Section 7-8: Volumes of Solids

1. 010802a, P.I. A.G.2
A block of wood is 5 inches long, 2 inches wide, and 3 inches high. What is the volume of this block of wood?

[A] 25 in^3 [B] 38 in^3

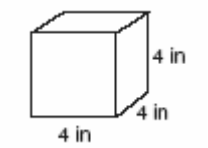
[C] 10 in^3 [D] 30 in^3

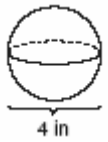
2. 060427a
A box in the shape of a cube has a volume of 64 cubic inches. What is the length of a side of the box?

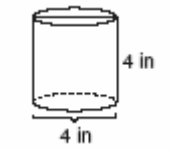
[A] 16 in [B] $21.\overline{3}$ in

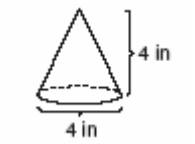
[C] 8 in [D] 4 in

3. 080403a
Which diagram represents the figure with the greatest volume?

[A] 

[B] 

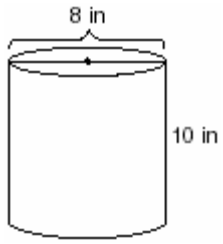
[C] 

[D] 

4. 080007a
The volume of a cube is 64 cubic inches. Its total surface area, in square inches, is

[A] 576 [B] 96 [C] 16 [D] 48

5. 060530a, P.I. A.G.2
A storage container in the shape of a right circular cylinder is shown in the accompanying diagram.

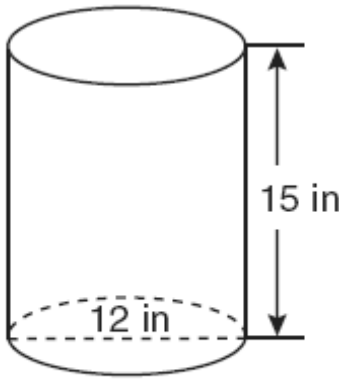


What is the volume of this container, to the *nearest hundredth*?

[A] 125.66 in^3 [B] 251.33 in^3

[C] 502.65 in^3 [D] 56.55 in^3

6. fall0712ia, P.I. A.G.2
A cylindrical container has a diameter of 12 inches and a height of 15 inches, as illustrated in the diagram below.



(Not drawn to scale)

What is the volume of this container to the *nearest tenth* of a cubic inch?

[A] 4,241.2 [B] 1,696.5

[C] 6,785.8 [D] 2,160.0

7. 060103a
If the length of a rectangular prism is doubled, its width is tripled, and its height remains the same, what is the volume of the new rectangular prism?

[A] triple the original volume

[B] nine times the original volume

[C] six times the original volume

[D] double the original volume

8. 010711a
A planned building was going to be 100 feet long, 75 feet deep, and 30 feet high. The owner decides to increase the volume of the building by 10% without changing the dimensions of the depth and the height. What will be the new length of this building?

[A] 108 ft [B] 112 ft

[C] 110 ft [D] 106 ft

9. 010123a, P.I. A.G.2
A cardboard box has length $x-2$, width $x+1$, and height $2x$.
a Write an expression, in terms of *x*, to represent the volume of the box.
b If $x = 8$ centimeters, what is the number of cubic centimeters in the volume of the box?

10. 010030a
The volume of a rectangular pool is 1,080 cubic meters. Its length, width, and depth are in the ratio 10:4:1. Find the number of meters in each of the three dimensions of the pool.

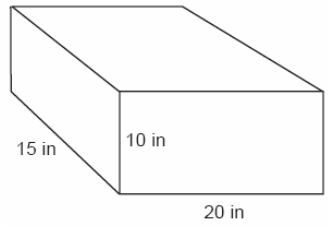
11. 010324a
A fish tank with a rectangular base has a volume of 3,360 cubic inches. The length and width of the tank are 14 inches and 12 inches, respectively. Find the height, in inches, of the tank.

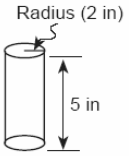
12. 069927a
The dimensions of a brick, in inches, are 2 by 4 by 8. How many such bricks are needed to have a total volume of exactly 1 cubic foot?

13. 060327a
Tina's preschool has a set of cardboard building blocks, each of which measures 9 inches by 9 inches by 4 inches. How many of these blocks will Tina need to build a wall 4 inches thick, 3 feet high, and 12 feet long?

14. 060028a, P.I. G.G.16
Tamika has a hard rubber ball whose circumference measures 13 inches. She wants to box it for a gift but can only find cube-shaped boxes of sides 3 inches, 4 inches, 5 inches, or 6 inches. What is the *smallest* box that the ball will fit into with the top on?

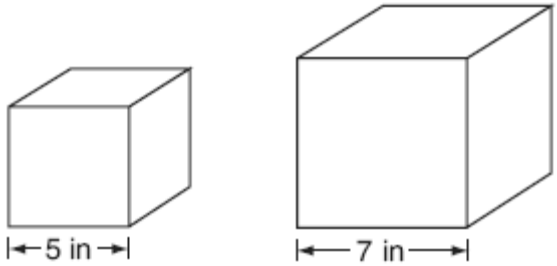
15. 010227a, P.I. A.G.2
In the accompanying diagram, a rectangular container with the dimensions 10 inches by 15 inches by 20 inches is to be filled with water, using a cylindrical cup whose radius is 2 inches and whose height is 5 inches. What is the maximum number of full cups of water that can be placed into the container without the water overflowing the container?





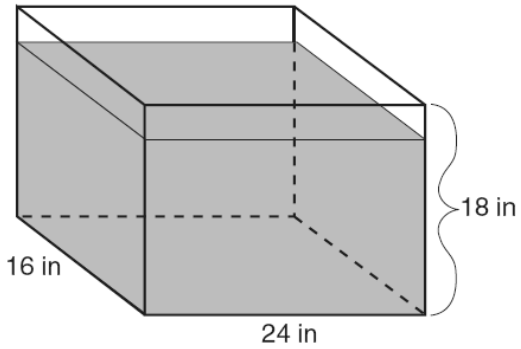
16. 060737a

Tracey has two empty cube-shaped containers with sides of 5 inches and 7 inches, as shown in the accompanying diagram. She fills the smaller container completely with water and then pours all the water from the smaller container into the larger container. How deep, to the *nearest tenth of an inch*, will the water be in the larger container?



17. 010537a

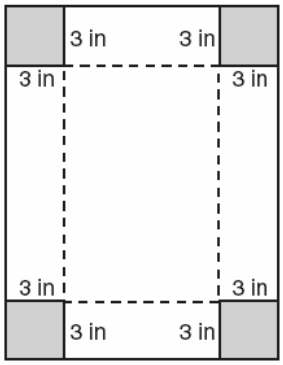
As shown in the accompanying diagram, the length, width, and height of Richard's fish tank are 24 inches, 16 inches, and 18 inches, respectively. Richard is filling his fish tank with water from a hose at the rate of 500 cubic inches per minute. How long will it take, to the *nearest minute*, to fill the tank to a depth of 15 inches?



(Not drawn to scale)

18. 060331a

Deborah built a box by cutting 3-inch squares from the corners of a rectangular sheet of cardboard, as shown in the accompanying diagram, and then folding the sides up. The volume of the box is 150 cubic inches, and the longer side of the box is 5 inches more than the shorter side. Find the number of inches in the shorter side of the *original* sheet of cardboard.



19. 080431b, P.I. A.A.8

A rectangular piece of cardboard is to be formed into an uncovered box. The piece of cardboard is 2 centimeters longer than it is wide. A square that measures 3 centimeters on a side is cut from each corner. When the sides are turned up to form the box, its volume is 765 cubic centimeters. Find the dimensions, in centimeters, of the original piece of cardboard.

20. 060724b

Denise is designing a storage box in the shape of a cube. Each side of the box has a length of 10 inches. She needs more room and decides to construct a larger box in the shape of a cube with a volume of 2,000 cubic inches. By how many inches, to the *nearest tenth*, should she *increase* the length of each side of the original box?

[1] D

[2] D

[3] A

[4] B

[5] C

[6] B

[7] C

[8] C

[9]
a [1] Either $(x - 2)(x + 1)(2x) = V$ or the same expression without "$= V$" is shown.
or [1] $2x^3 - 2x^2 - 4x$ or an equivalent expression is shown.
b [1] 864
or [1] The student substitutes appropriately into an incorrect part a equation.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[10]
[3] 3, 12, and 30 and an appropriate arithmetic method or equation is shown, such as $40x^3 = 1080$.
[2] An appropriate equation or method is shown, but not all three dimensions are found.
or [2] An appropriate method is shown, and although one computational mistake is made, the student does find three dimensions based on this mistake, such as dividing 1080 by 40 incorrectly.
[1] The student shows that multiplication is required to find volume but sets up an incorrect method and does not find three dimensions.
or [1] 3, 12, and 30 and no work is shown.
[0] The sum is used instead of the product,
or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[11]
[2] 20, and appropriate work is shown, such as $3{,}360 \div (14 \times 12)$.
[1] Appropriate work is shown, but one computational error is made.
or [1] 20, but no work is shown.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[12]
[3] 27 and an appropriate method or explanation is shown, such as $(\frac{1}{6})(\frac{1}{3})(\frac{2}{3}) = \frac{1}{27}$ of a cubic foot, thus 27 bricks needed or, in inches, $\frac{1728}{64} = 27$. A labeled drawing is an acceptable explanation.
[2] An appropriate method for finding volume is shown, but one computational mistake is made.
[1] Correct conversion into feet is shown.
or [1] The volume of 64 cubic inches is found.
or [1] 27 and no explanation is given.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[13]
[3] 64, and appropriate work is shown, such as calculating $\frac{(36 \times 144)}{(9 \times 9)}$ or drawing a labeled diagram.
[2] Appropriate work is shown, but one computational error is made.
[1] Appropriate work is shown, but more than one computational error is made.
or [1] Appropriate work is shown, but one conceptual error is made.
or [1] 64, but no work is shown.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[3] 5-inch box and appropriate work is shown, including showing a diameter between 4 and 5.
[2] The correct diameter is shown, but the wrong box size is chosen.
or [2] The correct radius is shown, but the 3-inch box is chosen.
[1] The correct diameter or radius is shown, but no box is chosen.
or [1] An appropriate radius between 2 and 3 is shown, using the incorrect formula $A = \pi r^2$, and the 3-inch box is chosen.
or [1] An appropriate diameter, using $A = \pi r^2$, is shown, but the appropriate box is chosen.
or [1] An appropriate radius, using $A = \pi r^2$, is shown, but no box is chosen.
or [1] The 5-inch box is chosen, but no work is shown.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[14]

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[3] 47, and appropriate work is shown.
[2] Appropriate work is shown, but one computational or rounding error is made.
or [2] The correct numerical value of the volume of the cup (20π or its equivalent) and the volume of the tank (3,000) are shown, but the solution is not completed.
[1] The correct volume of only the cup or only the tub is shown.
or [1] 47, but no work is shown.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[15]

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[3] 2.6, and appropriate work is shown, such as $(5 \bullet 5 \bullet 5) = (7 \bullet 7)h$.
[2] Appropriate work is shown, but one computational or rounding error is made.
[1] Appropriate work is shown, but two or more computational or rounding errors are made.
or [1] Appropriate work is shown, but one conceptual error is made, such as using an incorrect formula.
or [1] The volume of both of the cubes is found correctly, but no further correct work is shown.
or [1] 2.6, but no work is shown.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[16]

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[3] 12, and appropriate work is shown, such as calculating volume = $5{,}760\ \text{in}^3$ and dividing by $500\ \text{in}^3$.
[2] Appropriate work is shown, but one computational or rounding error is made.
or [2] The volume is found incorrectly by multiplying $24 \times 16 \times 18$, but it is divided by 500 and rounded appropriately, resulting in an answer of 14.
[1] Appropriate work is shown, but two or more computational or rounding errors are made.
or [1] Appropriate work is shown, but one conceptual error is made.
or [1] The volume of 5,760 is found correctly, but no further correct work is shown.
or [1] 12, but no work is shown.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[17]

[4] 11, and appropriate work is shown, such as solving the quadratic equation $3x(x+5)=150$ or trial and error with at least three trials and appropriate checks.
[3] Appropriate work is shown, but one computational error is made.
or [3] Appropriate work is shown to determine that 5 is the shorter side of the box, but the shorter side of the original sheet is not found or is found incorrectly.
or [3] An incorrect quadratic equation of equal difficulty is solved appropriately, and an appropriate shorter side of the original sheet is found.
[2] Appropriate work is shown, but more than one computational error is made.
or [2] Appropriate work is shown, but one conceptual error is made.
or [2] An incorrect quadratic equation of equal difficulty is solved appropriately, but the shorter side of the original sheet is not found.
or [2] A correct quadratic equation is set equal to zero, but no further correct work is shown.
or [2] The trial-and-error method is used to find a correct solution, but only two trials and appropriate checks are shown.
[1] Appropriate work is shown, but one conceptual error and one computational error are made.
or [1] One conceptual error is made in finding the shorter side of the box, and the corresponding shorter side of the original sheet is not found or is found incorrectly.
or [1] A correct quadratic equation is written, but it is not set equal to zero, and no further correct work is shown.
or [1] 11, but no work or only one trial with an appropriate check is shown.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[18]

[4] 21 by 23, and appropriate work is shown, such as solving the equation $765=3(x-4)(x-6)$.
[3] Appropriate work is shown, but one computational error is made.
or [3] Appropriate work is shown, but only one dimension is found.
[2] Appropriate work is shown, but two or more computational errors are made.
or [2] Appropriate work is shown, but one conceptual error is made.
or [2] An incorrect equation of equal difficulty is solved appropriately, and appropriate dimensions are found.
or [2] A correct quadratic equation is written in standard form, but no further correct work is shown.
[1] Appropriate work is shown, but one conceptual error and one computational error are made.
or [1] An incorrect equation of equal difficulty is written, and one computational error is made, but appropriate dimensions are found.
or [1] An incorrect equation of equal difficulty is solved appropriately, but one computational error is made when finding the length.
or [1] 21 by 23, but no work is shown.
[0] 21 or 23, but no work is shown.
or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[19]

[2] 2.6, and appropriate work is shown, such as solving the equation $(10+x)^3 = 2000$.
[1] Appropriate work is shown, but one computational or rounding error is made.
or [1] Appropriate work is shown, but one conceptual error is made.
or [1] The equation $(10+x)^3 = 2000$ is written, but no further correct work is shown.
or [1] An incorrect equation of equal difficulty is solved appropriately.
or [1] 2.6, but no work is shown.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[20]