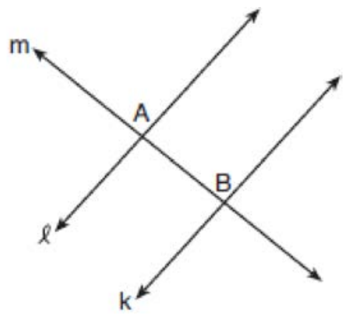
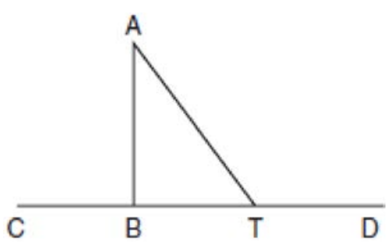
G.CO.C.9: Indirect Proofs

1 In the accompanying diagram, line ℓ is perpendicular to line m at A, line k is perpendicular to line m at B, and lines ℓ, m, and k are in the same plane.



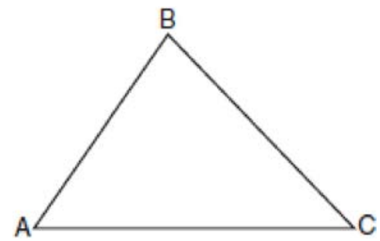
Which statement is the first step in an indirect proof to prove that ℓ is parallel to k?
1) Assume that ℓ, m, and k are not in the same plane.
2) Assume that ℓ is perpendicular to k.
3) Assume that ℓ is not perpendicular to m.
4) Assume that ℓ is not parallel to k.

2 Given: $\triangle ABT$, $\overline{CBTD}$, and $\overline{AB} \perp \overline{CD}$

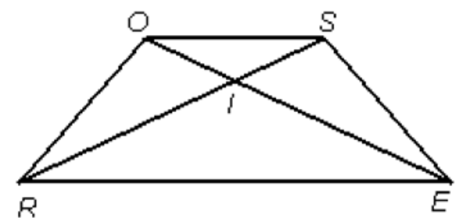


Write an indirect proof to show that $\overline{AT}$ is *not* perpendicular to $\overline{CD}$.

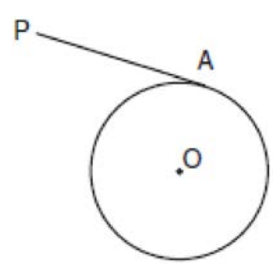
3 In the accompanying diagram, $\triangle ABC$ is *not* isosceles. Prove that if altitude $\overline{BD}$ were drawn, it would *not* bisect $\overline{AC}$.



4 Given trapezoid $ROSE$ with diagonals $\overline{RS}$ and $\overline{EO}$ intersecting at point I, prove that the diagonals of the trapezoid do *not* bisect each other.



5 In the accompanying diagram of circle O, $\overline{PA}$ is drawn tangent to the circle at A. Place B on $\overline{PA}$ anywhere between P and A and draw $\overline{OA}$, $\overline{OP}$, and $\overline{OA}$. Prove that $\overline{OB}$ is *not* perpendicular to $\overline{PA}$.



G.CO.C.9: Indirect Proofs
Answer Section

1 ANS: 4 REF: 010814b

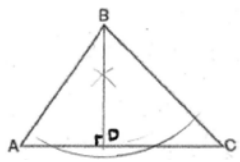
2 ANS:

Assume $\overline{AT} \perp \overline{CD}$. Then $m\angle ATB = 90°$. Since $\overline{AB} \perp \overline{CD}$, $m\angle ABT = 90°$. But a triangle may not have two right angles. Therefore the initial assumption is wrong and $\overline{AT}$ is *not* perpendicular to $\overline{CD}$.

REF: 060425b

3 ANS:

Assume $\overline{BD}$ bisects $\overline{AC}$ Since $\overline{BD}$ bisects $\overline{AC}$, $\overline{AD} \cong \overline{CD}$. Since $\overline{BD}$ is an altitude, $\overline{BD} \perp \overline{ADC}$. So $\angle ADB$ and $\angle CDB$ are right angles and congruent. $\overline{BD} \cong \overline{BD}$ because of the reflexive property. So $\triangle ABD \cong \triangle CBD$ by SAS. Corresponding parts of congruent triangles are congruent. Therefore $\overline{AB} \cong \overline{CB}$ But if $\overline{AB} \cong \overline{CB}$, then $\triangle ABC$ is isosceles. But the facts state $\triangle ABC$ is not isosceles. Therefore the initial assumption is wrong and $\overline{BD}$ does not bisect $\overline{AC}$.



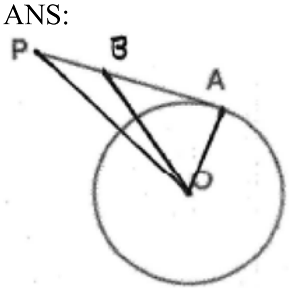
REF: 080230b

4 ANS:

A trapezoid has one and only one pair of opposite parallel sides, $\overline{OS}$ and $\overline{ER}$ Assume the diagonals of the trapezoid do bisect each other. Then $\overline{IS} \cong \overline{IR}$ and $\overline{IO} \cong \overline{IE}$ because of the definition of bisector. $\angle RIO \cong \angle EIS$ because they are vertical angles. Therefore $\triangle RIO \cong \triangle EIS$ because of SAS. Then, $\angle ORI \cong \angle ESI$ because of CPCTC. Because these alternate interior angles are congruent, $\overline{OR} \parallel \overline{ES}$. But a trapezoid can have only one pair of opposite parallel sides, which is a contradiction. Therefore the original assumption that the diagonals of the trapezoid bisect each other is false, proving that the diagonals of the trapezoid do not bisect each other.

REF: fall9933b

5 ANS:



Assume $\overline{OB} \perp \overline{PA}$. Then $m\angle OBA = 90°$. Since $\overline{PA}$ is a tangent and $\overline{OA}$ is a radius, $m\angle OAB = 90°$. But a triangle may not have two right angles. Therefore the initial assumption is wrong and $\overline{OA}$ is *not* perpendicular to $\overline{PA}$.

REF: 010432b