Regents Exam Questions
F.TF.C.8: Proving Trigonometric Identities 2
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Name: _____

F.TF.C.8: Proving Trigonometric Identities 2

1 The expression $\frac{1-\sin^2 x}{\cos^2 x}$ is equivalent to

1) 1
2) -1
3) $\cos x$
4) $\sin x$

2 The expression $\frac{1-\cos^2 x}{\sin^2 x}$ is equivalent to

1) 1
2) -1
3) $\sin x$
4) $\cos x$

3 If $\sin A = k$, then the value of the expression $(\sin A)(\cos A)(\tan A)$ is equivalent to

1) 1
2) $\frac{1}{k}$
3) k
4) k^2

4 The expression $\frac{\sin^2 A}{\tan A}$ is equivalent to

1) $\frac{\sin A}{\cos A}$
2) $\sin A \cos A$
3) $\frac{1}{\sin A \cos A}$
4) $\frac{\cos A}{\sin A}$

5 The expression $\frac{\sin x \cdot \cos x}{\tan x}$ is equivalent to

1) 1
2) $\sin^2 x$
3) $\cos x$
4) $\cos^2 x$

6 A crate weighing w pounds sits on a ramp positioned at an angle of θ with the horizontal. The forces acting on this crate are modeled by the equation $Mw\cos\theta = w\sin\theta$, where M is the coefficient of friction. What is an expression for M in terms of θ?

1) $M = \tan\theta$
2) $M = \cot\theta$
3) $M = \sec\theta$
4) $M = \csc\theta$

7 Show that $\sec\theta \sin\theta \cot\theta = 1$ is an identity.

8 For all values of θ for which the expressions are defined, prove the identity:
$\tan\theta + \cot\theta = \sec\theta\csc\theta$

9 Prove that the equation shown below is an identity for all values for which the functions are defined:
$\csc\theta \cdot \sin^2\theta \cdot \cot\theta = \cos\theta$

10 For all values of θ for which the expressions are defined, prove that the following is an identity.
$\sec\theta - \sin\theta\tan\theta = \cos\theta$

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11 For all values of x for which the expressions are defined, prove the following is an identity:
$\sec^2 x + \csc^2 x = (\tan x + \cot x)^2$

12 For all values of θ for which the expressions are defined, prove the following is an identity:
$(\cot\theta + \csc\theta)(1 - \cos\theta) = \sin\theta$

13 Express in simplest terms: $\dfrac{2 - 2\sin^2 x}{\cos x}$

14 For all values of x for which the expressions are defined, prove that the following is an identity:
$\dfrac{\cos x + \cot x}{1 + \csc x} = \cos x$

15 For all values of θ for which the expressions are defined, prove the identity:
$\dfrac{\tan\theta \csc^2\theta}{1 + \tan^2\theta} = \cot\theta$

16 For all values of x for which the expressions are defined, prove the following equation is an identity:
$\dfrac{2}{\sec x} = \dfrac{2 - 2\sin^2 x}{\cos x}$

17 Prove the following is an identity for all values of θ for which the expressions are defined:
$\dfrac{\sin\theta}{\cot\theta} + \cos\theta = \sec\theta$

18 Prove the following identity: $\dfrac{\tan\theta}{\cot\theta} + 1 = \sec^2\theta$

19 For all values of x for which the expressions are defined, prove that the following is an identity:
$\dfrac{\sec x + \csc x}{\tan x + \cot x} = \sin x + \cos x$

20 For all values of θ for which the expressions are defined, prove the following is an identity:
$\dfrac{\tan\theta - \cot\theta}{\tan\theta + \cot\theta} = 2\sin^2\theta - 1$

21 For all values of θ for which the expressions are defined, prove the following is an identity:
$\dfrac{\cos\theta \sin\theta + \cos\theta}{\cos^2\theta} = \tan\theta + \sec\theta$

22 Prove the following identity:
$\dfrac{\sin\theta}{1 + \cos\theta} + \dfrac{1 + \cos\theta}{\sin\theta} = 2\cot\theta\sec\theta$

23 For all values of θ for which the expressions are defined, prove that the following is an identity:
$\dfrac{(\cos\theta + \sin\theta)^2}{1 + 2\sin\theta\cos\theta} = \cos\theta\tan\theta\csc\theta$

24 For all values of θ for which the expressions are defined, prove that the following is an identity:
$\cos\theta(\cos\theta + 1) + \sin^2\theta = \dfrac{\sin\theta + \tan\theta}{\tan\theta}$

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Answer Section

1 ANS: 1

$$\frac{1-\sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} = 1$$

REF: 081616a2

2 ANS: 1

$$\frac{1-\cos^2 x}{\sin^2 x} = \frac{\sin^2 x}{\sin^2 x} = 1$$

REF: 060610b

3 ANS: 4 REF: 019026siii

4 ANS: 2 REF: 089023siii

5 ANS: 4 REF: 080032siii

6 ANS: 1

$$M w \cos\theta = w \sin\theta$$

$$M \cos\theta = \sin\theta$$

$$M = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

REF: 060515b

7 ANS:

$$\sec\theta \sin\theta \cot\theta = \frac{1}{\cos\theta} \cdot \sin\theta \cdot \frac{\cos\theta}{\sin\theta} = 1$$

REF: 011428a2

8 ANS:

$$\tan\theta + \cot\theta = \sec\theta \csc\theta$$

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta}$$

$$\frac{\sin^2\theta + \cos^2\theta}{\cos\theta \cdot \sin\theta} = \frac{1}{\cos\theta \cdot \sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

REF: 068036siii

9 ANS:

$$\frac{1}{\sin\theta} \cdot \sin^2\theta \cdot \frac{\cos\theta}{\sin\theta} = \cos\theta$$

$$\cos\theta = \cos\theta$$

REF: 011634a2

10 ANS:

$$\sec\theta - \sin\theta\tan\theta = \cos\theta$$

$$\frac{1}{\cos\theta} - \sin\theta\frac{\sin\theta}{\cos\theta} = \cos\theta$$

$$1 - \sin^2\theta = \cos^2\theta$$

$$\cos^2\theta = \cos^2\theta$$

REF: 088937siii

11 ANS:

$$\sec^2 x + \csc^2 x = (\tan x + \cot x)^2$$

$$\tan^2 x + 1 + 1 + \cot^2 x = \tan^2 x + 2\tan x\cot x + \cot^2 x$$

$$2 = 2\frac{\sin x}{\cos x}\frac{\cos x}{\sin x}$$

$$2 = 2$$

REF: 019542siii

12 ANS:

$$(\cot\theta + \csc\theta)(1 - \cos\theta) = \sin\theta$$

$$\left(\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}\right)(1 - \cos\theta) = \sin\theta$$

$$\sin\theta\cdot\left(\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}\right)(1 - \cos\theta) = \sin\theta\cdot\sin\theta$$

$$(\cos\theta + 1)(1 - \cos\theta) = \sin^2\theta$$

$$(1 - \cos^2\theta) = \sin^2\theta$$

$$\sin^2\theta = \sin^2\theta$$

REF: 060041siii

13 ANS:

$$2\cos x.\ \frac{2 - 2\sin^2 x}{\cos x} = \frac{2(1 - \sin^2 x)}{\cos x} = \frac{2\cos^2 x}{\cos x} = 2\cos x.$$

REF: 080526b

14 ANS:

$$\frac{\cos x + \cot x}{1 + \csc x} = \cos x$$

$$\frac{\cos x + \dfrac{\cos x}{\sin x}}{1 + \dfrac{1}{\sin x}} = \cos x$$

$$\cos x + \frac{\cos x}{\sin x} = \cos x + \frac{\cos x}{\sin x}$$

REF: 019439siii

15 ANS:

$$\frac{\tan\theta \csc^2\theta}{1 + \tan^2\theta} = \cot\theta$$

$$\frac{\dfrac{\sin\theta}{\cos\theta}\dfrac{1}{\sin^2\theta}}{\sec^2\theta} = \cot\theta$$

$$\frac{\dfrac{\sin\theta}{\cos\theta}\dfrac{1}{\sin^2\theta}}{\dfrac{1}{\cos^2\theta}} = \cot\theta$$

$$\frac{1}{\cos\theta\sin\theta}\cos^2\theta = \frac{\cos\theta}{\sin\theta}$$

$$\frac{\cos\theta}{\sin\theta} = \frac{\cos\theta}{\sin\theta}$$

REF: 068140siii

16 ANS:

$$\frac{2}{\sec x} = \frac{2 - 2\sin^2 x}{\cos x}$$

$$\frac{2}{\dfrac{1}{\cos x}} = \frac{2(1 - \sin^2 x)}{\cos x}$$

$$\cos x = \frac{\cos^2 x}{\cos x}$$

$$\cos x = \cos x$$

REF: 089341siii

17 ANS:

$\frac{\sin\theta}{\cot\theta} + \cos\theta = \sec\theta$

$\frac{\sin\theta + \cot\theta\cos\theta}{\cot\theta} = \frac{1}{\cos\theta}$

$\cos\theta(\sin\theta + \cot\theta\cos\theta) = \cot\theta$

$\cos\theta(\sin\theta + \cot\theta\cos\theta) = \frac{\cos\theta}{\sin\theta}$

$\sin\theta + \cot\theta\cos\theta = \frac{1}{\sin\theta}$

$\sin\theta(\sin\theta + \cot\theta\cos\theta) = 1$

$\sin\theta(\sin\theta + \frac{\cos\theta}{\sin\theta}\cos\theta) = 1$

$\sin^2\theta + \cos^2\theta = 1$

REF: 089940siii

18 ANS:

$\frac{\tan\theta}{\cot\theta} + 1 = \sec^2\theta$

$\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}} + 1 = \frac{1}{\cos^2\theta}$

$\frac{\sin^2\theta}{\cos^2\theta} + 1 = \frac{1}{\cos^2\theta}$

$\sin^2\theta + \cos^2\theta = 1$

REF: 080040siii

19 ANS:

$$\frac{\sec x + \csc x}{\tan x + \cot x} = \sin x + \cos x$$

$$\frac{\frac{1}{\cos x} + \frac{1}{\sin x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} = \sin x + \cos x$$

$$\frac{\frac{\sin x + \cos x}{\sin x \cos x}}{\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}} = \sin x + \cos x$$

$$\frac{\sin x + \cos x}{\sin x \cos x} \cdot \frac{\sin x \cos x}{1} = \sin x + \cos x$$

$$\sin x + \cos x = \sin x + \cos x$$

REF: 069541siii

20 ANS:

$$\frac{\tan \theta - \cot \theta}{\tan \theta + \cot \theta} = 2\sin^2 \theta - 1$$

$$\frac{\frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\sin \theta}}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}} = -(1 - 2\sin^2 \theta)$$

$$\frac{\frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\sin \theta}}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}} \times \frac{\cos \theta \sin \theta}{\cos \theta \sin \theta} = -(\cos 2\theta)$$

$$\frac{\sin^2 \theta - \cos^2 \theta}{\sin^2 \theta + \cos^2 \theta} = -(\cos 2\theta)$$

$$\sin^2 \theta - \cos^2 \theta = -(\cos 2\theta)$$

$$-(\cos 2\theta) = -(\cos 2\theta)$$

REF: 010042siii

21 ANS:

$$\frac{\cos\theta\sin\theta+\cos\theta}{\cos^2\theta}=\tan\theta+\sec\theta$$

$$\frac{\cos\theta(\sin\theta+1)}{\cos^2\theta}=\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}$$

$$\frac{\sin\theta+1}{\cos\theta}=\frac{\sin\theta+1}{\cos\theta}$$

REF: 068639siii

22 ANS:

$$\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=2\cot\theta\sec\theta$$

$$\frac{\sin^2\theta+(1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}=\frac{2\cos\theta}{\sin\theta}\frac{1}{\cos\theta}$$

$$\frac{\sin^2\theta+1+2\cos\theta+\cos^2\theta}{\sin\theta(1+\cos\theta)}=\frac{2}{\sin\theta}$$

$$\frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)}=\frac{2}{\sin\theta}$$

$$\frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}=\frac{2}{\sin\theta}$$

$$\frac{2}{\sin\theta}=\frac{2}{\sin\theta}$$

REF: 019938siii

23 ANS:

$$\frac{(\cos\theta+\sin\theta)^2}{1+2\sin\theta\cos\theta}=\cos\theta\tan\theta\csc\theta$$

$$\frac{\cos^2\theta+2\cos\theta\sin\theta+\sin^2\theta}{1+2\sin\theta\cos\theta}=\cos\theta\frac{\sin\theta}{\cos\theta}\frac{1}{\sin\theta}$$

$$\frac{1+2\cos\theta\sin\theta}{1+2\sin\theta\cos\theta}=1$$

REF: 068440siii

24 ANS:

$$\cos\theta(\cos\theta+1)+\sin^2\theta=\frac{\sin\theta+\tan\theta}{\tan\theta}$$

$$\cos^2\theta+\cos\theta+\sin^2\theta=\frac{\sin\theta}{\tan\theta}+1$$

$$\cos\theta+1=\frac{\sin\theta}{\frac{\sin\theta}{\cos\theta}}+1$$

$$\cos\theta+1=\cos\theta+1$$

REF: 088636siii