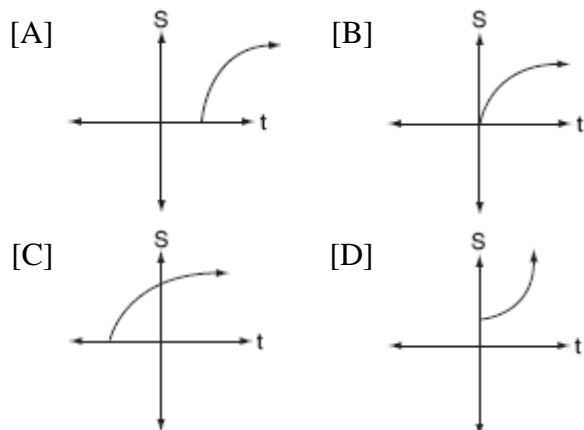


[A] absolute value [B] quadratic
[C] linear [D] exponential

10. 060718b, P.I. A2.A.13

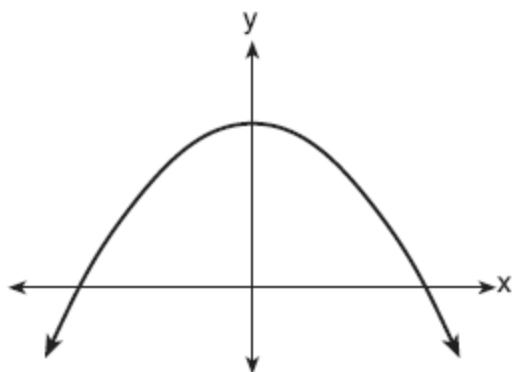
The formula $S = 20\sqrt{t + 273}$ is used to determine the speed of sound, S , in meters per second, near Earth's surface, where t is the surface temperature, in degrees Celsius.

Which graph best represents this function?



11. 060703b, P.I. A.G.4

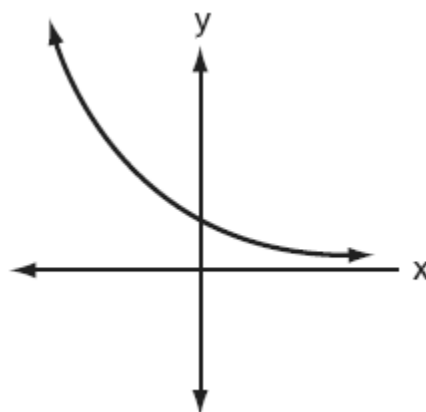
Which equation is best represented by the accompanying graph?



- [A] $y = 6x + 1$ [B] $y = 6x^2$
[C] $y = -x^2 + 1$ [D] $y = 6^x$

12. 010701b, P.I. A.G.4

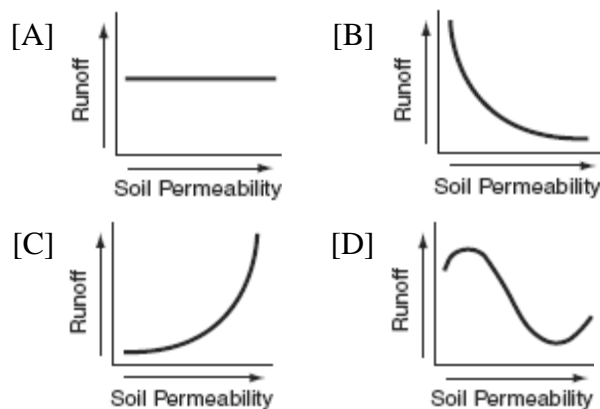
Which equation best represents the accompanying graph?



- [A] $y = x^2 + 2$ [B] $y = -2^x$
[C] $y = 2^{-x}$ [D] $y = 2^x$

13. 010603b

Which graph shows that soil permeability varies inversely to runoff?



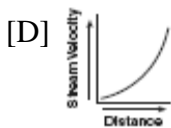
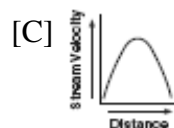
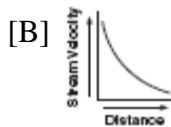
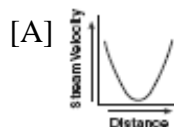
14. 060414b

Which function is symmetrical with respect to the origin?

- [A] $y = -\frac{5}{x}$ [B] $y = 5^x$
[C] $y = \sqrt{x+5}$ [D] $y = |5-x|$

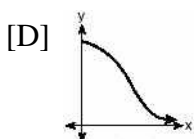
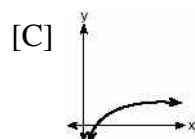
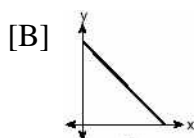
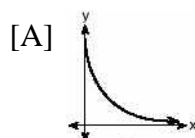
15. 060408b

Which graph represents an inverse variation between stream velocity and the distance from the center of the stream?



16. 080304b, P.I. A.G.4

The strength of a medication over time is represented by the equation $y = 200(1.5)^{-x}$, where x represents the number of hours since the medication was taken and y represents the number of micrograms per millimeter left in the blood. Which graph best represents this relationship?



17. 010310b

For a rectangular garden with a fixed area, the length of the garden varies inversely with the width. Which equation represents this situation for an area of 36 square units?

[A] $x - y = 36$

[B] $x + y = 36$

[C] $y = 36x$

[D] $y = \frac{36}{x}$

18. 060104b, P.I. A2.A.5

Camisha is paying a band \$330 to play at her graduation party. The amount each member earns, d , varies inversely as the number of members who play, n . The graph of the equation that represents the relationship between d and n is an example of

[A] a line

[B] a parabola

[C] an ellipse

[D] a hyperbola

Lesson 12-2: Simplifying Rational Expressions

Part 1: Simplifying Rational Expressions

19. 010109a, P.I. A.A.14

If $x \neq 0$, the expression $\frac{x^2 + 2x}{x}$ is equivalent to

[A] 2

[B] 4

[C] $x + 2$

[D] $3x$

20. 060102a, P.I. A.A.14

Which polynomial is the quotient of

$$\frac{6x^3 + 9x^2 + 3x}{3x}?$$

[A] $2x + 3$

[B] $2x^2 + 3x + 1$

[C] $6x^2 + 9x$

[D] $2x^2 + 3x$

21. fall0718ia, P.I. A.A.14

The expression $\frac{9x^4 - 27x^6}{3x^3}$ is equivalent to

[A] $9x^3(1 - x)$

[B] $3x(1 - 3x)$

[C] $3x(1 - 9x^5)$

[D] $3x(1 - 3x^2)$

22. 069924a, P.I. A.A.16

Simplify: $\frac{9x^2 - 15xy}{9x^2 - 25y^2}$

23. 010631a, P.I. A.A.16

Simplify: $\frac{x^2 + 6x + 5}{x^2 - 25}$

Lesson 12-3: Multiplying and Dividing Rational Expressions

Part 1: Multiplying Rational Expressions

24. 080117b, P.I. A.A.18

If the length of a rectangular garden is represented by $\frac{x^2 + 2x}{x^2 + 2x - 15}$ and its width is represented by $\frac{2x - 6}{2x + 4}$, which expression represents the area of the garden?

[A] $x + 5$

[B] $\frac{x}{x + 5}$

[C] x

[D] $\frac{x^2 + 2x}{2(x + 5)}$

25. 060124b, P.I. A.A.18

A rectangular prism has a length of $\frac{2x^2 + 2x - 24}{4x^2 + x}$, a width of $\frac{x^2 + x - 6}{x + 4}$, and a height of $\frac{8x^2 + 2x}{x^2 - 9}$. For all values of x for which it is defined, express, in terms of x , the volume of the prism in simplest form.

Part 2: Dividing Rational Expressions

26. 080022a, P.I. A.A.18

Perform the indicated operation and express the result in simplest terms: $\frac{x}{x + 3} \div \frac{3x}{x^2 - 9}$

27. 060727b, P.I. A2.A.16

If $f(x) = \frac{3x^2 - 27}{18x + 30}$ and $g(x) = \frac{x^2 - 7x + 12}{3x^2 - 7x - 20}$, find $f(x) \div g(x)$ for all values of x for which the expression is defined and express your answer in simplest form.

28. 010434b, P.I. A.A.18

Express in simplest form:

$$\frac{4x + 8}{x + 1} \cdot \frac{2 - x}{3x - 15} \div \frac{x^2 - 4}{2x^2 - 8x - 10}$$

29. 010733b, P.I. A.A.18

Perform the indicated operations and simplify completely:

$$\frac{x^2 - 9}{x^2 - 5x} \cdot \frac{5x - x^2}{x^2 - x - 12} \div \frac{x - 4}{x^2 - 8x + 16}$$

Lesson 12-4: Dividing Polynomials

Part 1: Dividing Polynomials

30. 060506a, P.I. A.A.14

When $3x^2 - 6x$ is divided by $3x$, the result is

[A] $x - 2$

[B] $x + 2$

[C] $-2x$

[D] $2x$

31. 010724a, P.I. A.A.14

The expression $(50x^3 - 60x^2 + 10x) \div 10x$ is equivalent to

[A] $5x^3 - 6x^2 + x$

[B] $5x^2 - 6x$

[C] $5x^2 - 6x + 1$

[D] $5x^2 - 60x^2 + 10x$

Lesson 12-5: Adding and Subtracting Rational Expressions

Part 2: Adding and Subtracting Rational Expressions with Unlike Denominators

32. What is the least common denominator of $\frac{1}{2}$,

$$\frac{2}{7x}, \text{ and } \frac{5}{x}?$$

- [A] $9x$ [B] $14x^2$ [C] $14x$ [D] $2x$

33. The sum of $\frac{3}{x} + \frac{2}{5}$, $x \neq 0$, is

[A] $\frac{2x+15}{5x}$ [B] $\frac{5}{x+5}$

[C] $\frac{1}{x}$ [D] $\frac{2x+15}{x+5}$

34. What is the sum of $\frac{2}{x}$ and $\frac{x}{2}$?

[A] $\frac{4+x^2}{2x}$ [B] 1

[C] $\frac{4+x}{2x}$ [D] $\frac{2+x}{2x}$

35. Which expression is equivalent to $\frac{a}{x} + \frac{b}{2x}$?

[A] $\frac{2a+b}{2x}$ [B] $\frac{2a+b}{x}$

[C] $\frac{a+b}{2x}$ [D] $\frac{a+b}{3x}$

36. What is the sum of $\frac{3}{7n}$ and $\frac{7}{3n}$?

[A] $\frac{42}{21n}$ [B] $\frac{10}{21n}$ [C] $\frac{1}{n}$ [D] $\frac{58}{21n}$

37. The expression $\frac{y}{x} - \frac{1}{2}$ is equivalent to

[A] $\frac{2y-x}{2x}$ [B] $\frac{x-2y}{2x}$

[C] $\frac{1-y}{2x}$ [D] $\frac{y-1}{x-2}$

38. Expressed as a single fraction, what is $\frac{1}{x+1} + \frac{1}{x}$, $x \neq 0, -1$?

[A] $\frac{2x+1}{x^2+x}$ [B] $\frac{2x+3}{x^2+x}$

[C] $\frac{3}{x^2}$ [D] $\frac{2}{2x+1}$

39. 060524b, P.I. A2.A.16

Express in simplest form: $\frac{1}{x} + \frac{1}{x+3}$

40. 010315b, P.I. A2.A.16

What is the sum of $\frac{3}{x-3}$ and $\frac{x}{3-x}$?

[A] 0 [B] -1 [C] $\frac{x+3}{x-3}$ [D] 1

41. 080505b, P.I. A2.A.16

What is the sum of $(y-5) + \frac{3}{y+2}$?

[A] $\frac{y-2}{y+2}$ [B] $y-5$

[C] $\frac{y^2-7}{y+2}$ [D] $\frac{y^2-3y-7}{y+2}$

42. 080733b, P.I. A2.A.16

Express in simplest form:

$$\frac{2x}{x^2-4} \div \frac{4}{x^2-4x+4} + \frac{12}{x^2-4} \cdot \frac{2-x}{3}$$

Lesson 12-6: Solving Rational Equations

Part 1: Solving Rational Equations

43. 010825a, P.I. A.A.26

If $\frac{5}{n} - \frac{1}{2} = \frac{3}{6n}$, what is the value of n ?

[A] 2 [B] -2 [C] $\frac{2}{7}$ [D] 9

44. 060429b, P.I. A2.A.23

Solve for x and express your answer in

simplest radical form: $\frac{4}{x} - \frac{3}{x+1} = 7$

45. 080529b, P.I. A2.A.23

Solve for all values of x : $\frac{9}{x} + \frac{9}{x-2} = 12$

46. 060212b, P.I. A2.A.23

What is the solution set of the equation

$$\frac{x}{x-4} - \frac{1}{x+3} = \frac{28}{x^2 - x - 12}?$$

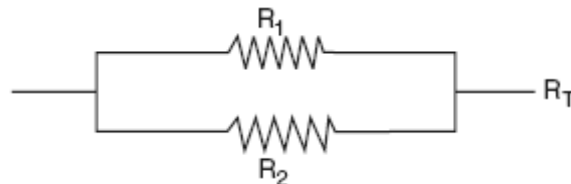
[A] {4,-6} [B] { } [C] {-6} [D] {4}

47. 080125b, P.I. A2.A.23

Working by herself, Mary requires 16 minutes more than Antoine to solve a mathematics problem. Working together, Mary and Antoine can solve the problem in 6 minutes. If this situation is represented by the equation $\frac{6}{t} + \frac{6}{t+16} = 1$, where t represents the number of minutes Antoine works alone to solve the problem, how many minutes will it take Antoine to solve the problem if he works by himself?

48. 080234b, P.I. A2.A.23

Electrical circuits can be connected in series, one after another, or in parallel circuits that branch off a main line. If circuits are hooked up in parallel, the reciprocal of the total resistance in the series is found by adding the reciprocals of each resistance, as shown in the accompanying diagram.



$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_T}$$

If $R_1 = x$, $R_2 = x + 3$, and the total resistance, R_T , is 2.25 ohms, find the positive value of R_1 to the nearest tenth of an ohm.

Part 2: Solving Proportions

49. 060612a, P.I. A.A.26

What is the value of x in the equation

$$\frac{x}{2x+1} = \frac{4}{3}?$$

[A] $-\frac{1}{5}$ [B] $-\frac{5}{4}$ [C] $-\frac{4}{5}$ [D] -5

50. 080439a, P.I. A.A.26

Solve for all values of x that satisfy the

$$\text{equation } \frac{x}{x+3} = \frac{5}{x+7}.$$

51. 010131a, P.I. A.A.26

Solve algebraically for x : $\frac{1}{x} = \frac{x+1}{6}$

52. 080722b, P.I. A.A.26

Solve for all values of x : $\frac{2}{x+1} = x$

53. 010224b, P.I. A2.A.23
A rectangle is said to have a golden ratio when $\frac{w}{h} = \frac{h}{w-h}$, where w represents width and h represents height. When $w = 3$, between which two consecutive integers will h lie?

54. fall0739ia, P.I. A.A.26
Solve for x : $\frac{x+1}{x} = \frac{-7}{x-12}$

Lesson 12-7: Counting Methods and Permutations

Part 1: Using the Multiplication Counting Principle

55. 060728a, P.I. A2.S.9
Max goes through the cafeteria line and counts seven different meals and three different desserts that he can choose. Which expression can be used to determine how many different ways Max can choose a meal and a dessert?
[A] $7 \cdot 3$ [B] ${}_7P_3$
[C] $7! \cdot 3!$ [D] ${}_7C_3$
56. 010612a, P.I. A2.S.9
Robin has 8 blouses, 6 skirts, and 5 scarves. Which expression can be used to calculate the number of different outfits she can choose, if an outfit consists of a blouse, a skirt, and a scarf?
[A] $8 + 6 + 5$ [B] $8 \cdot 6 \cdot 5$
[C] ${}_{19}C_3$ [D] $8!6!5!$
57. 060607a, P.I. A2.S.9
Leo purchased five shirts, three pairs of pants, and four pairs of shoes. Which expression represents how many different outfits consisting of one shirt, one pair of pants, and one pair of shoes Leo can make?
[A] ${}_{12}C_3$ [B] $5 \cdot 3 \cdot 4$
[C] ${}_{12}P_3$ [D] $5 + 3 + 4$
58. 080704a, P.I. A2.S.9
Jen and Barry's ice cream stand has three types of cones, six flavors of ice cream, and four kinds of sprinkles. If a serving consists of a cone, one flavor of ice cream, and one kind of sprinkles, how many different servings are possible?
[A] ${}_{13}P_3$ [B] 72 [C] ${}_{13}C_3$ [D] 90
59. 060403a, P.I. A.N.7
How many different outfits consisting of a hat, a pair of slacks, and a sweater can be made from two hats, three pairs of slacks, and four sweaters?
[A] 29 [B] 9 [C] 12 [D] 24
60. 080204a, P.I. A.N.7
Juan has three blue shirts, two green shirts, seven red shirts, five pairs of denim pants, and two pairs of khaki pants. How many different outfits consisting of one shirt and one pair of pants are possible?
[A] 19 [B] 420 [C] 84 [D] 130
61. 089923a, P.I. A.N.7
Paloma has 3 jackets, 6 scarves, and 4 hats. Determine the number of different outfits consisting of a jacket, a scarf, and a hat that Paloma can wear.

62. 080404a, P.I. A.N.7
The school cafeteria offers five sandwich choices, four desserts, and three beverages. How many different meals consisting of one sandwich, one dessert, and one beverage can be ordered?
[A] 12 [B] 60 [C] 3 [D] 1
63. 010503a, P.I. A.N.7
A deli has five types of meat, two types of cheese, and three types of bread. How many different sandwiches, consisting of one type of meat, one type of cheese, and one type of bread, does the deli serve?
[A] 75 [B] 30 [C] 10 [D] 25
64. 080502a, P.I. A.N.7
Cole's Ice Cream Stand serves sixteen different flavors of ice cream, three types of syrup, and seven types of sprinkles. If an ice cream sundae consists of one flavor of ice cream, one type of syrup, and one type of sprinkles, how many different ice cream sundaes can Cole serve?
[A] 26 [B] 3 [C] 336 [D] 10,836
65. 010405a, P.I. A.N.7
In a school building, there are 10 doors that can be used to enter the building and 8 stairways to the second floor. How many different routes are there from outside the building to a class on the second floor?
[A] 18 [B] 80 [C] 10 [D] 1
66. 060501a, P.I. A.N.7
Jeremy's bedroom has two doors leading into the hallway. His house has four doors leading to the outside. Using the doorways, in how many different ways can Jeremy leave his room and go outside?
[A] 4 [B] 6 [C] 5 [D] 8
67. 080111a, P.I. A.N.7
A certain car comes in three body styles with a choice of two engines, a choice of two transmissions, and a choice of six colors. What is the minimum number of cars a dealer must stock to have one car of every possible combination?
[A] 13 [B] 36 [C] 72 [D] 42
68. 080636a, P.I. A.N.7
Debbie goes to a diner famous for its express lunch menu. The menu has five appetizers, three soups, seven entrees, six vegetables, and four desserts. How many different meals consisting of either an appetizer *or* a soup, one entree, one vegetable, and one dessert can Debbie order?
69. 010218a
When Kimberly bought her new car, she found that there were 72 different ways her car could be equipped. Her choices included four choices of engine and three choices of transmission. If her only other choice was color, how many choices of color did she have?
[A] 65 [B] 6 [C] 60 [D] 12
- Part 2: Finding Permutations
70. 080107a, P.I. A.N.6
The value of $5!$ is
[A] 5 [B] 120 [C] 20 [D] $\frac{1}{5}$
71. 080503a, P.I. A.N.6
The value of $\frac{7!}{3!}$ is
[A] 7 [B] 840 [C] 24 [D] 4

72. 060605a, P.I. A.N.6
What is the value of $\frac{8!}{4!}$?
[A] $2!$ [B] $4!$ [C] 2 [D] 1,680
73. 010713a, A2.S.10
Which value is equivalent to ${}_3P_3$?
[A] $3!$ [B] 1 [C] 9 [D] 27
74. 089917a, P.I. A.N.8
How many different 6-letter arrangements can be formed using the letters in the word "ABSENT," if each letter is used only once?
[A] 6 [B] 720 [C] 36 [D] 46,656
75. 010013a, P.I. A.N.8
How many different 4-letter arrangements can be formed using the letters of the word "JUMP," if each letter is used only once?
[A] 4 [B] 24 [C] 12 [D] 16
76. 060723a, P.I. A.N.8
What is the total number of different four-letter arrangements that can be formed from the letters in the word "VERTICAL," if each letter is used only once in an arrangement?
[A] 40,320 [B] 1,680
[C] 8 [D] 6,720
77. 080727a, P.I. A2.S.10
Which expression represents the number of different 8-letter arrangements that can be made from the letters of the word "SAVANNAH" if each letter is used only once?
[A] $8!$ [B] ${}_8P_5$ [C] $\frac{8!}{3!2!}$ [D] $\frac{8!}{5!}$
78. 010829a, P.I. A2.S.10
What is the total number of different seven-letter arrangements that can be formed using the letters in the word "MILLION"?
[A] 30 [B] 2,520 [C] 1,260 [D] 210
79. 010114a, P.I. A.N.8
A locker combination system uses three digits from 0 to 9. How many different three-digit combinations with no digit repeated are possible?
[A] 30 [B] 720 [C] 1,000 [D] 504
80. 060016a, P.I. A.N.8
How many different five-digit numbers can be formed from the digits 1, 2, 3, 4, and 5 if each digit is used only once?
[A] 20 [B] 120 [C] 60 [D] 24
81. 060023a, P.I. A.N.8
All seven-digit telephone numbers in a town begin with 245. How many telephone numbers may be assigned in the town if the last four digits do *not* begin or end in a zero?
82. 080616a, P.I. A.N.8
Julia has four different flags that she wants to hang on the wall of her room. How many different ways can the flags be arranged in a row?
[A] 24 [B] 16 [C] 1 [D] 10
83. 010323a, P.I. A.N.8
Six members of a school's varsity tennis team will march in a parade. How many different ways can the players be lined up if Angela, the team captain, is always at the front of the line?
84. 060125a, P.I. A.N.8
There were seven students running in a race. How many different arrangements of first, second, and third place are possible?

85. 080034a, P.I. A.N.8

The telephone company has run out of seven-digit telephone numbers for an area code. To fix this problem, the telephone company will introduce a new area code. Find the number of new seven-digit telephone numbers that will be generated for the new area code if both of the following conditions must be met:

- o The first digit cannot be a zero or a one.
- o The first three digits cannot be the emergency number (911) or the number used for information (411).

86. 010435a, P.I. A.N.8

In Jackson County, Wyoming, license plates are made with two letters (A through Z) followed by three digits (0 through 9). The plates are made according to the following restrictions:

- o the first letter must be *J* or *W*, and the second letter can be any of the 26 letters in the alphabet

- o no digit can be repeated

How many different license plates can be made with these restrictions?

87. 060329a, P.I. A.N.8

A certain state is considering changing the arrangement of letters and numbers on its license plates. The two options the state is considering are:

Option 1: three letters followed by a four-digit number with repetition of both letters and digits allowed

Option 2: four letters followed by a three-digit number without repetition of either letters or digits

[Zero may be chosen as the first digit of the number in either option.]

Which option will enable the state to issue more license plates? How many *more* different license plates will that option yield?

Lesson 12-8: Combinations

Part 1: Combinations

88. 080527a, P.I. A2.S.11

The expression ${}_9C_2$ is equivalent to

[A] ${}_9C_7$ [B] ${}_9P_7$ [C] ${}_9P_2$ [D] $\frac{9!}{2!}$

89. 080720a, P.I. A2.S.11

The expression ${}_8C_3$ is equivalent to

[A] ${}_8P_3$ [B] ${}_8C_5$ [C] ${}_8P_5$ [D] $\frac{8!}{3!}$

90. 010515a, P.I. A2.S.11

How many different three-member teams can be selected from a group of seven students?

[A] 5,040 [B] 1 [C] 35 [D] 210

91. 010729a, P.I. A2.S.11

If the Math Olympiad Club consists of eighteen students, how many different teams of four students can be formed for competitions?

[A] 3,060 [B] 66 [C] 73,440 [D] 72

92. 069907a, P.I. A2.S.11

How many different three-member teams can be formed from six students?

[A] 720 [B] 120 [C] 216 [D] 20

93. 010307a, P.I. A2.S.9

There are 12 people on a basketball team, and the coach needs to choose 5 to put into a game. How many different possible ways can the coach choose a team of 5 if each person has an equal chance of being selected?

[A] ${}_{12}C_5$ [B] ${}_{12}P_5$ [C] ${}_5P_{12}$ [D] ${}_5C_{12}$

94. 060320a, P.I. A2.S.11
How many different five-member teams can be made from a group of eight students, if each student has an equal chance of being chosen?
[A] 6,720 [B] 336 [C] 56 [D] 40
95. 080626a, P.I. A2.S.11
In the next Olympics, the United States can enter four athletes in the diving competition. How many different teams of four divers can be selected from a group of nine divers?
[A] 36 [B] 126 [C] 6,561 [D] 3,024
96. 080025a, P.I. A2.S.11
Alan, Becky, Jesus, and Mariah are four students in the chess club. If two of these students will be selected to represent the school at a national convention, how many combinations of two students are possible?
97. 010424a, P.I. A2.S.11
Five people have volunteered to work on an awards dinner at Madison High School. How many different committees of four can be formed from the five people?
[A] 5 [B] 20 [C] 10 [D] 1
98. 010628a, P.I. A2.S.9
A committee of five members is to be randomly selected from a group of nine freshmen and seven sophomores. Which expression represents the number of different committees of three freshmen and two sophomores that can be chosen?
[A] ${}_{16}C_3 \cdot {}_{16}C_2$ [B] ${}_9C_3 + {}_7C_2$
[C] ${}_9C_3 \cdot {}_7C_2$ [D] ${}_9P_3 \cdot {}_7P_2$
99. 060534a, P.I. A2.S.11
An algebra class of 21 students must send 5 students to meet with the principal. How many different groups of 5 students could be formed from this class?
100. 060426a, P.I. A2.S.11
In a game, each player receives 5 cards from a deck of 52 different cards. How many different groupings of cards are possible in this game?
[A] 5! [B] ${}_{52}C_5$ [C] ${}_{52}P_5$ [D] $\frac{52!}{5!}$
101. 080126a, P.I. A2.S.11
Megan decides to go out to eat. The menu at the restaurant has four appetizers, three soups, seven entrees, and five desserts. If Megan decides to order an appetizer or a soup, and one entree, and two different desserts, how many different choices can she make?
102. 080229a, P.I. A2.S.11
On a bookshelf, there are five different mystery books and six different biographies. How many different sets of four books can Emilio choose if two of the books must be mystery books and two of the books must be biographies?
103. 060114a, P.I. A2.S.11
If there are four teams in a league, how many games will have to be played so that each team plays every other team once?
[A] 3 [B] 6 [C] 8 [D] 16
104. 060632a, P.I. A2.S.11
Five friends met for lunch, and they all shook hands. Each person shook the other person's right hand only once. What was the total number of handshakes?

Part 2: Probability with Counting Techniques

105. 010034a, P.I. A2.S.12
Three roses will be selected for a flower vase. The florist has 1 red rose, 1 white rose, 1 yellow rose, 1 orange rose and 1 pink rose from which to choose.
a How many different three rose selections can be formed from the 5 roses?
b What is the probability that 3 roses selected at random will contain 1 red rose, 1 white rose, and 1 pink rose?
c What is the probability that 3 roses selected at random will *not* contain an orange rose?
106. 060034a, P.I. A2.S.12
Paul orders a pizza. Chef Carl randomly chooses two different toppings to put on the pizza from the following: pepperoni, onion, sausage, mushrooms, and anchovies. If Paul will not eat pizza with mushrooms, determine the probability that Paul will *not* eat the pizza Chef Carl has made.
107. 010126a, P.I. A2.S.12
Sal has a small bag of candy containing three green candies and two red candies. While waiting for the bus, he ate two candies out of the bag, one after another, without looking. What is the probability that both candies were the same color?
108. 060234a, P.I. A2.S.12
Alexi's wallet contains four \$1 bills, three \$5 bills, and one \$10 bill. If Alexi randomly removes two bills without replacement, determine whether the probability that the bills will total \$15 is greater than the probability that the bills will total \$2.
109. 069932a, P.I. A2.S.12
A bookshelf contains six mysteries and three biographies. Two books are selected at random without replacement.
a What is the probability that both books are mysteries?
b What is the probability that one book is a mystery and the other is a biography?

[1] B _____

[2] B _____

[3] C _____

[4] D _____

[5] C _____

[6] B _____

[7] D _____

[8] A _____

[9] B _____

[10] C _____

[11] C _____

[12] C _____

[13] B _____

[14] A _____

[15] B _____

[16] A _____

[17] D _____

[18] D _____

[19] C _____

[20] B _____

[21] D _____

[2] $\frac{3x}{3x+5y}$

[1] One correct factoring is shown, either $3x(3x - 5y)$ or $(3x - 5y)(3x + 5y)$.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[22] incorrect procedure. _____

[2] $\frac{x+1}{x-5}$, and appropriate work is shown.

[1] Only one expression is factored correctly, such as $(x + 5)(x + 1)$ or $(x + 5)(x - 5)$, but an appropriate simplification is done.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[23] incorrect procedure. _____

[24] B _____

[2] $4(x - 2)$ or $4x - 8$, and appropriate work is shown.

[1] The problem is factored correctly but not reduced to simplest form.

or [1] Only two of the expressions are factored correctly, but an appropriate answer is found.

or [1] $4(x - 2)$ or $4x - 8$, but no work is shown.

[0] Only the formula for volume is shown.

or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an

[25] obviously incorrect procedure. _____

[2] $\frac{x-3}{3}$ and multiplication by the reciprocal,

correct factoring, and canceling are shown.

[1] The difference of two squares,

$x^2 - 9 = (x + 3)(x - 3)$, is factored correctly.

or [1] Appropriate work is shown, but the final answer is incorrect.

or [1] $\frac{x-3}{3}$ but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[26] incorrect procedure. _____

[4] $\frac{x+3}{2}$, and appropriate work is shown.

[3] Appropriate work is shown, but one computational, factoring, or simplification error is made.

[2] Appropriate work is shown, but two or more computational, factoring, or simplification errors are made.

or [2] Appropriate work is shown, but one conceptual error is made, such as failing to multiply by the reciprocal of $g(x)$ or trying to solve for x .

[1] Appropriate work is shown, but one conceptual error and one computational, factoring, or simplification error are made.

or [1] $\frac{x+3}{2}$, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[27] incorrect procedure.

[6] $-\frac{8}{3}$, and appropriate work is shown.

[5] Appropriate work is shown, but one computational error is made.

[4] Appropriate work is shown, but two or more computational errors are made.

[3] Appropriate work is shown, but one conceptual error is made, such as not factoring out -1 when canceling out $2 - x$.

[2] Appropriate work is shown, but one conceptual error and one computational error are made.

[1] $-\frac{8}{3}$, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[28] incorrect procedure.

[6] $-(x-3)$, $-x+3$, or $3-x$, and appropriate work is shown.

[5] Appropriate work is shown, but one computational, factoring, or simplification error is made.

[4] Appropriate work is shown, but two computational, factoring, or simplification errors are made.

or [4] $x-3$, and appropriate work is shown.

[3] Appropriate work is shown, but three or more computational, factoring, or simplification errors are made.

or [3] Appropriate work is shown, but one conceptual error is made, such as not multiplying by the multiplicative inverse.

[2] Appropriate work is shown, but one conceptual error and one computational, factoring, or simplification error are made.

[1] $-(x-3)$, $-x+3$, or $3-x$, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[29] incorrect procedure.

[30] A

[31] C

[32] C

[33] A

[34] A

[35] A

[36] D

[37] A

[38] A

[2] $\frac{2x+3}{x(x+3)}$ or $\frac{2x+3}{x^2+3x}$, and appropriate

work is shown.

[1] Appropriate work is shown, but one computational error is made or the answer is not simplified completely.

or [1] Appropriate work is shown, but one conceptual error is made.

or [1] $\frac{2x+3}{x(x+3)}$ or $\frac{2x+3}{x^2+3x}$, but no work is

shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[39] incorrect procedure.

[40] B

[41] D

[6] $\frac{x-4}{2}$, and appropriate work is shown.

[5] Appropriate work is shown, but one computational error is made.

[4] Appropriate work is shown, but two or more computational errors are made.

or [4] Appropriate work is shown, but -1 is not factored out.

[3] Appropriate work is shown, but one conceptual error is made, such as not following the correct order of operations.

[2] Appropriate work is shown, but one conceptual error and one computational error are made.

[1] Appropriate work is shown, but one conceptual error and two or more computational errors are made.

or [1] Appropriate work is shown, but two conceptual errors are made.

or [1] $\frac{x-4}{2}$, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[42] incorrect procedure.

[43] D

[4] $\frac{-3 \pm \sqrt{37}}{7}$ or an equivalent answer, and

appropriate work is shown.

[3] A correct quadratic equation is written and appropriate work is shown, but one computational or simplification error is made.

or [3] An incorrect quadratic equation of equal difficulty is solved appropriately.

[2] A correct quadratic equation is written and appropriate work is shown, but two or more computational or simplification errors are made.

or [2] Appropriate work is shown, but one conceptual error is made.

or [2] A correct quadratic equation is written in standard form, but no further correct work is shown.

[1] An incorrect equation of a lesser degree of difficulty is solved appropriately.

or [1] $\frac{-3 \pm \sqrt{37}}{7}$ or an equivalent answer, but

no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[44] incorrect procedure.

[4] 3 and $\frac{1}{2}$, and appropriate work is shown.

[3] Appropriate work is shown, but one computational error is made.

or [3] Appropriate work is shown, but only one of the values is found.

[2] Appropriate work is shown, but two or more computational errors are made.

or [2] Appropriate work is shown, but one conceptual error is made.

or [2] The correct quadratic equation is written in standard form, but no further correct work is shown.

or [2] An incorrect quadratic equation of equal difficulty is solved appropriately.

[1] Appropriate work is shown, but one conceptual error and one computational error are made.

or [1] An incorrect equation of a lesser degree of difficulty is solved appropriately.

or [1] 3 and $\frac{1}{2}$ but no work is shown.

[0] 3 or $\frac{1}{2}$, but no work is shown.

or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an

[45] obviously incorrect procedure.

[46] C

[2] 8 or an equivalent answer, and appropriate work is shown.

[1] The denominators are cleared correctly, such as $6(t+16)+6t=t(t+16)$, but the factoring is incorrect, or one error is made using the quadratic formula.

or [1] The denominators are not cleared correctly, but an equation of equal difficulty is solved.

or [1] 8 or an equivalent answer, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[47] incorrect procedure.

[6] 3.5, and appropriate work is shown.

[5] Appropriate work is shown, but one computational or rounding error is made.

[4] A substitution error is made, resulting in an incorrect quadratic equation of equal difficulty, but the incorrect equation is solved appropriately.

[3] A correct substitution is made, resulting in the correct quadratic equation in standard form, but the equation is not solved.

[2] A substitution error is made, resulting in an incorrect equation of equal difficulty, and one computational or rounding error is made.

[1] 3.5, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[48] incorrect procedure.

[49] C

- [4] 3 and -5 , and appropriate work is shown, such as $x(x + 7) = 5(x + 3)$ or trial and error with at least three trials and appropriate checks for each solution.
- [3] Appropriate work is shown, but one computational or factoring error is made.
or [3] Appropriate work is shown, but only one correct solution is found.
or [3] The trial-and-error method is used to find both correct solutions, but only two trials and appropriate checks are shown for each solution.
- [2] Appropriate work is shown, but two or more computational or factoring errors are made.
or [2] A correct quadratic equation is written and factored, but no further correct work is shown.
or [2] The trial-and-error method is attempted and at least six systematic trials and appropriate checks are shown, but neither solution is found.
- [1] A correct quadratic equation is written, but no further correct work is shown.
or [1] 3 and -5 , but no work or only one trial with an appropriate check is shown.
- [0] 3 or -5 , but no work or only one trial with an appropriate check is shown.
or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
- [50] _____
- [4] 2 and -3 , and a correct quadratic equation is shown, such as $x(x + 1) = 6$, and solved algebraically.
- [3] The student shows a correct quadratic equation but makes one algebraic error and carries it to solution or no solution for the equation generated.
or [3] Correct work is shown, but only one root is found as the answer.
- [2] A correct quadratic equation is used, but two or more errors are made.
or [2] An incorrect quadratic equation of equal difficulty is shown and solved appropriately.
- [1] The student cross multiplies but produces only a linear equation that is solved appropriately.
or [1] 2 and -3 , but no algebraic work is shown.
- [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
- [51] _____
- [2] 1 and -2 , and appropriate work is shown.
- [1] Appropriate work is shown, but one computational error is made.
or [1] Appropriate work is shown, but one conceptual error is made.
or [1] Appropriate work is shown, but only one value is found.
or [1] 1 and -2 , but no work is shown.
- [0] 1 or -2 , but no work is shown.
or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
- [52] _____

[2] 1 and 2, $1 < h < 2$, or $1 < 1.854 < 2$, and appropriate work is shown.

[1] $\frac{3}{h} = \frac{h}{3-h}$ is shown, but one

computational error is made.

or [1] The positive root, 1.854, is obtained from the quadratic, but the two correct consecutive integers are not stated.

or [1] 1 and 2, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[53] incorrect procedure.

[4] 6 and -2, and appropriate work is shown, such as an algebraic solution or trial and error with at least three trials and appropriate checks.

[3] Appropriate work is shown, but one computational or factoring error is made.

or [3] Appropriate work is shown, but only one solution is found.

[2] The correct quadratic equation is written in standard form.

or [2] Appropriate work is shown, but two or more computational or factoring errors are made.

or [2] Appropriate work is shown, but one conceptual error is made.

or [2] The trial-and-error method is used to find at least one solution, but only two trials and appropriate checks are shown.

or [2] The trial-and-error method is attempted and at least six systematic trials and appropriate checks are shown, but no solution is found.

or [2] An incorrect quadratic equation of equal difficulty is solved appropriately.

[1] $x^2 - 11x - 12 = -7x$, but no further correct work is shown.

or [1] 6 and -2, but no work or only one trial with an appropriate check is shown.

or [1] An incorrect equation of a lesser degree of difficulty is solved appropriately.

or [1] Appropriate work is shown, but one conceptual error and one computational or factoring error are made.

[0] 6 or -2, but no work is shown.

or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an

[54] obviously incorrect procedure.

[55] A

[56] B

[57] B

[58] B

[59] D

[60] C

[2] 72 and an appropriate method, such as $3 \times 6 \times 4$, is shown.
[1] 72 and no explanation is given.
or [1] An appropriate method is shown, but the student has one computational mistake or an incomplete listing, such as 2 of the 3 clothing categories.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[61] incorrect procedure.

[62] B

[63] B

[64] C

[65] B

[66] D

[67] C

[3] 1,344, and appropriate work is shown, such as $8 \cdot 7 \cdot 6 \cdot 4$.

[2] Appropriate work is shown, but one computational error is made.

[1] Appropriate work is shown, but two or more computational errors are made.

or [1] Appropriate work is shown, but one conceptual error is made, such as basing the answer on ordering an appetizer and a soup, using $5 \cdot 3 \cdot 7 \cdot 6 \cdot 4$.

or [1] 1,344, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[68] incorrect procedure.

[69] B

[70] B

[71] B

[72] D

[73] A

[74] B

[75] B

[76] B

[77] C

[78] C

[79] B

[80] B

[2] 8,100 and appropriate work is shown, such as $9 \times 10 \times 10 \times 9$.

[1] 10,000 but appropriate work is shown.

or [1] Appropriate work is shown, but the student multiplies incorrectly.

or [1] An appropriate pattern is shown, such as $9 \times 10 \times 10 \times 9$.

or [1] 8,100 but no work is shown.

[0] 38 is shown.

or [0] The student attempts to use the counting principle, but adds.

or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[81] incorrect procedure.

[82] A

[2] 120, and appropriate work is shown, such as $1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$.

[1] Appropriate work is shown, but one computational error is made.

or [1] 720 and ${}_6P_6$ or $6!$ is shown.

or [1] 120, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[83] incorrect procedure.

[2] 210, and appropriate work is shown, such as $7 \cdot 6 \cdot 5$ or ${}_7P_3$.
 [1] Appropriate work is shown, but no answer or an incorrect answer is found.
 or [1] 210, but no work is shown.
 [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[4] 7.98×10^6 or 7,980,000 and appropriate work is shown, such as $8 \times 10^6 - 2 \times 10^4$.
 [3] Appropriate work is shown, but one computational error is made.
 or [3] The student uses 1–9 instead of 0–9 as the number of digits in $8 \times 9^6 - 2 \times 9^4$.
 [2] The student correctly produces only one part, 8×10^6 or 2×10^4 , but carries the process to an appropriate result.
 or [2] Appropriate work is shown, but more than one error is made.
 [1] The student produces only one part, 8×9^6 or 2×9^4 .
 or [1] 7,980,000 but no work is shown.
 [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[2] 37,440 and appropriate work is shown, such as $2 \times 26 \times 10 \times 9 \times 8$ or ${}_2P_1 \times {}_{26}P_1 \times {}_{10}P_1 \times {}_9P_1 \times {}_8P_1$.
 [1] Appropriate work is shown, but one computational or conceptual error is made.
 or [1] Appropriate work is shown for at least one restriction, such as 2×26 or $10 \times 9 \times 8$.
 or [1] 37,440 but no work is shown.
 [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[3] Option 2 will yield 82,576,000 more possibilities, and appropriate work is shown, such as $26^3 \cdot 10^4$ and ${}_{26}P_4 \cdot {}_{10}P_3$.

[2] Appropriate work is shown, but one computational error is made, but the appropriate option is identified.
 or [2] The correct numbers of arrangements are found for both Option 1 and Option 2, but the question of which option will yield more arrangements is not answered or is answered incorrectly.

[1] Appropriate work is shown, but more than one computational error is made, but the appropriate option is identified.

or [1] Appropriate work is shown, but one conceptual error is made, but the appropriate option is identified.

or [1] Either Option 1 or Option 2 is found correctly, but no further correct work is shown.

or [1] Option 2 will yield 82,576,000 more possibilities, but no work is shown.

[0] Option 2, but no work or inappropriate work is shown.

or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[87] _____

[88] A

[89] B

[90] C

[91] A

[92] D

[93] A

[94] C

[95] B

[2] 6 and appropriate work is shown, such as using the combination ${}_4C_2$, listing all six possible outcomes, or drawing a correct tree diagram.

[1] A correct setup of combinations is shown, but an incorrect solution, such as leaving ${}_4C_2$, or no integral solution is found.

or [1] An appropriate list or tree diagram is shown, but an incorrect solution is found, such as 5, by omitting one of the possible combinations.

or [1] 12 but a complete list or tree diagram is shown.

or [1] 6 but no work is shown.

[0] The answer is completely incorrect, such as ${}_4P_2$ or 4×3 .

or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an

[96] obviously incorrect procedure.

[97] A

[98] C

[2] 20,349, and appropriate work is shown, such as ${}_{21}C_5 = 20,349$.

[1] Appropriate work is shown, but one computational error is made.

or [1] Appropriate work is shown, but one conceptual error is made, such as determining the value of ${}_{21}P_5$.

or [1] 20,349, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[99] incorrect procedure.

[100] B

[3] 490, and appropriate work is shown, such as $7 \cdot 7 \cdot 10$.

[2] Appropriate work is shown, but one computational error is made.

or [2] Appropriate work is shown, but an incorrect answer is found, based on an incorrect

number of possible dessert combinations or an incorrect number of soup or appetizer choices.

or [2] Appropriate work is shown, but an incorrect answer is found, based on one error in the tree diagram.

or [2] $\frac{1}{490}$, but appropriate work is shown.

[1] 7, 7, and 10 are added instead of multiplied.

or [1] The counting principle is used correctly, but incorrect substitutions are made, but

an appropriate answer is shown.

or [1] 490, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[101] incorrect procedure.

[3] 150, and appropriate work is shown, such as ${}_5C_2 \cdot {}_6C_2$.

[2] Appropriate work is shown, but one computational error is made.

or [2] All the possible combinations of two mystery books and all the possible combinations of two biographies are calculated, but the answers are not multiplied.

[1] Appropriate work is shown, but more than one computational error is made.

or [1] Appropriate work is shown, but one conceptual error is made, such as the computation ${}_{11}C_4 = 330$.

or [1] 150, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[102] incorrect procedure.

[103] B

[2] 10, and appropriate work is shown, such as ${}_5C_2$ or a diagram or a list.

[1] Appropriate work is shown, but one computational error is made.

or [1] Appropriate work is shown, but one conceptual error is made.

or [1] 10, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[104] incorrect procedure.

a [2] 10 and an appropriate tree diagram, list, sample space, or ${}_5C_3 = 10$ is shown.

[1] 10 and no work is shown.

or [1] An appropriate method is shown, but not all 10 possible combinations are listed

b [1] $\frac{1}{10}$

or [1] An appropriate answer is found for an incorrect response in part a.

c [1] $\frac{4}{10}$ or $\frac{2}{5}$ or 0.4

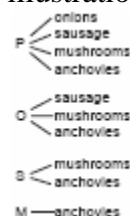
or [1] An appropriate answer is found for an incorrect response in part a.

a and b and c [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by

[105] an obviously incorrect procedure.

[4] $\frac{4}{10}$ and appropriate work is shown, such

as the following illustration or any other correct method:



[3] Appropriate work is shown, but one computational error is made.

or [3] Appropriate work and complement $\frac{6}{10}$ are shown.

or [3] Appropriate work is shown, but the answer is incomplete.

[2] ${}_5C_2$ and the work is appropriate but incomplete.

or [2] 10 but appropriate work is shown.

or [2] A correct sample space or tree diagram is shown.

[1] Incorrect work leading to $0 \leq \text{fraction} \leq 1$ or $0 \leq \text{percent} \leq 100$ is shown.

or [1] $\frac{4}{10}$ but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[106] incorrect procedure.

[3] $\frac{8}{20}$ or an equivalent answer, and

appropriate work is shown, such as using a tree diagram or writing the equation

$$\frac{6}{20} + \frac{2}{20} = \frac{8}{20}.$$

[2] One computational error is made in

finding $\frac{6}{20}$ or $\frac{2}{20}$, but an appropriate sum is found.

or [2] $\frac{2}{20}$ and $\frac{6}{20}$ are found, but no sum is shown.

[1] $\frac{6}{20}$ or $\frac{2}{20}$, and appropriate work is shown.

or [1] An appropriate answer is found, using replacement with a tree diagram or an

equation such as $\frac{3}{5} \cdot \frac{3}{5} + \frac{2}{5} \cdot \frac{2}{5} = \frac{13}{25}$.

or [1] $\frac{8}{20}$, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[107] incorrect procedure.

[4] No, and an appropriate explanation is given, such as $P(15) = \frac{6}{56} < P(2) = \frac{12}{56}$.

[3] One of the two probabilities is found correctly, but one computational error is made in finding the other, but an appropriate conclusion is drawn, based on the values found.

or [3] Replacement is used to conclude

$$P(15) = \frac{6}{64} < P(2) = \frac{12}{64}.$$

or [3] The two probabilities are found correctly, but no conclusion or the incorrect conclusion is drawn.

[2] One of the probabilities is found correctly, but one computational error is made in finding the other, and no conclusion or the incorrect conclusion is drawn.

[1] An appropriate method is used, such as a tree diagram or sample space, but the probabilities are not determined or are determined incorrectly.

or [1] $P(15) = \frac{6}{56} < P(2) = \frac{12}{56}$, but no work is shown.

[0] No, but no work is shown.

or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct

response that was obtained by an obviously

[108] incorrect procedure.

a [2] $\frac{30}{72}$ or an equivalent value is found and

an appropriate explanation is given.

[1] An acceptable method is used correctly, such as a tree diagram, sample space, or combinations, but the correct answer is not given.

or [1] Replacement is used, and an answer of $\frac{36}{81}$ or an equivalent is found.

or [1] $\frac{30}{72}$ and no work is shown.

b [2] $\frac{36}{72}$ or an equivalent value is found and

an appropriate explanation is given.

or [2] An appropriate probability for an incorrect denominator for part a is found.

[1] An appropriate method is shown, but one computational mistake is made.

or [1] Replacement is shown, and the answer $\frac{36}{81}$ or an equivalent is found.

or [1] The student does not take into account both orders and answers $\frac{18}{72}$ or an equivalent.

or [1] $\frac{36}{72}$ and no work is shown.

or a and b

[1] An error in method is made but the erroneous answer is interpreted correctly in either part a or b or both.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[109] incorrect procedure.
