F.TF.A.2: Unit Circle

1 Point $M\left(t, \frac{4}{7}\right)$ is located in the second quadrant on the unit circle. Determine the exact value of $t$.

2 In the accompanying diagram of a unit circle, the ordered pair $(x, y)$ represents the locus of points forming the circle. Which ordered pair is equivalent to $(x, y)$?

3 Point $A\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ is on the unit circle whose center is the origin. If $\theta$ is an angle in standard position whose terminal ray passes through point $A$, what is the value of $\sin \theta$?

4 The terminal side of $\theta$, an angle in standard position, intersects the unit circle at $P\left(\frac{1}{3}, \frac{\sqrt{8}}{3}\right)$. What is the value of $\sec \theta$?

5 In the accompanying diagram of circle $O$, $COA$ is a diameter, $O$ is the origin, $OA = 1$, and $m\angle BOA = 30$. What are the coordinates of $B$?
6. What is the image of (1,0) after a counterclockwise rotation of 60°?

1) \( \left( \frac{1}{2}, \frac{\sqrt{3}}{2} \right) \)

2) \( \left( \frac{\sqrt{3}}{2}, \frac{1}{2} \right) \)

3) \( \left( \frac{1}{2}, \frac{-\sqrt{3}}{2} \right) \)

4) \( \left( \frac{\sqrt{3}}{2}, \frac{-1}{2} \right) \)

7. In the accompanying diagram of a unit circle, the ordered pair \((x,y)\) represents the point where the terminal side of \(\theta\) intersects the unit circle.

If \(m\angle\theta = 120\), what is the value of \(x\) in simplest form?

1) \( \frac{\sqrt{3}}{2} \)

2) \( \frac{-\sqrt{3}}{2} \)

3) \( \frac{1}{2} \)

4) \( \frac{-1}{2} \)

8. In the accompanying diagram of a unit circle, the ordered pair \((x,y)\) represents the point where the terminal side of \(\theta\) intersects the unit circle.

If \(\theta = 150°\), what is the value of \(x\)?

1) \( 1 \)

2) \( \frac{\sqrt{3}}{2} \)

3) \( \frac{-1}{2} \)

4) \( \frac{-\sqrt{2}}{2} \)

9. In the unit circle shown in the accompanying diagram, what are the coordinates of \((x,y)\)?

If \(m\angle\theta = 120\), what is the value of \(x\) in simplest form?

1) \( \left( \frac{\sqrt{3}}{2}, -0.5 \right) \)

2) \( \left( -0.5, \frac{-\sqrt{3}}{2} \right) \)

3) \( (-30, -210) \)

4) \( \left( \frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2} \right) \)
10. On the unit circle shown in the diagram below, sketch an angle, in standard position, whose degree measure is $240^\circ$ and find the exact value of $\sin 240^\circ$.

11. In the accompanying diagram of circle $O$, point $O$ is the origin, $YO = 1$, $JO = 1$, and $TOY$ is a diameter. If the coordinates of point $J$ are $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$, how many degrees are in $\angle JOY$?

12. If $\theta$ is an angle in standard position and its terminal side passes through the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ on a unit circle, a possible value of $\theta$ is
   1) $30^\circ$
   2) $60^\circ$
   3) $120^\circ$
   4) $150^\circ$

13. If $\theta$ is an angle in standard position and its terminal side passes through point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ on the unit circle, then a possible value of $\theta$ is
   1) $60^\circ$
   2) $120^\circ$
   3) $150^\circ$
   4) $330^\circ$

14. In the accompanying diagram of a unit circle, the ordered pair $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ represents the point where the terminal side of $\theta$ intersects the unit circle. What is $\angle \theta$?
   1) $210^\circ$
   2) $225^\circ$
   3) $233^\circ$
   4) $240^\circ$
15 In the diagram below of a unit circle, the ordered pair \( \left( -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right) \) represents the point where the terminal side of \( \theta \) intersects the unit circle.

What is \( m \angle \theta \)?
1) 45
2) 135
3) 225
4) 240

16 In the accompanying diagram, point \( P(-0.6, -0.8) \) is on unit circle \( O \).

What is the measure of angle \( \theta \) to the nearest degree?
1) 143
2) 217
3) 225
4) 233

17 In the accompanying diagram, point \( P(0.6, -0.8) \) is on unit circle \( O \). What is the value of \( \theta \), to the nearest degree?

18 Which diagram represents an angle of \( \frac{7}{4} \pi \) radians in standard position?
1)
2)
3)
4)
19 In the accompanying diagram of a unit circle, the ordered pair \((x,y)\) represents the point where the terminal side of \(\theta\) intersects the unit circle. If \(\theta = -\frac{\pi}{3}\), what is the value of \(y\)?

1) \(-\frac{3}{2}\)  
2) \(-\frac{2}{2}\)  
3) \(-\sqrt{3}\)  
4) \(-\frac{1}{2}\)

20 In the accompanying diagram, \(PR\) is tangent to circle \(O\) at \(R\), \(QS \perp OR\), and \(PR \perp OR\). Which measure represents \(\sin \theta\)?

1) \(SO\)  
2) \(RO\)  
3) \(PR\)  
4) \(QS\)

21 The accompanying diagram shows unit circle \(O\), with radius \(OB = 1\). Which line segment has a length equivalent to \(\cos \theta\)?

1) \(\overline{AB}\)  
2) \(\overline{CD}\)  
3) \(\overline{OC}\)  
4) \(\overline{OA}\)

22 In the diagram below, the length of which line segment is equal to the exact value of \(\sin \theta\)?

1) \(\overline{TO}\)  
2) \(\overline{TS}\)  
3) \(\overline{OR}\)  
4) \(\overline{OS}\)
23 The accompanying diagram shows unit circle $O$, with radius $OD = 1$.

Which line segment has a length equivalent to $\tan \theta$?

1) $\overline{AD}$
2) $\overline{BC}$
3) $\overline{OA}$
4) $\overline{OB}$
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Answer Section

1. ANS:
   \[ r^2 + \left( \frac{4}{7} \right)^2 = 1 \]
   \[ r^2 + \frac{16}{49} = \frac{49}{49} \]
   \[ r^2 = \frac{33}{49} \]
   \[ t = \pm \frac{\sqrt{33}}{7} \]

   REF: 011931aii

2. ANS: 4
   REF: 068724siii

3. ANS:
   \[ \frac{\sqrt{3}}{2} \]

   REF: 018514siii

4. ANS: 1
   REF: 011815aii

5. ANS: 2
   REF: 068926siii

6. ANS: 1
   REF: 089028siii

7. ANS: 3
   REF: 069728siii

8. ANS: 2
   REF: 010226siii

9. ANS: 1

   \[ \cos(180^\circ + 30^\circ) = -\frac{\sqrt{3}}{2} \]
   \[ \sin(180^\circ + 30^\circ) = -0.5 \]

   REF: 010718b
10 ANS:

\[ \begin{align*}
\cos \theta &= \frac{1}{2} \\
\sin \theta &= \frac{\sqrt{3}}{2}
\end{align*} \]

\[ \theta = \cos^{-1} \frac{1}{2} \quad \text{or} \quad \theta = \sin^{-1} \frac{\sqrt{3}}{2} \]

\[ \theta = 60^\circ \quad \theta = 60^\circ \]

REF: 010205b

11 ANS: 45

REF: 089502siii

12 ANS: 2

\[ \cos \theta = \frac{-\sqrt{3}}{2} \]

\[ \theta = \cos^{-1} \left( \frac{-\sqrt{3}}{2} \right) = \pm 150^\circ \]

Since the terminal side of \( \theta \) lies in Quadrant III, \( \theta = -150^\circ \). Coterminal angles differ by multiples of 360°. \(-150 + 360 = 210\). or

\[ \begin{align*}
\sin \theta &= \frac{-1}{2} \\
\theta &= \sin^{-1} \left( \frac{-1}{2} \right) = -30^\circ
\end{align*} \]

\[ \sin(-30) = -\sin 30 = \sin(180 + 30) = \sin 210. \]

REF: 080510b

13 ANS: 2 REF: 069932siii

14 ANS: 1

\[ \begin{align*}
\sin \theta &= \frac{-1}{2} \\
\theta &= \sin^{-1} \left( \frac{-1}{2} \right) = -30^\circ
\end{align*} \]

15 ANS: 3 REF: 011104a2

16 ANS: 4 REF: 060028siii
17 ANS:

\[ \cos \theta = 0.6 \]

\[ \cos \theta = \cos (360^\circ - \theta) \]

307. \( \theta = \cos^{-1} 0.6 \). Since the terminal side of \( \theta \) lies in Quadrant IV, \( \theta \equiv 53^\circ \)

\[ \theta = \cos^{-1} (360 - 53) \]

or

\[ \theta = \cos 307 \]

\[ \theta = 307^\circ \]

\[ \sin \theta = -0.8 \]

\[ \theta = \sin^{-1} (-0.8) \). Coterminal angles differ by multiples of 360°. \( -53 + 360 = 307 \).

\[ \theta \equiv -53^\circ \]

REF: 010422b

18 ANS: 1 REF: 081609a2
19 ANS: 1 REF: 019828siii
20 ANS: 4 REF: 060520b
21 ANS: 4 REF: 080618b
22 ANS: 2 REF: 011315a2
23 ANS: 2 REF: 080335siii