# Lesson 2-2: Subtracting Real Numbers

## Part 2: Applying Subtraction

1. 010403a, P.I. 7.N.13
On February 18, from 9 a.m. until 2 p.m., the temperature rose from $-14°$ F to $36°$ F. What was the total increase in temperature during this time period?

[A] 50° [B] 32° [C] 36° [D] 22°

# Lesson 2-4: The Distributive Property

## Part 2: Simplifying Algebraic Expressions

2. 060625a, P.I. A.A.17
The expression $\frac{5x}{6}+\frac{x}{4}$ is equivalent to

[A] $\frac{13x}{12}$ [B] $\frac{3x}{5}$ [C] $\frac{5x^2}{10}$ [D] $\frac{5x}{24}$

3. 080623a
The expression $2x^2 - x^2$ is equivalent to

[A] $x^2$ [B] $-2x^4$ [C] 2 [D] $x^0$

# Lesson 2-5: Properties of Numbers

## Part 1: Identifying and Using Properties

4. 010107a, P.I. A.N.1
If $a$ and $b$ are integers, which equation is always true?

[A] $a-b=b-a$ [B] $a+b=b+a$

[C] $\frac{a}{b}=\frac{b}{a}$ [D] $a+2b=b+2a$

5. 010720a, P.I. A.N.1
If $M$ and $A$ represent integers, $M+A=A+M$ is an example of which property?

[A] closure [B] commutative

[C] associative [D] distributive

6. 060424a, P.I. A.N.1
Which expression is an example of the associative property?

[A] $(x+y)+z=x+(y+z)$

[B] $x\cdot 1=x$ [C] $x+y+z=z+y+x$

[D] $x(y+z)=xy+xz$

7. 010428a, P.I. A.N.1
Which equation illustrates the associative property of addition?

[A] $3(x + 2) = 3x + 6$

[B] $(3 + x) + y = 3 + (x + y)$

[C] $3 + x = 0$ [D] $x + y = y + x$

8. 080725a, P.I. A.N.1
Which equation illustrates the associative property?

[A] $a(b+c)=(ab)+(ac)$ [B] $a(1)=a$

[C] $(a+b)+c=a+(b+c)$

[D] $a+b=b+a$

9. fall0705ia, P.I. A.N.1
Which property is illustrated by the equation $ax+ay=a(x+y)$?

[A] distributive [B] commutative

[C] identity [D] associative

10. 080504a, P.I. A.N.1
The equation $*(\Delta + \Diamond) = *\Delta + *\Diamond$ is an example of the

[A] distributive law [B] associative law

[C] transitive law [D] commutative law

11. 080413a, P.I. A.N.1
Which equation illustrates the distributive property of multiplication over addition?

[A] $6(3a + 4b) = 18a + 24b$

[B] $6(3a + 4b) = (3a + 4b)6$

[C] $6(3a + 4b) = 6(4b + 3a)$

[D] $6(3a + 4b) = 18a + 4b$

12. 010812a, P.I. A.N.1
Which property is represented by the statement $\frac{1}{2}(6a + 4b) = 3a + 2b$?

[A] distributive [B] commutative

[C] identity [D] associative

13. 060503a, P.I. A.N.1
Which equation illustrates the distributive property?

[A] $a + (b + c) = (a + b) + c$

[B] $a + b = b + a$ [C] $5(a + b) = 5a + 5b$

[D] $a + 0 = a$

14. 060108a, P.I. A.N.1
Which equation illustrates the distributive property for real numbers?

[A] $-3(5 + 7) = (-3)(5) + (-3)(7)$

[B] $(1.3\times0.07)\times0.63=1.3\times(0.07\times0.63)$

[C] $\sqrt{3}+0=\sqrt{3}$ [D] $\frac{1}{3}+\frac{1}{2}=\frac{1}{2}+\frac{1}{3}$

15. 060306a, P.I. A.N.1
Tori computes the value of $8\times95$ in her head by thinking $8(100-5)=8\times100-8\times5$.
Which number property is she using?

[A] associative [B] commutative

[C] closure [D] distributive

16. 080601a, P.I. A.N.1
While solving the equation $4(x+2)=28$, Becca wrote $4x+8=28$. Which property did she use?

[A] distributive [B] commutative

[C] identity [D] associative

17. 060714a, P.I. A.N.1
Which property is illustrated by the equation $\frac{3}{2}x+0=\frac{3}{2}x$?

[A] additive identity property

[B] commutative property of addition

[C] additive inverse property

[D] distributive property

18. 089907a, P.I. A.N.1
Which equation is an illustration of the additive identity property?

[A] $x\cdot\frac{1}{x}=1$ [B] $x+0=x$

[C] $x-x=0$ [D] $x\cdot1=x$

19. 060624a, P.I. A.N.1
Which statement best illustrates the additive identity property?

[A] $6 + (-6) = 0$ [B] $6 + 0 = 6$

[C] $6(2) = 2(6)$ [D] $6 + 2 = 2 + 6$

20. 010314a, P.I. A.N.1
Which equation illustrates the multiplicative identity element?

[A] $x \cdot \frac{1}{x} = 1$ [B] $x + 0 = x$

[C] $x \cdot 1 = x$ [D] $x - x = 0$

21. 010207a, P.I. A.N.1
Which expression must be added to $3x$ - 7 to equal 0?

[A] 0 [B] $3x$ + 7

[C] -$3x$ + 7 [D] -$3x$ - 7

22. 060315a, P.I. A.N.1

What is the additive inverse of $\frac{2}{3}$?

[A] $-\frac{3}{2}$ [B] $\frac{3}{2}$ [C] $\frac{1}{3}$ [D] $-\frac{2}{3}$

23. 010821a, P.I. A.N.1

The additive inverse of $\frac{1}{a}$ is

[A] $a$ [B] $-a$ [C] $-\frac{1}{a}$ [D] 0

24. 060413a, P.I. A.N.1
Which property of real numbers is illustrated by the equation $-\sqrt{3} + \sqrt{3} = 0$

[A] additive identity

[B] commutative property of addition

[C] associative property of addition

[D] additive inverse

25. 060011a, P.I. A.N.1

If $a \neq 0$ and the sum of $x$ and $\frac{1}{a}$ is 0, then

[A] $x = -\frac{1}{a}$ [B] $x$ = -$a$

[C] $x$ = 1 - $a$ [D] $x = a$

26. 010516a, P.I. A.N.1

What is the multiplicative inverse of $\frac{3}{4}$?

[A] $\frac{4}{3}$ [B] -1 [C] $-\frac{3}{4}$ [D] $-\frac{4}{3}$

27. 010730a, P.I. A.N.1

The multiplicative inverse of $-\frac{1}{3}$ is

[A] -3 [B] $\frac{1}{3}$ [C] 3 [D] $-\frac{1}{3}$

28. 010630a, P.I. A.N.1
Which equation illustrates the multiplicative inverse property?

[A] $-1 \cdot x = -x$ [B] $1 \cdot 0 = 0$

[C] $1 \cdot x = x$ [D] $x \cdot \frac{1}{x} = 1$

29. 080129a, P.I. A.N.1
Ramón said that the set of integers is *not* closed for one of the basic operations (addition, subtraction, multiplication, or division). You want to show Ramón that his statement is correct. For the operation for which the set of integers is *not* closed, write an example using:
- a positive even integer and a zero
- a positive and a negative even integer
- two negative even integers

Be sure to explain why *each* of your examples illustrates that the set of integers is *not* closed for that operation.

30. 010217a, P.I. A.N.1
Which set is closed under division?

[A] whole numbers [B] integers

[C] {1} [D] counting numbers

31. 060224a, P.I. A.N.1
An addition table for a subset of real numbers is shown below. Which number is the identity element? Explain your answer.

| + | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 |
| 1 | 1 | 2 | 3 | 4 |
| 2 | 2 | 3 | 4 | 0 |
| 3 | 3 | 4 | 0 | 1 |

32. 080112a, P.I. A.N.1
The operation element @ is determined by the following table:

| @ | *a* | *b* | *c* |
|---|---|---|---|
| *a* | *a* | *b* | *c* |
| *b* | *b* | *c* | *a* |
| *c* | *c* | *a* | *b* |

What is the identity element of this operation?

[A] *a* and *b* [B] *c*

[C] *b*, only [D] *a*, only

33. 080514a, P.I. A.N.1
What is the identity element for ♣ in the accompanying table?

| ♣ | *r* | *s* | *t* | *u* |
|---|---|---|---|---|
| *r* | *t* | *r* | *u* | *s* |
| *s* | *r* | *s* | *t* | *u* |
| *t* | *u* | *t* | *s* | *r* |
| *u* | *s* | *u* | *r* | *t* |

[A] *u* [B] *r* [C] *t* [D] *s*

34. 080222a, P.I. A.N.1
In the addition table for a subset of real numbers shown below, which number is the inverse of 3? Explain your answer.

| ⊕ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 1 |
| 2 | 3 | 4 | 1 | 2 |
| 3 | 4 | 1 | 2 | 3 |
| 4 | 1 | 2 | 3 | 4 |

35. 080010a, P.I. A.N.1
The operation * for the set {*p*,*r*,*s*,*v*} is defined in the accompanying table. What is the inverse element of *r* under the operation *?

| * | *p* | *r* | *s* | *v* |
|---|---|---|---|---|
| *p* | *s* | *v* | *p* | *r* |
| *r* | *v* | *p* | *r* | *s* |
| *s* | *p* | *r* | *s* | *v* |
| *v* | *r* | *s* | *v* | *p* |

[A] *v* [B] *r* [C] *s* [D] *p*

# Lesson 2-6: Theoretical and Experimental Probability

## Part 1: Theoretical Probability

36. 060630a, P.I. A.S.20
Which inequality represents the probability, $x$, of any event happening?

[A] $0 \le x \le 1$ [B] $0 < x < 1$

[C] $x < 1$ [D] $x \ge 0$

37. 069901a, P.I. A.S.20
A fair coin is thrown in the air four times. If the coin lands with the head up on the first three tosses, what is the probability that the coin will land with the head up on the fourth toss?

[A] $\frac{1}{8}$ [B] 0 [C] $\frac{1}{2}$ [D] $\frac{1}{16}$

38. 010209a, P.I. A.S.20
A fair coin is tossed three times. What is the probability that the coin will land tails up on the second toss?

[A] $\frac{1}{2}$ [B] $\frac{1}{3}$ [C] $\frac{2}{3}$ [D] $\frac{3}{4}$

39. 060712a, P.I. A.S.20
When a fair coin was tossed ten times, it landed heads up the first seven times. What is the probability that on the eighth toss the coin will land with tails up?

[A] $\frac{3}{7}$ [B] $\frac{7}{10}$ [C] $\frac{1}{2}$ [D] $\frac{3}{10}$

40. 010709a, P.I. A.S.20
Seth tossed a fair coin five times and got five heads. The probability that the next toss will be a tail is

[A] 0 [B] $\frac{1}{2}$ [C] $\frac{1}{6}$ [D] $\frac{5}{6}$

41. 010832a, P.I. A.S.20
As captain of his football team, Jamal gets to call heads or tails for the toss of a fair coin at the beginning of each game. At the last three games, the coin has landed with heads up. What is the probability that the coin will land with heads up at the next game? Explain your answer.

42. 060415a, P.I. A.S.20
Mary chooses an integer at random from 1 to 6. What is the probability that the integer she chooses is a prime number?

[A] $\frac{4}{6}$ [B] $\frac{3}{6}$ [C] $\frac{2}{6}$ [D] $\frac{5}{6}$

43. 080011a, P.I. A.S.20
A box contains six black balls and four white balls. What is the probability of selecting a black ball at random from the box?

[A] $\frac{6}{10}$ [B] $\frac{1}{10}$ [C] $\frac{4}{6}$ [D] $\frac{6}{4}$

44. 060705a, P.I. A.S.20
A six-sided number cube has faces with the numbers 1 through 6 marked on it. What is the probability that a number less than 3 will occur on one toss of the number cube?

[A] $\frac{4}{6}$ [B] $\frac{3}{6}$ [C] $\frac{2}{6}$ [D] $\frac{1}{6}$

45. 080604a, P.I. A.S.20
The faces of a cube are numbered from 1 to 6. What is the probability of *not* rolling a 5 on a single toss of this cube?

[A] $\frac{5}{6}$ [B] $\frac{1}{5}$ [C] $\frac{1}{6}$ [D] $\frac{4}{5}$

46. 060202a, P.I. A.S.20
If the probability that it will rain on Thursday is $\frac{5}{6}$, what is the probability that it will *not* rain on Thursday?

[A] $\frac{5}{6}$ [B] 1 [C] 0 [D] $\frac{1}{6}$

47. 010017a, P.I. A.S.20
The party registration of the voters in Jonesville is shown in the table below.

| Registered Voters in Jonesville | |
|---|---|
| **Party Registration** | **Number of Voters Registered** |
| Democrat | 6,000 |
| Republican | 5,300 |
| Independent | 3,700 |

If one of the registered Jonesville voters is selected at random, what is the probability that the person selected is *not* a Democrat?

[A] 0.667 [B] 0.400

[C] 0.333 [D] 0.600

48. 010805a, P.I. A.S.20
A box contains 6 dimes, 8 nickels, 12 pennies, and 3 quarters. What is the probability that a coin drawn at random is *not* a dime?

[A] $\frac{23}{29}$ [B] $\frac{6}{29}$ [C] $\frac{12}{29}$ [D] $\frac{8}{29}$

49. fall0702ia, P.I. A.S.20
Throughout history, many people have contributed to the development of mathematics. These mathematicians include Pythagoras, Euclid, Hypatia, Euler, Einstein, Agnesi, Fibonacci, and Pascal. What is the probability that a mathematician's name selected at random from those listed will start with either the letter *E* or the letter *A*?

[A] $\frac{3}{8}$ [B] $\frac{2}{8}$ [C] $\frac{6}{8}$ [D] $\frac{4}{8}$

50. 010811a, P.I. A.S.20
Which event has a probability of zero?

[A] choosing a triangle that is both isosceles and right

[B] choosing a letter from the alphabet that has line symmetry

[C] choosing a pair of parallel lines that have unequal slopes

[D] choosing a number that is greater than 6 and is even

51. 010321a, P.I. A.S.19
If Laquisha can enter school by any one of three doors and the school has two staircases to the second floor, in how many different ways can Laquisha reach a room on the second floor? Justify your answer by drawing a tree diagram or listing a sample space.

52. 010731a, P.I. A.S.19
Kimberly has three pair of pants: one black, one red, and one tan. She also has four shirts: one pink, one white, one yellow, and one green. Draw a tree diagram or list the sample space showing all possible outfits that she could wear, if an outfit consists of one pair of pants and one shirt. How many different outfits can Kimberly wear?

53. 089922a, P.I. A.S.19
The Grimaldis have three children born in different years.
*a* Draw a tree diagram or list a sample space to show all the possible arrangements of boy and girl children in the Grimaldi family.
*b* Using your information from part *a*, what is the probability that the Grimaldis have three boys?

54. fall0736ia, P.I. A.S.19
Mr. Laub has three children: two girls (Sue and Karen) and one boy (David). After each meal, one child is chosen at random to wash dishes. If the same child can be chosen for both lunch and dinner, construct a tree diagram or list a sample space of all the possible outcomes of who will wash dishes after lunch and dinner on Saturday. Determine the probability that one boy and one girl will wash dishes after lunch and dinner on Saturday.

## Lesson 2-7: Probability of Compound Events

### Part 1: Finding the Probability of Independent Events

55. 080430a, P.I. A.S.23
Selena and Tracey play on a softball team. Selena has 8 hits out of 20 times at bat, and Tracey has 6 hits out of 16 times at bat. Based on their past performance, what is the probability that both girls will get a hit next time at bat?

[A] $\frac{48}{320}$ [B] $\frac{14}{36}$ [C] $\frac{31}{40}$ [D] 1

56. 060529a, P.I. A.S.23
The probability that the Cubs win their first game is $\frac{1}{3}$. The probability that the Cubs win their second game is $\frac{3}{7}$. What is the probability that the Cubs win both games?

[A] $\frac{16}{21}$ [B] $\frac{6}{7}$ [C] $\frac{1}{7}$ [D] $\frac{2}{5}$

### Part 2: Finding the Probability of Dependent Events

57. 060305a, P.I. A.S.23
Bob and Laquisha have volunteered to serve on the Junior Prom Committee. The names of twenty volunteers, including Bob and Laquisha, are put into a bowl. If two names are randomly drawn from the bowl without replacement, what is the probability that Bob's name will be drawn first and Laquisha's name will be drawn second?

[A] $\frac{1}{20}\cdot\frac{1}{20}$ [B] $\frac{2}{20!}$

[C] $\frac{2}{20}$ [D] $\frac{1}{20}\cdot\frac{1}{19}$

58. 010525a, P.I. A.S.23
A student council has seven officers, of which five are girls and two are boys. If two officers are chosen at random to attend a meeting with the principal, what is the probability that the first officer chosen is a girl and the second is a boy?

[A] $\frac{2}{7}$ [B] $\frac{10}{42}$ [C] $\frac{7}{14}$ [D] $\frac{7}{13}$

59. 080127a, P.I. A.S.23
There are four students, all of different heights, who are to be randomly arranged in a line. What is the probability that the tallest student will be first in line and the shortest student will be last in line?

60. 060130a, P.I. A.S.23
Mr. Yee has 10 boys and 15 girls in his mathematics class. If he chooses two students at random to work on the blackboard, what is the probability that both students chosen are girls?

[1] A

[2] A

[3] A

[4] B

[5] B

[6] A

[7] B

[8] C

[9] A

[10] A

[11] A

[12] A

[13] C

[14] A

[15] D

[16] A

[17] A

[18] B

[19] B

[20] C

[21] C

[22] D

[23] C

[24] D

[25] A

[26] A

[27] A

[28] D

[29]
[3] All three examples are illustrated under division correctly, such as $2 \div 0$, $-2 \div 4$, $-2 \div -4$, and correct explanations are given.
[2] Only two of the three examples are illustrated and explained correctly.
or [2] All three examples are illustrated correctly, but only one explanation is given or is correct.
or [2] The division examples and explanations are correct, but at most two incorrect examples are also shown, such as examples for addition, subtraction, or multiplication.
[1] The division examples and explanations are correct, but more than two incorrect examples are shown, such as examples for addition, subtraction, or multiplication.
or [1] All three examples are illustrated correctly, but no correct explanation is given.
or [1] Only one correct example with a correct explanation is given.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[30] C

[31]
[2] 0, and an appropriate explanation is given, such as 0 is the number that when added to any number results in that number or does not change it, or 1 + 0 = 1, 2 + 0 = 2, and 3 + 0 = 3.
[1] 0, but no explanation or an incorrect explanation is given.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[32] D

[33] D

[2] 1, and an appropriate explanation is given, such as when 1 is added to 3, the result is the identity element, 4; therefore 1 is the inverse of 3.
[1] $1 + 3 = 4$, but the identity element is not identified.
or [1] 4 is identified as the inverse because the identity element and inverse element are confused.
or [1] 1, but no explanation or an incorrect explanation is given.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[34]

[35] A

[36] A

[37] C

[38] A

[39] C

[40] B

[2] $\frac{1}{2}$ or an equivalent answer, and an appropriate explanation is written.
[1] A correct explanation is written, but the probability is not stated.
or [1] $\frac{1}{2}$ or an equivalent answer, but no explanation is written.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[41]

[42] B

[43] A

[44] C

[45] A

[46] D

[47] D

[48] A

[49] D

[50] C

[2] 6, and a correct tree diagram is drawn or sample space is listed.
[1] A correct tree diagram is drawn or sample space is listed, but no answer or an incorrect answer is found.
or [1] An appropriate answer is found, based on an incorrect tree diagram or sample space.
or [1] 6, but no tree diagram is drawn or sample space is listed.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[51]

[2] 12, and a correct tree diagram or a correct sample space is shown.
[1] An incomplete tree diagram or sample space is shown with at least 8 possible combinations shown, and an appropriate number of outfits is found.
or [1] A correct tree diagram or sample space is shown, but the number of possible outfits is missing or is incorrect.
or [1] 12, but $3 \times 4$ is used to find the number of outfits.
[0] 12, but no work is shown.
or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[52]

a [1] A correct tree diagram or listing of all 8 possibilities is shown.

b [1] $\frac{1}{8}$

or [1] An appropriate answer is given for an incorrect part a tree diagram or listing.

a and b

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[53]

[3] $\frac{4}{9}$, and a correct tree diagram or sample space is shown.

[2] A correct tree diagram or sample space is shown, but no probability or an incorrect probability is given.

or [2] An incorrect tree diagram or sample space is shown, but an appropriate probability is found.

[1] Appropriate work is shown, but one conceptual error is made.

or [1] $\frac{4}{9}$, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[54]

[55] A

[56] C

[57] D

[58] B

[3] $\frac{2}{24}$ or an equivalent answer, and an appropriate explanation is given or appropriate work is shown, such as a tree diagram, sample space, or permutations.

[2] Appropriate work is shown, but one computational error is made.

or [2] Appropriate work is shown, but only a numerator or a denominator is determined correctly.

or [2] $\frac{2}{24}$ or an equivalent answer, but only work for either the numerator or the denominator is shown.

[1] The probability of the tallest or the probability of the shortest student being in the proper position is correct, such as .

or [1] Only a tree diagram, sample space, or permutations are shown.

or [1] $\frac{2}{24}$ or an equivalent answer, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[59]

[3] $\frac{7}{20}$ or an equivalent answer, and appropriate work is shown, such as $\frac{15}{25} \bullet \frac{14}{24}$ or $\frac{{}_{15}C_2}{{}_{25}C_2}$.

[2] $\frac{15}{25} \bullet \frac{14}{24}$ or $\frac{{}_{15}C_2}{{}_{25}C_2}$ is shown, but one computational error is made or no further work is shown.

or [2] ${}_{15}C_2$ and ${}_{25}C_2$ are computed correctly, but no further work is shown.

or [2] Appropriate work is shown, but one computational error is made.

[1] The correct probabilities are found, but they are added instead of multiplied.

or [1] Only one of the two parts of the probability is correct.

or [1] Appropriate work is shown, but more than one error is made.

or [1] $\frac{7}{20}$ or an equivalent answer, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[60]