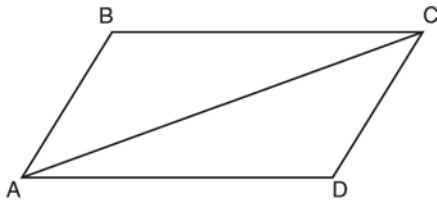


G.SRT.B.5: Quadrilateral Proofs

- 1 Given that $ABCD$ is a parallelogram, a student wrote the proof below to show that a pair of its opposite angles are congruent.

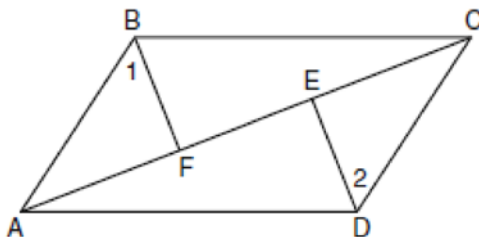


Statement	Reason
1. $ABCD$ is a parallelogram.	1. Given
2. $\overline{BC} \cong \overline{AD}$ $\overline{AB} \cong \overline{DC}$	2. Opposite sides of a parallelogram are congruent.
3. $\overline{AC} \cong \overline{CA}$	3. Reflexive Postulate of Congruency
4. $\triangle ABC \cong \triangle CDA$	4. Side-Side-Side
5. $\angle B \cong \angle D$	5. _____

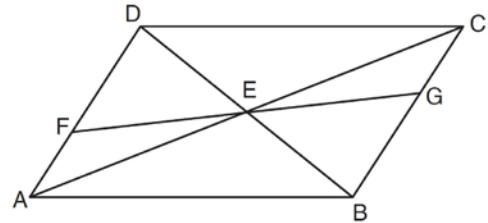
What is the reason justifying that $\angle B \cong \angle D$?

- 1) Opposite angles in a quadrilateral are congruent.
- 2) Parallel lines have congruent corresponding angles.
- 3) Corresponding parts of congruent triangles are congruent.
- 4) Alternate interior angles in congruent triangles are congruent.

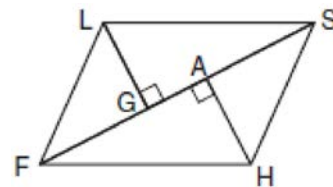
- 2 Given: Quadrilateral $ABCD$, diagonal \overline{AFEC} , $\overline{AE} \cong \overline{FC}$, $\overline{BF} \perp \overline{AC}$, $\overline{DE} \perp \overline{AC}$, $\angle 1 \cong \angle 2$
Prove: $ABCD$ is a parallelogram.



- 3 In the diagram below of quadrilateral $ABCD$, $\overline{AD} \cong \overline{BC}$ and $\angle DAE \cong \angle BCE$. Line segments \overline{AC} , \overline{DB} , and \overline{FG} intersect at E . Prove: $\triangle AEF \cong \triangle CEG$

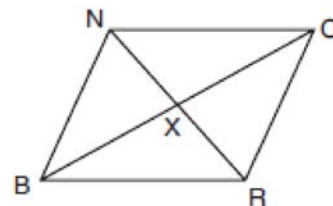


- 4 Given: parallelogram $FLSH$, diagonal \overline{FGAS} , $\overline{LG} \perp \overline{FS}$, $\overline{HA} \perp \overline{FS}$



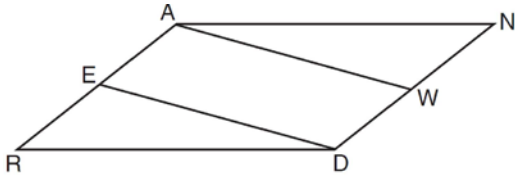
Prove: $\triangle LGS \cong \triangle HAF$

- 5 The accompanying diagram shows quadrilateral $BRON$, with diagonals \overline{NR} and \overline{BO} , which bisect each other at X .



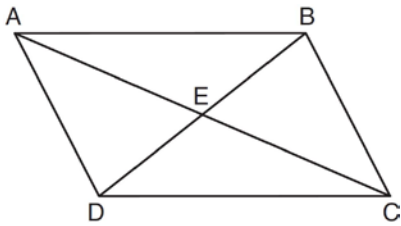
Prove: $\triangle BNX \cong \triangle ORX$

- 6 Given: Parallelogram \overline{ANDR} with \overline{AW} and \overline{DE} bisecting \overline{NWD} and \overline{REA} at points W and E , respectively



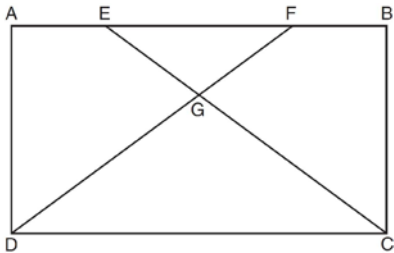
Prove that $\triangle ANW \cong \triangle DRE$. Prove that quadrilateral $AWDE$ is a parallelogram.

- 7 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals \overline{AC} and \overline{BD} intersecting at E

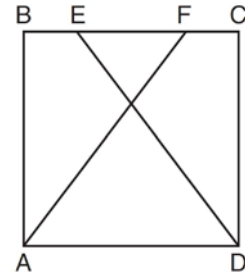


Prove: $\triangle AED \cong \triangle CEB$
Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

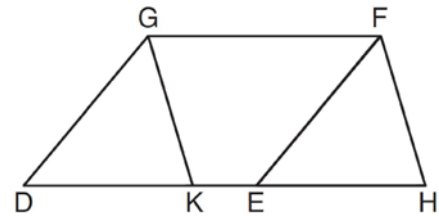
- 8 The diagram below shows rectangle $ABCD$ with points E and F on side \overline{AB} . Segments \overline{CE} and \overline{DF} intersect at G , and $\angle ADG \cong \angle BCG$. Prove: $\overline{AE} \cong \overline{BF}$



- 9 The diagram below shows square $ABCD$ where E and F are points on \overline{BC} such that $\overline{BE} \cong \overline{FC}$, and segments \overline{AF} and \overline{DE} are drawn. Prove that $\overline{AF} \cong \overline{DE}$.

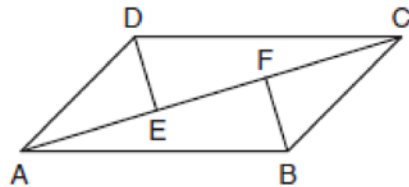


- 10 Given: Parallelogram $DEFG$, K and H are points on \overline{DE} such that $\angle DGK \cong \angle EFH$ and \overline{GK} and \overline{FH} are drawn.



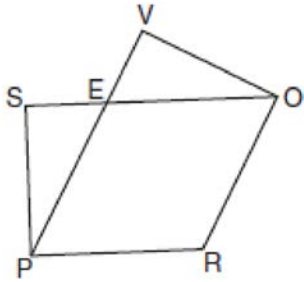
Prove: $\overline{DK} \cong \overline{EH}$

- 11 In quadrilateral $ABCD$, $\overline{AB} \cong \overline{CD}$, $\overline{AB} \parallel \overline{CD}$, and \overline{BF} and \overline{DE} are perpendicular to diagonal \overline{AC} at points F and E .



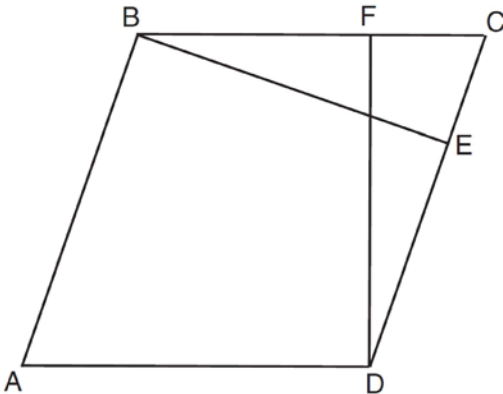
Prove: $\overline{AE} \cong \overline{CF}$

- 12 Given: $PROE$ is a rhombus, \overline{SEO} , \overline{PEV} ,
 $\angle SPR \cong \angle VOR$



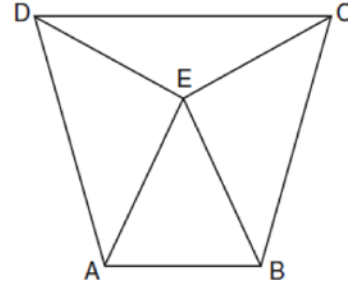
Prove: $\overline{SE} \cong \overline{EV}$

- 13 In the diagram of parallelogram $ABCD$ below,
 $\overline{BE} \perp \overline{CED}$, $\overline{DF} \perp \overline{BFC}$, $\overline{CE} \cong \overline{CF}$.



Prove $ABCD$ is a rhombus.

- 14 Isosceles trapezoid $ABCD$ has bases \overline{DC} and \overline{AB} with nonparallel legs \overline{AD} and \overline{BC} . Segments \overline{AE} , \overline{BE} , \overline{CE} , and \overline{DE} are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.



Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

- 15 Given: Quadrilateral $ABCD$ with $\overline{AB} \cong \overline{CD}$,
 $\overline{AD} \cong \overline{BC}$, and diagonal \overline{BD} is drawn
Prove: $\angle BDC \cong \angle ABD$

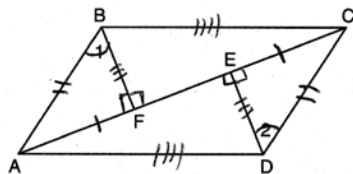
- 16 Prove that the diagonals of a parallelogram bisect each other.

- 17 A tricolored flag is made out of a rectangular piece of cloth whose corners are labeled A , B , C , and D . The colored regions are separated by two line segments, \overline{BM} and \overline{CM} , that meet at point M , the midpoint of side \overline{AD} . Prove that the two line segments that separate the regions will always be equal in length, regardless of the size of the flag.

G.SRT.B.5: Quadrilateral Proofs

Answer Section

- 1 ANS: 3 REF: 081208ge
 2 ANS:



$\overline{FE} \cong \overline{FE}$ (Reflexive Property); $\overline{AE} - \overline{FE} \cong \overline{FC} - \overline{EF}$ (Line Segment Subtraction Theorem); $\overline{AF} \cong \overline{CE}$ (Substitution); $\angle BFA \cong \angle DEC$ (All right angles are congruent); $\triangle BFA \cong \triangle DEC$ (AAS); $\overline{AB} \cong \overline{CD}$ and $\overline{BF} \cong \overline{DE}$ (CPCTC); $\angle BFC \cong \angle DEA$ (All right angles are congruent); $\triangle BFC \cong \triangle DEA$ (SAS); $\overline{AD} \cong \overline{CB}$ (CPCTC); $ABCD$ is a parallelogram (opposite sides of quadrilateral $ABCD$ are congruent)

REF: 080938ge

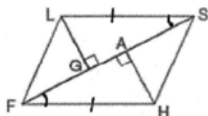
- 3 ANS:

Quadrilateral $ABCD$, $\overline{AD} \cong \overline{BC}$ and $\angle DAE \cong \angle BCE$ are given. $\overline{AD} \parallel \overline{BC}$ because if two lines are cut by a transversal so that a pair of alternate interior angles are congruent, the lines are parallel. $ABCD$ is a parallelogram because if one pair of opposite sides of a quadrilateral are both congruent and parallel, the quadrilateral is a parallelogram. $\overline{AE} \cong \overline{CE}$ because the diagonals of a parallelogram bisect each other. $\angle FEA \cong \angle GEC$ as vertical angles. $\triangle AEF \cong \triangle CEG$ by ASA.

REF: 011238ge

- 4 ANS:

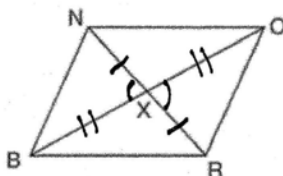
Because $FLSH$ is a parallelogram, $\overline{FH} \cong \overline{SL}$. Because $FLSH$ is a parallelogram, $\overline{FH} \parallel \overline{SL}$ and since \overline{FGAS} is a transversal, $\angle AFH$ and $\angle LSG$ are alternate interior angles and congruent. Therefore $\triangle LGS \cong \triangle HAF$ by AAS.



REF: 010634b

- 5 ANS:

Because diagonals \overline{NR} and \overline{BO} bisect each other, $\overline{NX} \cong \overline{RX}$ and $\overline{BX} \cong \overline{OX}$. $\angle BXN$ and $\angle OXR$ are congruent



vertical angles. Therefore $\triangle BNX \cong \triangle ORX$ by SAS.

REF: 080731b

6 ANS:

Parallelogram $ANDR$ with \overline{AW} and \overline{DE} bisecting \overline{NWD} and \overline{REA} at points W and E (Given). $\overline{AN} \cong \overline{RD}$, $\overline{AR} \cong \overline{DN}$ (Opposite sides of a parallelogram are congruent). $AE = \frac{1}{2}AR$, $WD = \frac{1}{2}DN$, so $\overline{AE} \cong \overline{WD}$ (Definition of bisect and division property of equality). $\overline{AR} \parallel \overline{DN}$ (Opposite sides of a parallelogram are parallel). $AWDE$ is a parallelogram (Definition of parallelogram). $RE = \frac{1}{2}AR$, $NW = \frac{1}{2}DN$, so $\overline{RE} \cong \overline{NW}$ (Definition of bisect and division property of equality). $\overline{ED} \cong \overline{AW}$ (Opposite sides of a parallelogram are congruent). $\triangle ANW \cong \triangle DRE$ (SSS).

REF: 011635geo

7 ANS:

Quadrilateral $ABCD$ is a parallelogram with diagonals \overline{AC} and \overline{BD} intersecting at E (Given). $\overline{AD} \cong \overline{BC}$ (Opposite sides of a parallelogram are congruent). $\angle AED \cong \angle CEB$ (Vertical angles are congruent). $\overline{BC} \parallel \overline{DA}$ (Definition of parallelogram). $\angle DBC \cong \angle BDA$ (Alternate interior angles are congruent). $\triangle AED \cong \triangle CEB$ (AAS). 180° rotation of $\triangle AED$ around point E .

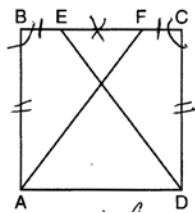
REF: 061533geo

8 ANS:

Rectangle $ABCD$ with points E and F on side \overline{AB} , segments \overline{CE} and \overline{DF} intersect at G , and $\angle ADG \cong \angle BCE$ are given. $\overline{AD} \cong \overline{BC}$ because opposite sides of a rectangle are congruent. $\angle A$ and $\angle B$ are right angles and congruent because all angles of a rectangle are right and congruent. $\triangle ADF \cong \triangle BCE$ by ASA. $\overline{AF} \cong \overline{BE}$ per CPCTC. $\overline{EF} \cong \overline{FE}$ under the Reflexive Property. $\overline{AF} - \overline{EF} \cong \overline{BE} - \overline{FE}$ using the Subtraction Property of Segments. $\overline{AE} \cong \overline{BF}$ because of the Definition of Segments.

REF: 011338ge

9 ANS:

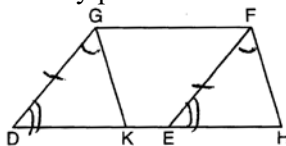


Square $ABCD$; E and F are points on \overline{BC} such that $\overline{BE} \cong \overline{FC}$; \overline{AF} and \overline{DE} drawn (Given). $\overline{AB} \cong \overline{CD}$ (All sides of a square are congruent). $\angle ABF \cong \angle DCE$ (All angles of a square are equiangular). $\overline{EF} \cong \overline{FE}$ (Reflexive property). $\overline{BE} + \overline{EF} \cong \overline{FC} + \overline{FE}$ (Additive property of line segments). $\overline{BF} \cong \overline{CE}$ (Angle addition). $\triangle ABF \cong \triangle DCE$ (SAS). $\overline{AF} \cong \overline{DE}$ (CPCTC).

REF: 061538ge

10 ANS:

Parallelogram $DEFG$, K and H are points on \overline{DE} such that $\angle DGK \cong \angle EFH$ and \overline{GK} and \overline{FH} are drawn (given). $\overline{DG} \cong \overline{EF}$ (opposite sides of a parallelogram are congruent). $\overline{DG} \parallel \overline{EF}$ (opposite sides of a parallelogram are parallel). $\angle D \cong \angle FEH$ (corresponding angles formed by parallel lines and a transversal are congruent).



$\triangle DGK \cong \triangle EFH$ (ASA). $\overline{DK} \cong \overline{EH}$ (CPCTC).

REF: 081538ge

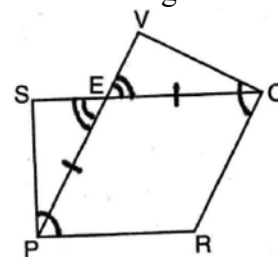
11 ANS:

Quadrilateral $ABCD$, $\overline{AB} \cong \overline{CD}$, $\overline{AB} \parallel \overline{CD}$, and \overline{BF} and \overline{DE} are perpendicular to diagonal \overline{AC} at points F and E (given). $\angle AED$ and $\angle CFB$ are right angles (perpendicular lines form right angles). $\angle AED \cong \angle CFB$ (All right angles are congruent). $ABCD$ is a parallelogram (A quadrilateral with one pair of sides congruent and parallel is a parallelogram). $\overline{AD} \parallel \overline{BC}$ (Opposite sides of a parallelogram are parallel). $\angle DAE \cong \angle BCF$ (Parallel lines cut by a transversal form congruent alternate interior angles). $\overline{DA} \cong \overline{BC}$ (Opposite sides of a parallelogram are congruent). $\triangle ADE \cong \triangle CBF$ (AAS). $\overline{AE} \cong \overline{CF}$ (CPCTC).

REF: 011735geo

12 ANS:

Because $PROE$ is a rhombus, $\overline{PE} \cong \overline{OE}$. $\angle SEP \cong \angle VEO$ are congruent vertical angles. $\angle EPR \cong \angle EOR$ because opposite angles of a rhombus are congruent. $\angle SPE \cong \angle VOE$ because of the Angle Subtraction



Theorem. $\triangle SEP \cong \triangle VEO$ because of ASA. $\overline{SE} \cong \overline{EV}$ because of CPCTC.

REF: 010934b

13 ANS:

Parallelogram $ABCD$, $\overline{BE} \perp \overline{CED}$, $\overline{DF} \perp \overline{BFC}$, $\overline{CE} \cong \overline{CF}$ (given). $\angle BEC \cong \angle DFC$ (perpendicular lines form right angles, which are congruent). $\angle FCD \cong \angle BCE$ (reflexive property). $\triangle BEC \cong \triangle DFC$ (ASA). $\overline{BC} \cong \overline{CD}$ (CPCTC). $ABCD$ is a rhombus (a parallelogram with consecutive congruent sides is a rhombus).

REF: 081535geo

14 ANS:

Isosceles trapezoid $ABCD$, $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$ (given); $\overline{AD} \cong \overline{BC}$ (congruent legs of isosceles trapezoid); $\angle DEA$ and $\angle CEB$ are right angles (perpendicular lines form right angles); $\angle DEA \cong \angle CEB$ (all right angles are congruent); $\angle CDA \cong \angle DCB$ (base angles of an isosceles trapezoid are congruent); $\angle CDA - \angle CDE \cong \angle DCB - \angle DCE$ (subtraction postulate); $\triangle ADE \cong \triangle BCE$ (AAS); $\overline{EA} \cong \overline{EB}$ (CPCTC);

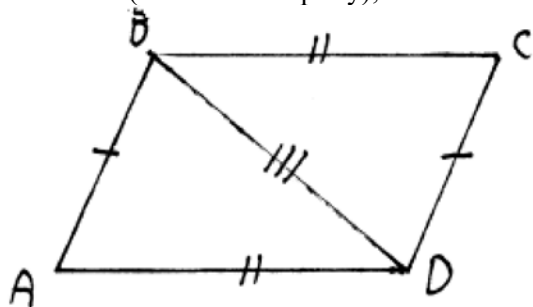
$$\angle EDA \cong \angle ECB$$

$\triangle AEB$ is an isosceles triangle (an isosceles triangle has two congruent sides).

REF: 081735geo

15 ANS:

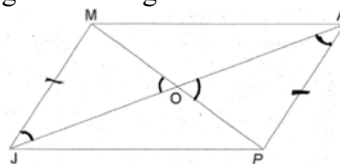
$\overline{BD} \cong \overline{DB}$ (Reflexive Property); $\triangle ABD \cong \triangle CDB$ (SSS); $\angle BDC \cong \angle ABD$ (CPCTC).



REF: 061035ge

16 ANS:

Assume parallelogram $JMAP$ with diagonals intersecting at O . Opposite sides of a parallelogram are congruent, so $\overline{JM} \cong \overline{AP}$. $\angle JOM$ and $\angle AOP$ are congruent vertical angles. Because $JMAP$ is a parallelogram, $\overline{JM} \parallel \overline{AP}$ and since \overline{JOA} is a transversal, $\angle MJO$ and $\angle PAO$ are alternate interior angles and congruent. Therefore $\triangle MJO \cong \triangle PAO$ by AAS. Corresponding parts of congruent triangles are congruent. Therefore $\overline{JO} \cong \overline{AO}$ and

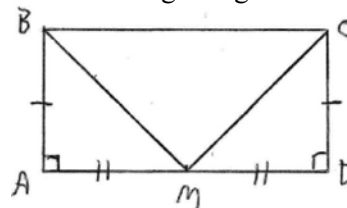


$\overline{MO} \cong \overline{PO}$ and the diagonals of a parallelogram bisect each other.

REF: 010233b

17 ANS:

$\overline{AB} \cong \overline{CD}$, because opposite sides of a rectangle are congruent. $\overline{AM} \cong \overline{DM}$, because of the definition of midpoint. $\angle A$ and $\angle D$ are right angles because a rectangle has four right angles. $\angle A \cong \angle D$, because all right angles are



congruent. $\triangle ABM \cong \triangle DCM$, because of SAS. $\overline{BM} \cong \overline{CM}$ because of CPCTC.

REF: 080834b