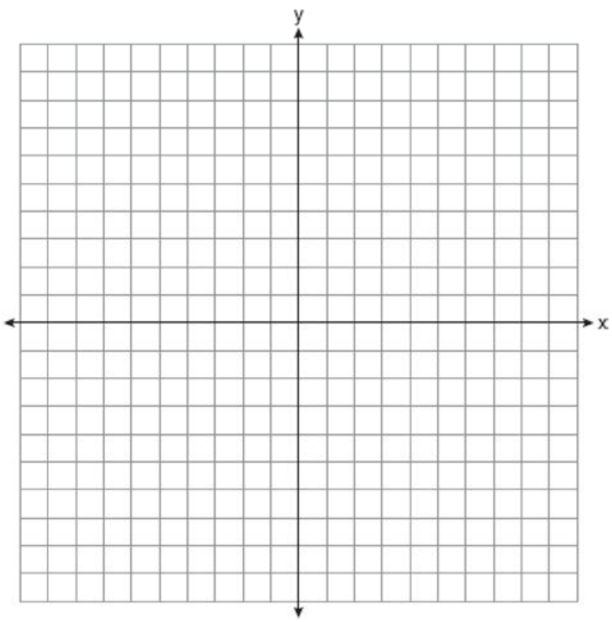
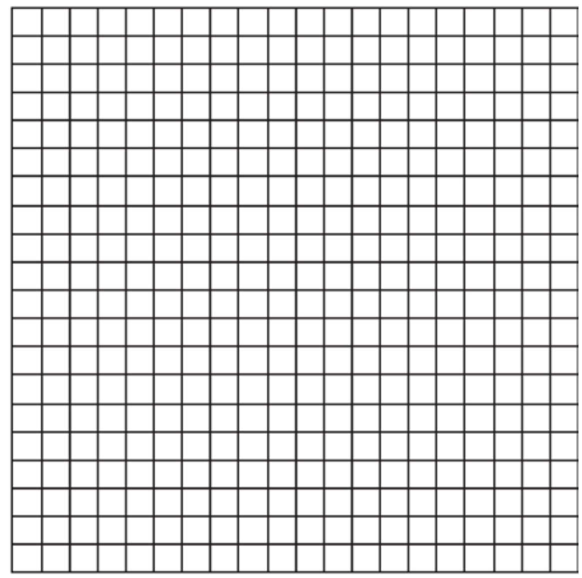
G.GPE.B.4: Quadrilaterals in the Coordinate Plane 2

1 In square *GEOM*, the coordinates of *G* are $(2,-2)$ and the coordinates of *O* are $(-4,2)$. Determine and state the coordinates of vertices *E* and *M*. [The use of the set of axes below is optional.]

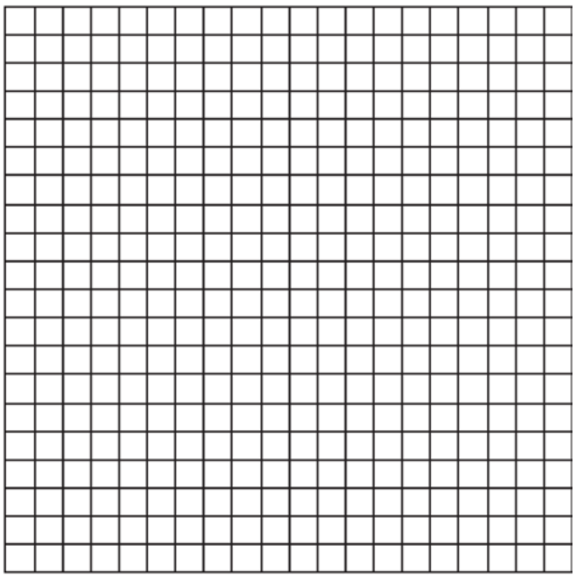


2 The coordinates of quadrilateral *ABCD* are $A(-1,-5)$, $B(8,2)$, $C(11,13)$, and $D(2,6)$. Using coordinate geometry, prove that quadrilateral *ABCD* is a rhombus. [The use of the grid is optional.]

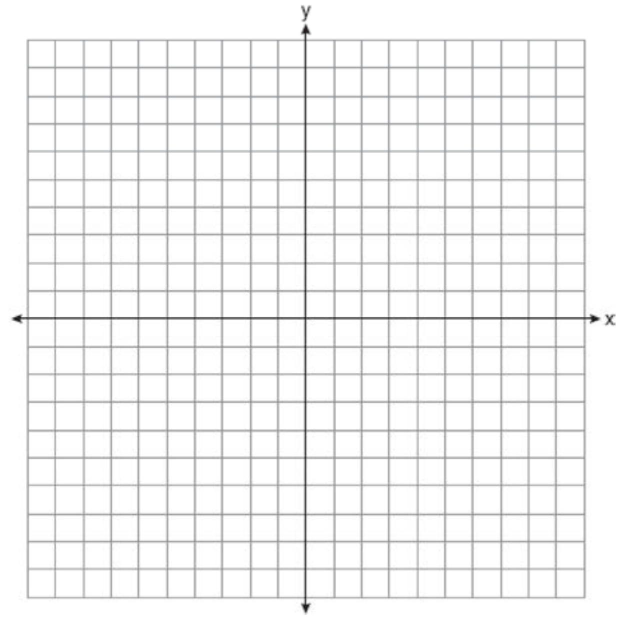


Name: ________________________

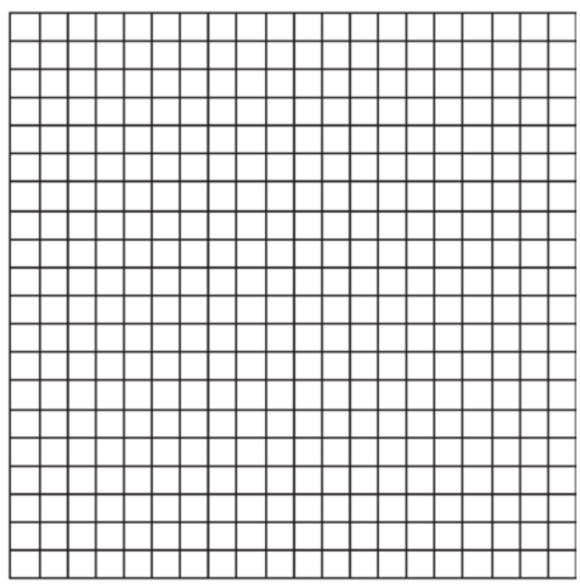
3 Jim is experimenting with a new drawing program on his computer. He created quadrilateral $TEAM$ with coordinates $T(-2,3)$, $E(-5,-4)$, $A(2,-1)$, and $M(5,6)$. Jim believes that he has created a rhombus but not a square. Prove that Jim is correct. [The use of the grid is optional.]



4 Quadrilateral $PQRS$ has vertices $P(-2,3)$, $Q(3,8)$, $R(4,1)$, and $S(-1,-4)$. Prove that $PQRS$ is a rhombus. Prove that $PQRS$ is *not* a square. [The use of the set of axes below is optional.]



5 Given: $A(-2,2)$, $B(6,5)$, $C(4,0)$, $D(-4,-3)$
Prove: $ABCD$ is a parallelogram but not a rectangle. [The use of the grid is optional.]



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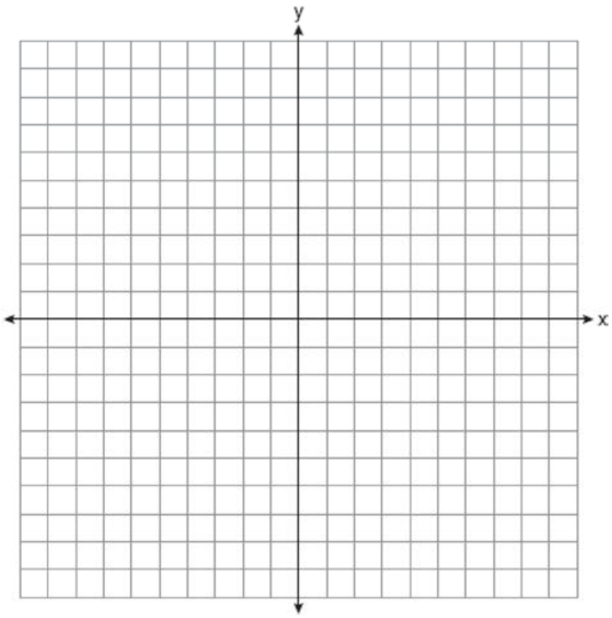
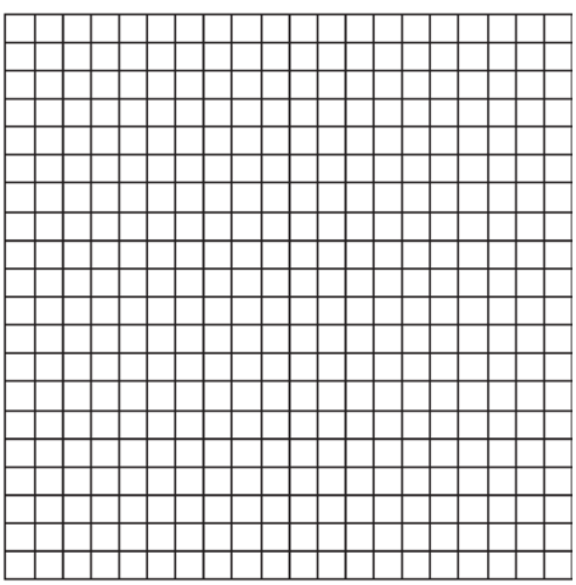
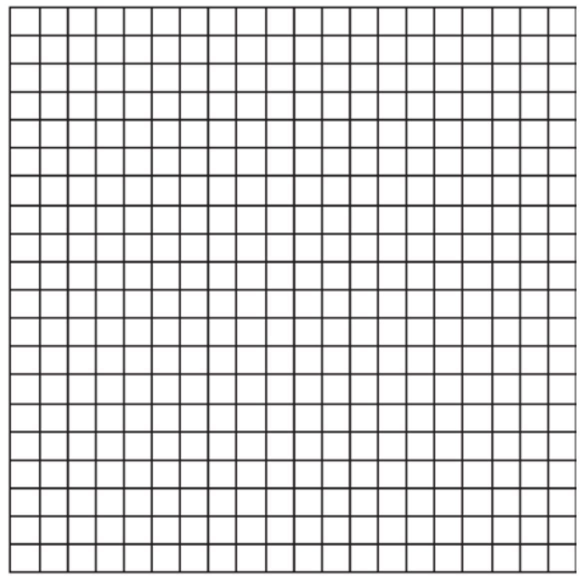
6 The vertices of quadrilateral *JKLM* have coordinates $J(-3,1)$, $K(1,-5)$, $L(7,-2)$, and $M(3,4)$. Prove that *JKLM* is a parallelogram. Prove that *JKLM* is *not* a rhombus. [The use of the set of axes below is optional.]



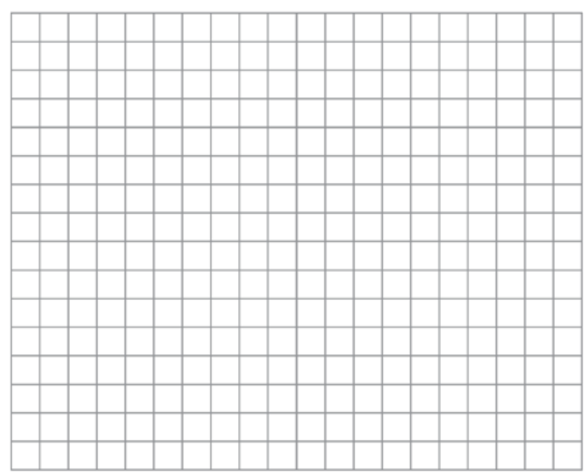
7 Quadrilateral *MATH* has coordinates $M(1,1)$, $A(-2,5)$, $T(3,5)$, and $H(6,1)$. Prove that quadrilateral *MATH* is a rhombus and prove that it is *not* a square. [The use of the grid is optional.]



8 Quadrilateral *ABCD* has vertices $A(2,3)$, $B(7,10)$, $C(9,4)$, and $D(4,-3)$. Prove that *ABCD* is a parallelogram but *not* a rhombus. [The use of the grid is optional.]



9 Given: Quadrilateral *ABCD* has vertices $A(-5,6)$, $B(6,6)$, $C(8,-3)$, and $D(-3,-3)$.
Prove: Quadrilateral *ABCD* is a parallelogram but is neither a rhombus nor a rectangle. [The use of the grid below is optional.]



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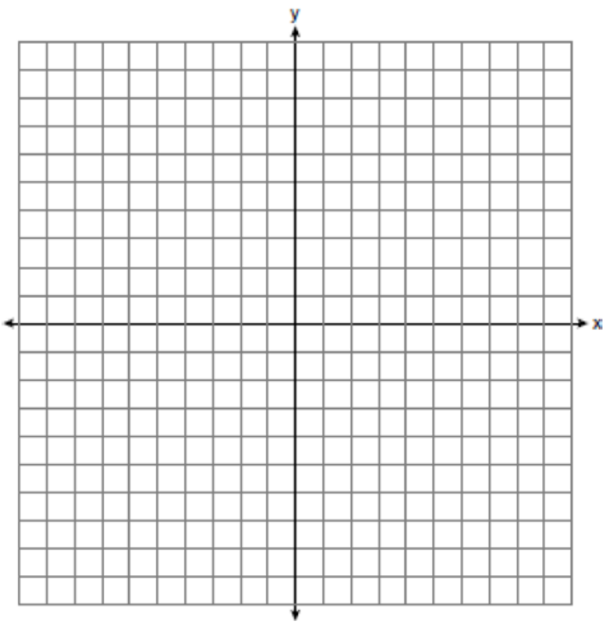
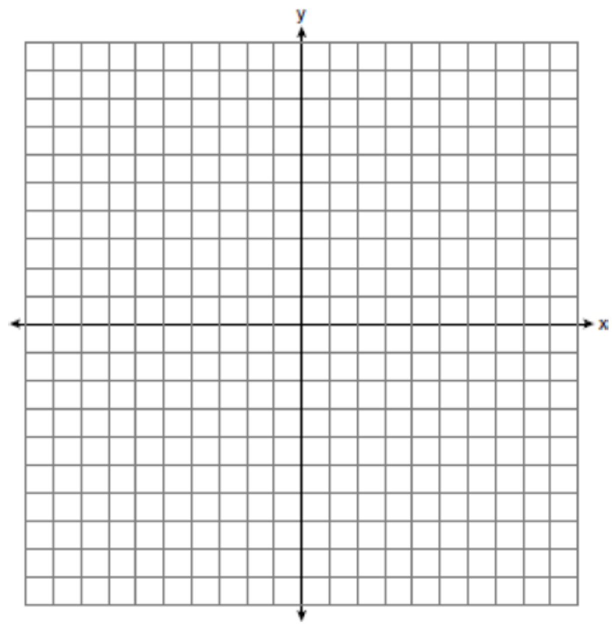
10 In rhombus $MATH$, the coordinates of the endpoints of the diagonal $\overline{MT}$ are $M(0,-1)$ and $T(4,6)$. Write an equation of the line that contains diagonal $\overline{AH}$. [Use of the set of axes below is optional.] Using the given information, explain how you know that your line contains diagonal $\overline{AH}$.



11 In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$. Prove that $\triangle RST$ is a right triangle. State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle. Prove that your quadrilateral $RSTP$ is a rectangle. [The use of the set of axes below is optional.]



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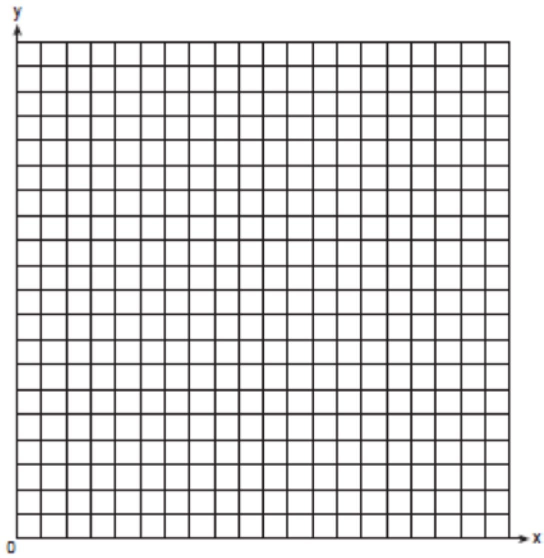
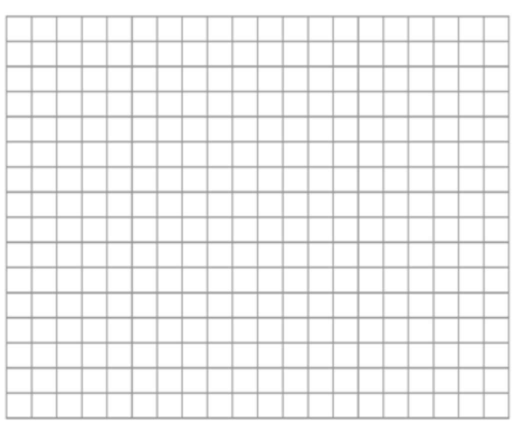
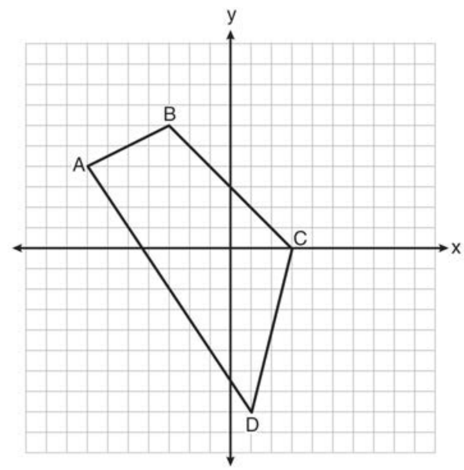
12 Ashanti is surveying for a new parking lot shaped like a parallelogram. She knows that three of the vertices of parallelogram *ABCD* are $A(0,0)$, $B(5,2)$, and $C(6,5)$. Find the coordinates of point *D* and sketch parallelogram *ABCD* on the accompanying set of axes. Justify mathematically that the figure you have drawn is a parallelogram.



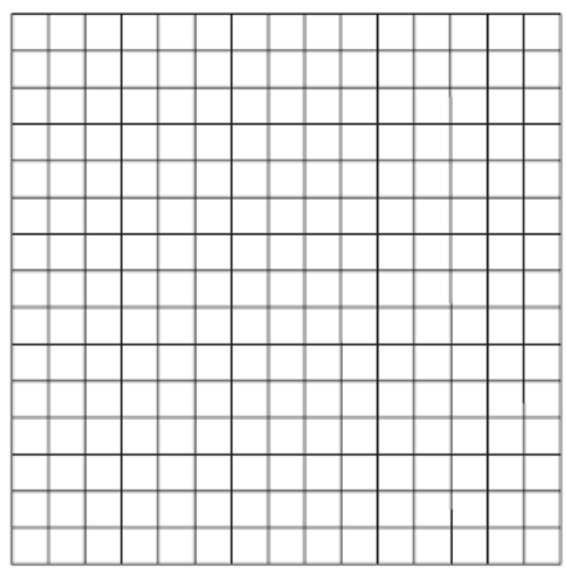
13 Given: $\triangle ABC$ with vertices $A(-6,-2)$, $B(2,8)$, and $C(6,-2)$. $\overline{AB}$ has midpoint *D*, $\overline{BC}$ has midpoint *E*, and $\overline{AC}$ has midpoint *F*.
Prove: *ADEF* is a parallelogram
ADEF is *not* a rhombus
[The use of the grid is optional.]



14 Quadrilateral *ABCD* with vertices $A(-7,4)$, $B(-3,6)$, $C(3,0)$, and $D(1,-8)$ is graphed on the set of axes below. Quadrilateral *MNPQ* is formed by joining *M*, *N*, *P*, and *Q*, the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{AD}$, respectively. Prove that quadrilateral *MNPQ* is a parallelogram. Prove that quadrilateral *MNPQ* is *not* a rhombus.



15 Given: $A(1,6)$, $B(7,9)$, $C(13,6)$, and $D(3,1)$
Prove: *ABCD* is a trapezoid. [*The use of the accompanying grid is optional*.]



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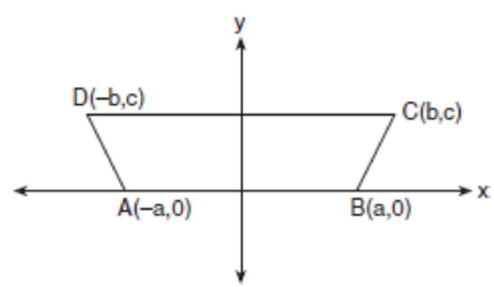
16 Quadrilateral *KATE* has vertices $K(1,5)$, $A(4,7)$, $T(7,3)$, and $E(1,-1)$.
a Prove that *KATE* is a trapezoid. [The use of the grid is optional.]
b Prove that *KATE* is *not* an isosceles trapezoid.

17 The coordinates of quadrilateral *JKLM* are $J(1,-2)$, $K(13,4)$, $L(6,8)$, and $M(-2,4)$. Prove that quadrilateral *JKLM* is a trapezoid but *not* an isosceles trapezoid. [The use of the grid is optional.]

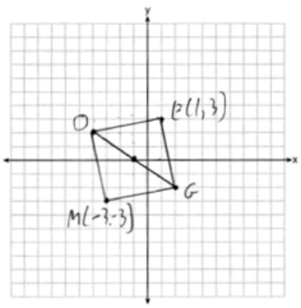
18 Given: $T(-1,1)$, $R(3,4)$, $A(7,2)$, and $P(-1,-4)$
Prove: *TRAP* is a trapezoid.
TRAP is not an isosceles trapezoid.
[The use of the grid is optional.]

19 In the accompanying diagram of *ABCD*, where $a \neq b$, prove *ABCD* is an isosceles trapezoid.



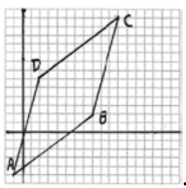
G.GPE.B.4: Quadrilaterals in the Coordinate Plane 2
Answer Section

1 ANS:



REF: 011731geo

2 ANS:



. To prove that *ABCD* is a rhombus, show that all sides are congruent using the distance formula:

$d_{\overline{AB}} = \sqrt{(8-(-1))^2 + (2-(-5))^2} = \sqrt{130}$.

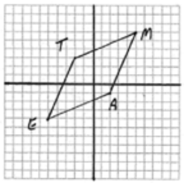
$d_{\overline{BC}} = \sqrt{(11-8)^2 + (13-2)^2} = \sqrt{130}$

$d_{\overline{CD}} = \sqrt{(11-2)^2 + (13-6)^2} = \sqrt{130}$

$d_{\overline{AD}} = \sqrt{(2-(-1))^2 + (6-(-5))^2} = \sqrt{130}$

REF: 060327b

3 ANS:



. To prove that *TEAM* is a rhombus, show that all sides are congruent using the distance formula: $d_{\overline{ET}}=\sqrt{(-2-(-5))^2+(3-(-4))^2}=\sqrt{58}$. A square has four right angles. If *TEAM* is a square, then $\overline{ET}\perp\overline{AE}$,

$d_{\overline{AM}}=\sqrt{(2-5)^2+((-1)-6)^2}=\sqrt{58}$

$d_{\overline{AE}}=\sqrt{(-5-2)^2+(-4-(-1))^2}=\sqrt{58}$

$d_{\overline{MT}}=\sqrt{(-2-5)^2+(3-6)^2}=\sqrt{58}$

$\overline{AE}\perp\overline{AM}$, $\overline{AM}\perp\overline{AT}$ and $\overline{MT}\perp\overline{ET}$. Lines that are perpendicular have slopes that are opposite reciprocals of each other. The slopes of sides of *TEAM* are: $m_{\overline{ET}}=\dfrac{-4-3}{-5-(-2)}=\dfrac{7}{3}$ $m_{\overline{AE}}=\dfrac{-4-(-1)}{-5-2}=\dfrac{3}{7}$ Because $\dfrac{7}{3}$ and $\dfrac{3}{7}$ are not

$$m_{\overline{AM}}=\frac{6-(-1)}{5-2}=\frac{7}{3}\quad m_{\overline{MT}}=\frac{3-6}{-2-5}=\frac{3}{7}$$

opposite reciprocals, consecutive sides of *TEAM* are not perpendicular, and *TEAM* is not a square.

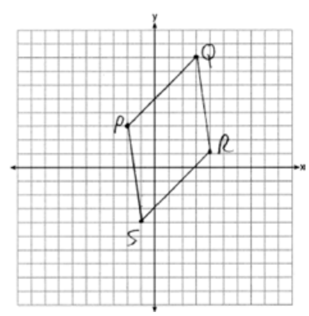
REF: 010533b

4 ANS:

$\overline{PQ}$ $\sqrt{(8-3)^2+(3--2)^2}=\sqrt{50}$ $\overline{QR}$ $\sqrt{(1-8)^2+(4-3)^2}=\sqrt{50}$ $\overline{RS}$ $\sqrt{(-4-1)^2+(-1-4)^2}=\sqrt{50}$

$\overline{PS}$ $\sqrt{(-4-3)^2+(-1--2)^2}=\sqrt{50}$ *PQRS* is a rhombus because all sides are congruent. $m_{\overline{PQ}}=\dfrac{8-3}{3--2}=\dfrac{5}{5}=1$

$m_{\overline{QR}}=\dfrac{1-8}{4-3}=-7$ Because the slopes of adjacent sides are not opposite reciprocals, they are not perpendicular and do not form a right angle. Therefore *PQRS* is not a square.

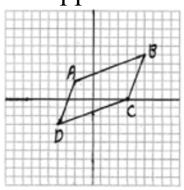


REF: 061735geo

5 ANS:
To prove that $ABCD$ is a parallelogram, show that both pairs of opposite sides of the parallelogram are parallel by showing the opposite sides have the same slope: $m_{\overline{AB}} = \dfrac{5-2}{6-(-2)} = \dfrac{3}{8}$ $m_{\overline{AD}} = \dfrac{-3-2}{-4-(-2)} = \dfrac{5}{2}$

$$m_{\overline{CD}} = \frac{-3-0}{-4-4} = \frac{3}{8} \quad m_{\overline{BC}} = \frac{5-0}{6-4} = \frac{5}{2}$$

A rectangle has four right angles. If $ABCD$ is a rectangle, then $\overline{AB} \perp \overline{BC}$, $\overline{BC} \perp \overline{CD}$, $\overline{CD} \perp \overline{AD}$, and $\overline{AD} \perp \overline{AB}$. Lines that are perpendicular have slopes that are the opposite and reciprocal of each other. Because $\dfrac{3}{8}$ and $\dfrac{5}{2}$ are not opposite reciprocals, the consecutive sides of $ABCD$ are not perpendicular, and $ABCD$ is not a rectangle.



REF: 060633b

6 ANS:

$m_{\overline{JM}} = \dfrac{1-4}{-3-3} = \dfrac{-3}{-6} = \dfrac{1}{2}$ Since both opposite sides have equal slopes and are parallel, $JKLM$ is a parallelogram.

$m_{=\overline{ML}} = \dfrac{4--2}{3-7} = \dfrac{6}{-4} = -\dfrac{3}{2}$

$m_{\overline{LK}} = \dfrac{-2--5}{7-1} = \dfrac{3}{6} = \dfrac{1}{2}$

$m_{\overline{KJ}} = \dfrac{-5-1}{1--3} = \dfrac{-6}{4} = -\dfrac{3}{2}$

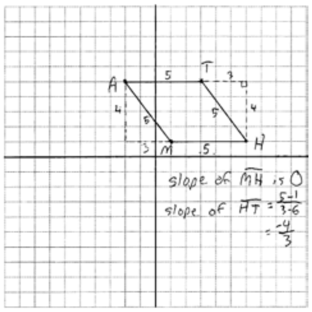
$\overline{JM} = \sqrt{(-3-3)^2 + (1-4)^2} = \sqrt{45}$. $\overline{JM}$ is not congruent to $\overline{ML}$, so $JKLM$ is not a rhombus since not all sides

$\overline{ML} = \sqrt{(7-3)^2 + (-2-4)^2} = \sqrt{52}$

are congruent.

REF: 061438ge

7 ANS:



The length of each side of quadrilateral is 5. Since each side is congruent, quadrilateral *MATH* is a rhombus. The slope of $\overline{MH}$ is 0 and the slope of $\overline{HT}$ is $-\frac{4}{3}$. Since the slopes are not negative reciprocals, the sides are not perpendicular and do not form rights angles. Since adjacent sides are not perpendicular, quadrilateral *MATH* is not a square.

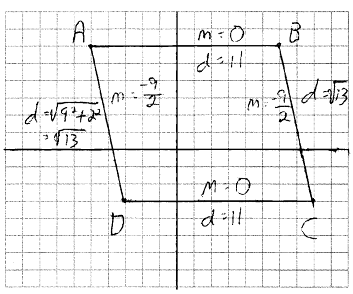
REF: 011138ge

8 ANS:

$m_{\overline{AB}} = \frac{10-3}{7-2} = \frac{7}{5}$, $m_{\overline{CD}} = \frac{4-(-3)}{9-4} = \frac{7}{5}$, $m_{\overline{AD}} = \frac{3-(-3)}{2-4} = \frac{6}{-2} = -3$, $m_{\overline{BC}} = \frac{10-4}{7-9} = \frac{6}{-2} = -3$ (Definition of slope). $\overline{AB} \| \overline{CD}$, $\overline{AD} \| \overline{BC}$ (Parallel lines have equal slope). Quadrilateral *ABCD* is a parallelogram (Definition of parallelogram). $d_{\overline{AD}} = \sqrt{(2-4)^2 + (3-(-3))^2} = \sqrt{40}$, $d_{\overline{AB}} = \sqrt{(7-2)^2 + (10-3)^2} = \sqrt{74}$ (Definition of distance). $\overline{AD}$ is not congruent to $\overline{AB}$ (Congruent lines have equal distance). *ABCD* is not a rhombus (A rhombus has four equal sides).

REF: 061031b

9 ANS:



$\overline{AB} \| \overline{CD}$ and $\overline{AD} \| \overline{CB}$ because their slopes are equal. *ABCD* is a parallelogram because opposite side are parallel. $\overline{AB} \neq \overline{BC}$. *ABCD* is not a rhombus because all sides are not equal. $\overline{AB} \sim\perp \overline{BC}$ because their slopes are not opposite reciprocals. *ABCD* is not a rectangle because $\angle ABC$ is not a right angle.

REF: 081038ge

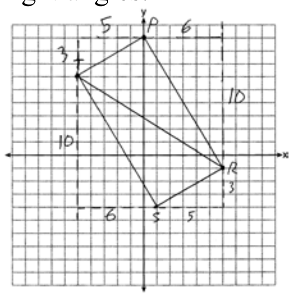
10 ANS:

$M\left(\frac{4+0}{2}, \frac{6-1}{2}\right) = M\left(2, \frac{5}{2}\right)$ $m = \frac{6--1}{4-0} = \frac{7}{4}$ $m_{\perp} = -\frac{4}{7}$ $y - 2.5 = -\frac{4}{7}(x-2)$ The diagonals, $\overline{MT}$ and $\overline{AH}$, of rhombus *MATH* are perpendicular bisectors of each other.

REF: fall1411geo

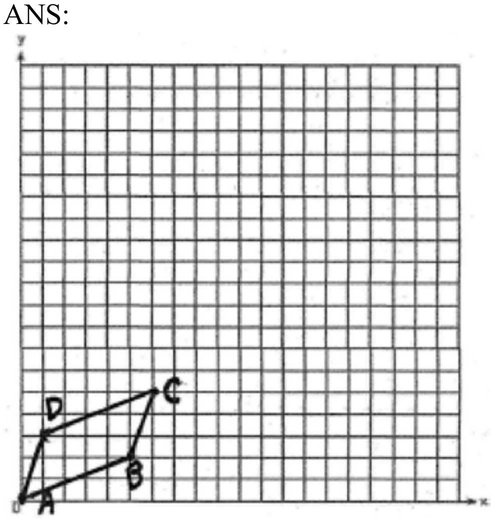
11 ANS:

$m_{\overline{TS}} = \frac{-10}{6} = -\frac{5}{3}$ $m_{\overline{SR}} = \frac{3}{5}$ Since the slopes of $\overline{TS}$ and $\overline{SR}$ are opposite reciprocals, they are perpendicular and form a right angle. $\triangle RST$ is a right triangle because $\angle S$ is a right angle. $P(0,9)$ $m_{\overline{RP}} = \frac{-10}{6} = -\frac{5}{3}$ $m_{\overline{PT}} = \frac{3}{5}$ Since the slopes of all four adjacent sides ($\overline{TS}$ and $\overline{SR}$, $\overline{SR}$ and $\overline{RP}$, $\overline{PT}$ and $\overline{TS}$, $\overline{RP}$ and $\overline{PT}$) are opposite reciprocals, they are perpendicular and form right angles. Quadrilateral $RSTP$ is a rectangle because it has four right angles.



REF: 061536geo

12 ANS:



Both pairs of opposite sides of a parallelogram are parallel. Parallel lines have the same slope. The slope of side $\overline{BC}$ is 3. For side $\overline{AD}$ to have a slope of 3, the coordinates of point D must be (1,3). $m\overline{\text{AB}} = \frac{2-0}{5-0} = \frac{2}{5}$ $m\overline{\text{AD}} = \frac{3-0}{1-0} = 3$

$$m\overline{CD} = \frac{5-3}{6-1} = \frac{2}{5} \quad m\overline{BC} = \frac{5-2}{6-5} = 3$$

REF: 080032a

13 ANS:

$m_{\overline{AB}} = \left(\frac{-6+2}{2}, \frac{-2+8}{2}\right) = D(2,3)$ $m_{\overline{BC}} = \left(\frac{2+6}{2}, \frac{8+-2}{2}\right) = E(4,3)$ $F(0,-2)$. To prove that *ADEF* is a parallelogram, show that both pairs of opposite sides of the parallelogram are parallel by showing the opposite sides have the same slope: $\text{m}_{\overline{AD}} = \frac{3--2}{-2--6} = \frac{5}{4}$ $\overline{AF} \parallel \overline{DE}$ because all horizontal lines have the same slope. *ADEF*

$\text{m}_{FE} = \frac{3--2}{4-0} = \frac{5}{4}$

is not a rhombus because not all sides are congruent. $AD = \sqrt{5^2+4^2} = \sqrt{41}$ $AF = 6$

REF: 081138ge

14 ANS:

$M\left(\frac{-7+-3}{2}, \frac{4+6}{2}\right) = M(-5,5)$. $m_{\overline{MN}} = \frac{5-3}{-5-0} = \frac{2}{-5}$. Since both opposite sides have equal slopes and are

$N\left(\frac{-3+3}{2}, \frac{6+0}{2}\right) = N(0,3)$ $m_{\overline{PQ}} = \frac{-4--2}{2--3} = \frac{-2}{5}$

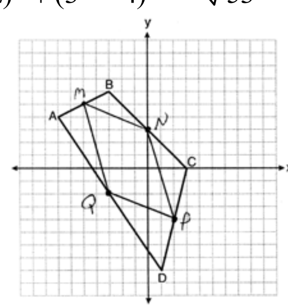
$P\left(\frac{3+1}{2}, \frac{0+-8}{2}\right) = P(2,-4)$ $m_{\overline{NA}} = \frac{3--4}{0-2} = \frac{7}{-2}$

$Q\left(\frac{-7+1}{2}, \frac{4+-8}{2}\right) = Q(-3,-2)$ $m_{\overline{QM}} = \frac{-2-5}{-3--5} = \frac{-7}{2}$

parallel, *MNPQ* is a parallelogram. $\overline{MN} = \sqrt{(-5-0)^2+(5-3)^2} = \sqrt{29}$. $\overline{MN}$ is not congruent to $\overline{NP}$, so *MNPQ*

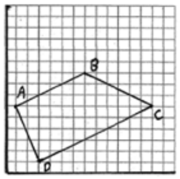
$\overline{NA} = \sqrt{(0-2)^2+(3--4)^2} = \sqrt{53}$

is not a rhombus since not all sides are congruent.



REF: 081338ge

15 ANS:

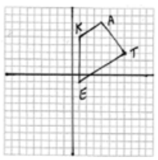


. To prove that *ABCD* is a trapezoid, show that one pair of opposite sides of the figure is parallel by showing they have the same slope and that the other pair of opposite sides is not parallel by showing they do not have the same slope: $m_{\overline{AB}} = \dfrac{9-6}{7-1} = \dfrac{3}{6} = \dfrac{1}{2}$ $m_{\overline{AD}} = \dfrac{6-1}{1-3} = -\dfrac{5}{2}$

$$m_{\overline{CD}} = \frac{6-1}{13-3} = \frac{5}{10} = \frac{1}{2} \quad m_{\overline{BC}} = \frac{9-6}{7-13} = -\frac{3}{6} = -\frac{1}{2}$$

REF: 080134b

16 ANS:



. To prove that *KATE* is a trapezoid, show that one pair of opposite sides of the figure is parallel by showing they have the same slope and that the other pair of opposite sides is not parallel by showing they do not have the same slope: $m_{\overline{AK}} = \dfrac{7-5}{4-1} = \dfrac{2}{3}$ $m_{\overline{EK}} = \dfrac{-1-5}{1-1} = \text{undefined}$

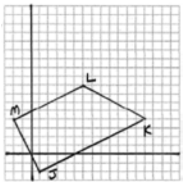
$$m_{\overline{ET}} = \frac{3-(-1)}{7-1} = \frac{4}{6} = \frac{2}{3} \quad m_{\overline{AT}} = \frac{7-3}{4-7} = -\frac{4}{3}$$

To prove that a trapezoid is not an isosceles trapezoid, show that the opposite sides that are not parallel are also not congruent using the distance formula: $d_{\overline{EK}} = \sqrt{(1-1)^2 + (5-(-1))^2} = 6$

$$d_{\overline{AT}} = \sqrt{(4-7)^2 + (7-3)^2} = 5$$

REF: 010333b

17 ANS:



. To prove that *JKLM* is a trapezoid, show that one pair of opposite sides of the figure is parallel by showing they have the same slope and that the other pair of opposite sides is not parallel by showing they do not have the same slope: $m_{\overline{JK}} = \dfrac{4-(-2)}{13-1} = \dfrac{1}{2}$ $m_{\overline{JM}} = \dfrac{-2-4}{1-(-2)} = -2$

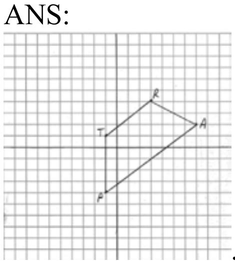
$$m_{\overline{LM}} = \frac{8-4}{6-(-2)} = \frac{1}{2} \quad m_{\overline{KL}} = \frac{4-8}{13-6} = -\frac{4}{7}$$

To prove that a trapezoid is not an isosceles trapezoid, show that the opposite sides that are not parallel are also not congruent using the distance formula: $d_{\overline{JM}} = \sqrt{(1-(-2))^2 + (-2-4)^2} = \sqrt{45}$

$$d_{\overline{KL}} = \sqrt{(13-6)^2 + (4-8)^2} = \sqrt{65}$$

REF: 080434b

18 ANS:



. To prove that *TRAP* is a trapezoid, show that one pair of opposite sides of the figure is parallel by showing they have the same slope and that the other pair of opposite sides is not parallel by showing they do not have the same slope: $m_{\overline{TR}} = \dfrac{1-4}{-1-3} = \dfrac{3}{4}$ $m_{\overline{TP}} = \dfrac{1-(-4)}{-1-(-1)} = \text{undefined}$

$$m_{\overline{PA}} = \frac{-4-2}{-1-7} = \frac{3}{4} \quad m_{\overline{RA}} = \frac{4-2}{3-7} = -\frac{1}{2}$$

To prove that a trapezoid is not an isosceles trapezoid, show that the opposite sides that are not parallel are also not congruent using the distance formula: $d_{\overline{TP}} = \sqrt{(-1-(-1))^2 + (1-(-4))^2} = 5$

$$d_{\overline{RA}} = \sqrt{(3-7)^2 + (4-2)^2} = \sqrt{20} = 2\sqrt{5}$$

REF: 080933b

19 ANS:

To prove that *ABCD* is a trapezoid, show that one pair of opposite sides of the figure is parallel by showing they have the same slope and that the other pair of opposite sides is not parallel by showing they do not have the same slope: $m_{\overline{AB}} = \frac{0-0}{-a-a} = \frac{0}{-2a} = 0$ $m_{\overline{AD}} = \frac{c-0}{-b-(-a)} = \frac{c}{-b+a}$ If $\overline{AD}$ and $\overline{BC}$ are parallel, then: $\frac{c}{-b+a} = \frac{c}{b-a}$

$m_{\overline{CD}} = \frac{c-c}{-b-b} = \frac{0}{-2b} = 0$ $m_{\overline{BC}} = \frac{c-0}{b-a} = \frac{c}{b-a}$

$$c(b-a) = c(-b+a)$$
$$b-a = -b+a$$
$$2a = 2b$$
$$a = b$$

But the facts of the problem indicate $a \neq b$, so $\overline{AD}$ and $\overline{BC}$ are not parallel.

To prove that a trapezoid is an isosceles trapezoid, show that the opposite sides that are not parallel are congruent using the distance formula: $d_{\overline{BC}} = \sqrt{(b-a)^2 + (c-0)^2}$ $d_{\overline{AD}} = \sqrt{(-b-(-a))^2 + (c-0)^2}$

$$d_{\overline{BC}} = \sqrt{b^2 - 2ab + a^2 + c^2} = \sqrt{a^2 + b^2 - 2ab + c^2}$$

$$d_{\overline{AD}} = \sqrt{(a-b)^2 + c^2} = \sqrt{a^2 - 2ab + b^2 + c^2} = \sqrt{a^2 + b^2 - 2ab + c^2}$$

REF: 080534b