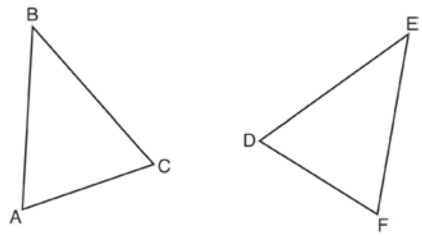
# G.CO.B.7: Triangle Congruency

1 Which statement is sufficient evidence that $\triangle DEF$ is congruent to $\triangle ABC$?



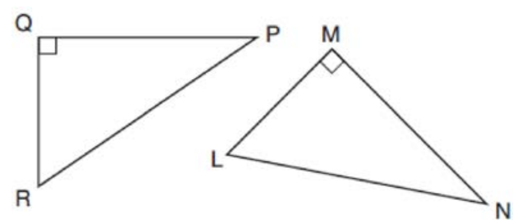

1) $AB = DE$ and $BC = EF$
2) $\angle D \cong \angle A$, $\angle B \cong \angle E$, $\angle C \cong \angle F$
3) There is a sequence of rigid motions that maps $\overline{AB}$ onto $\overline{DE}$, $\overline{BC}$ onto $\overline{EF}$, and $\overline{AC}$ onto $\overline{DF}$.
4) There is a sequence of rigid motions that maps point $A$ onto point $D$, $\overline{AB}$ onto $\overline{DE}$, and $\angle B$ onto $\angle E$.

2 Triangles $JOE$ and $SAM$ are drawn such that $\angle E \cong \angle M$ and $\overline{EJ} \cong \overline{MS}$. Which mapping would *not* always lead to $\triangle JOE \cong \triangle SAM$?
1) $\angle J$ maps onto $\angle S$
2) $\angle O$ maps onto $\angle A$
3) $\overline{EO}$ maps onto $\overline{MA}$
4) $\overline{JO}$ maps onto $\overline{SA}$

3 In the two distinct acute triangles $ABC$ and $DEF$, $\angle B \cong \angle E$. Triangles $ABC$ and $DEF$ are congruent when there is a sequence of rigid motions that maps
1) $\angle A$ onto $\angle D$, and $\angle C$ onto $\angle F$
2) $\overline{AC}$ onto $\overline{DF}$, and $\overline{BC}$ onto $\overline{EF}$
3) $\angle C$ onto $\angle F$, and $\overline{BC}$ onto $\overline{EF}$
4) point $A$ onto point $D$, and $\overline{AB}$ onto $\overline{DE}$

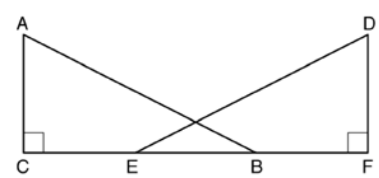
4 Triangles $YEG$ and $POM$ are two distinct non-right triangles such that $\angle G \cong \angle M$. Which statement is sufficient to prove $\triangle YEG$ is always congruent to $\triangle POM$?
1) $\angle E \cong \angle O$ and $\angle Y \cong \angle P$
2) $\overline{YG} \cong \overline{PM}$ and $\overline{YE} \cong \overline{PO}$
3) There is a sequence of rigid motions that maps $\angle E$ onto $\angle O$ and $\overline{YE}$ onto $\overline{PO}$.
4) There is a sequence of rigid motions that maps point $Y$ onto point $P$ and $\overline{YG}$ onto $\overline{PM}$.

5 In the diagram below, right triangle $PQR$ is transformed by a sequence of rigid motions that maps it onto right triangle $NML$.

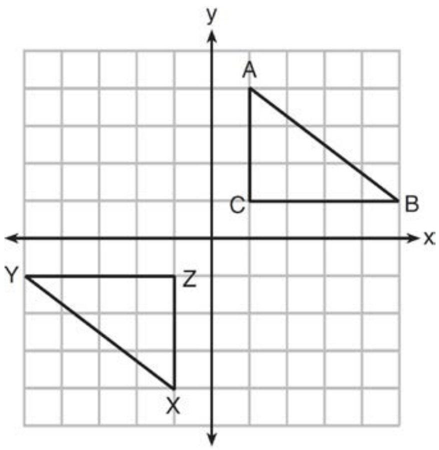



Write a set of three congruency statements that would show $ASA$ congruency for these triangles.

6 Given right triangles $ABC$ and $DEF$ where $\angle C$ and $\angle F$ are right angles, $\overline{AC} \cong \overline{DF}$ and $\overline{CB} \cong \overline{FE}$. Describe a precise sequence of rigid motions which would show $\triangle ABC \cong \triangle DEF$.

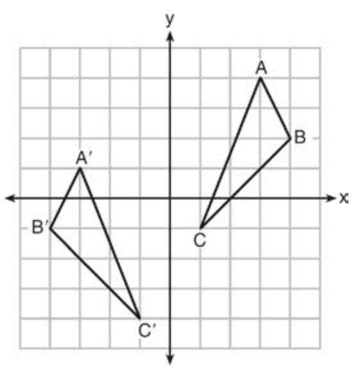


7 In the diagram below, $\triangle ABC$ and $\triangle XYZ$ are graphed.



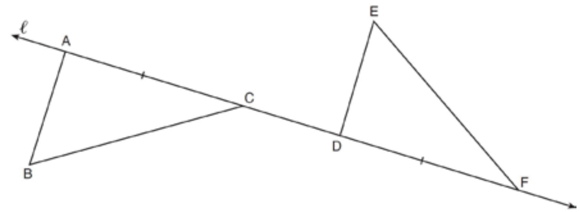

Use the properties of rigid motions to explain why $\triangle ABC \cong \triangle XYZ$.

8 As graphed on the set of axes below, $\triangle A'B'C'$ is the image of $\triangle ABC$ after a sequence of transformations.



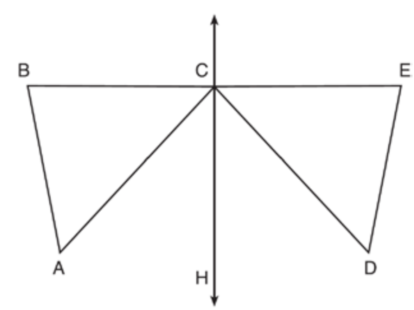

Is $\triangle A'B'C'$ congruent to $\triangle ABC$? Use the properties of rigid motion to explain your answer.

9 In the diagram below, $\overline{AC} \cong \overline{DF}$ and points $A$, $C$, $D$, and $F$ are collinear on line $\ell$.



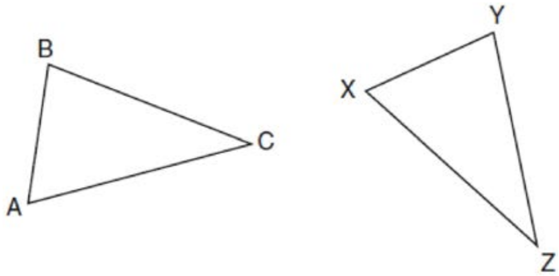

Let $\triangle D'E'F'$ be the image of $\triangle DEF$ after a translation along $\ell$, such that point $D$ is mapped onto point $A$. Determine and state the location of $F'$. Explain your answer. Let $\triangle D''E''F''$ be the image of $\triangle D'E'F'$ after a reflection across line $\ell$. Suppose that $E''$ is located at $B$. Is $\triangle DEF$ congruent to $\triangle ABC$? Explain your answer.

10 Given: $D$ is the image of $A$ after a reflection over $\overleftrightarrow{CH}$.
$\overleftrightarrow{CH}$ is the perpendicular bisector of $\overline{BCE}$
$\triangle ABC$ and $\triangle DEC$ are drawn
Prove: $\triangle ABC \cong \triangle DEC$



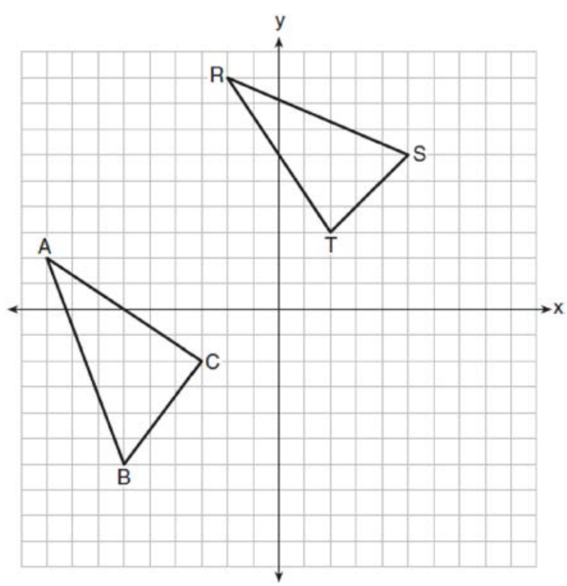

11 After a reflection over a line, $\triangle A'B'C'$ is the image of $\triangle ABC$. Explain why triangle $ABC$ is congruent to triangle $\triangle A'B'C'$.

12 In the diagram below of $\triangle ABC$ and $\triangle XYZ$, a sequence of rigid motions maps $\angle A$ onto $\angle X$, $\angle C$ onto $\angle Z$, and $\overline{AC}$ onto $\overline{XZ}$.




Determine and state whether $\overline{BC} \cong \overline{YZ}$. Explain why.

13 In the graph below, $\triangle ABC$ has coordinates $A(-9,2)$, $B(-6,-6)$, and $C(-3,-2)$, and $\triangle RST$ has coordinates $R(-2,9)$, $S(5,6)$, and $T(2,3)$.




Is $\triangle ABC$ congruent to $\triangle RST$? Use the properties of rigid motions to explain your reasoning.

# G.CO.B.7: Triangle Congruency
## Answer Section

1 ANS: 3 REF: 061524geo

2 ANS: 4
d) is SSA

REF: 061914geo

3 ANS: 3
NYSED has stated that all students should be awarded credit regardless of their answer to this question.

REF: 061722geo

4 ANS: 3
(3) is AAS, which proves congruency. (1) is AAA, (2) is SSA and (4) is AS.

REF: 012422geo

5 ANS:
$\angle Q \cong \angle M$ $\angle P \cong \angle N$ $\overline{QP} \cong \overline{MN}$

REF: 012025geo

6 ANS:
Translate $\triangle ABC$ along $\overline{CF}$ such that point $C$ maps onto point $F$, resulting in image $\triangle A'B'C'$. Then reflect $\triangle A'B'C'$ over $\overline{DF}$ such that $\triangle A'B'C'$ maps onto $\triangle DEF$.
or
Reflect $\triangle ABC$ over the perpendicular bisector of $\overline{EB}$ such that $\triangle ABC$ maps onto $\triangle DEF$.

REF: fall1408geo

7 ANS:
The transformation is a rotation, which is a rigid motion.

REF: 081530geo

8 ANS:
Yes. The sequence of transformations consists of a reflection and a translation, which are isometries which preserve distance and congruency.

REF: 011628geo

9 ANS:
Translations preserve distance. If point $D$ is mapped onto point $A$, point $F$ would map onto point $C$. $\triangle DEF \cong \triangle ABC$ as $\overline{AC} \cong \overline{DF}$ and points are collinear on line $\ell$ and a reflection preserves distance.

REF: 081534geo

10 ANS:

It is given that point $D$ is the image of point $A$ after a reflection in line $CH$. It is given that $\overleftrightarrow{CH}$ is the perpendicular bisector of $\overline{BCE}$ at point $C$. Since a bisector divides a segment into two congruent segments at its midpoint, $\overline{BC} \cong \overline{EC}$. Point $E$ is the image of point $B$ after a reflection over the line $CH$, since points $B$ and $E$ are equidistant from point $C$ and it is given that $\overleftrightarrow{CH}$ is perpendicular to $\overline{BE}$. Point $C$ is on $\overleftrightarrow{CH}$, and therefore, point $C$ maps to itself after the reflection over $\overleftrightarrow{CH}$. Since all three vertices of triangle $ABC$ map to all three vertices of triangle $DEC$ under the same line reflection, then $\triangle ABC \cong \triangle DEC$ because a line reflection is a rigid motion and triangles are congruent when one can be mapped onto the other using a sequence of rigid motions.

REF: spr1414geo

11 ANS:

Reflections are rigid motions that preserve distance.

REF: 061530geo

12 ANS:

Yes. $\angle A \cong \angle X$, $\angle C \cong \angle Z$, $\overline{AC} \cong \overline{XZ}$ after a sequence of rigid motions which preserve distance and angle measure, so $\triangle ABC \cong \triangle XYZ$ by ASA. $\overline{BC} \cong \overline{YZ}$ by CPCTC.

REF: 081730geo

13 ANS:

No. Since $\overline{BC} = 5$ and $\overline{ST} = \sqrt{18}$ are not congruent, the two triangles are not congruent. Since rigid motions preserve distance, there is no rigid motion that maps $\triangle ABC$ onto $\triangle RST$.

REF: 011830geo