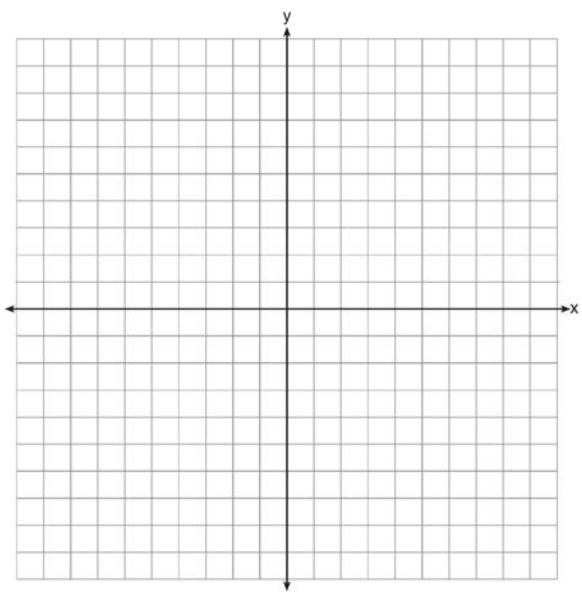
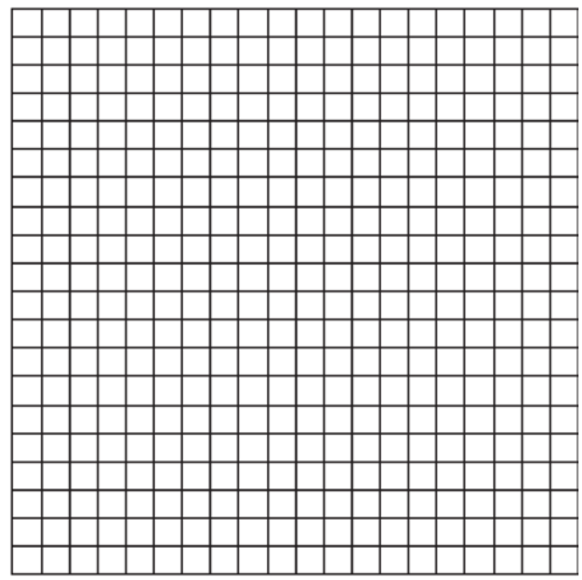
F.IF.C.7: Graphing Trigonometric Functions 4

1 In the interval $0 \le x \le 2\pi$, in how many points will the graphs of the equations $y = \sin x$ and $y = \frac{1}{2}$ intersect?
1) 1
2) 2
3) 3
4) 4

2 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point $(0,2)$.

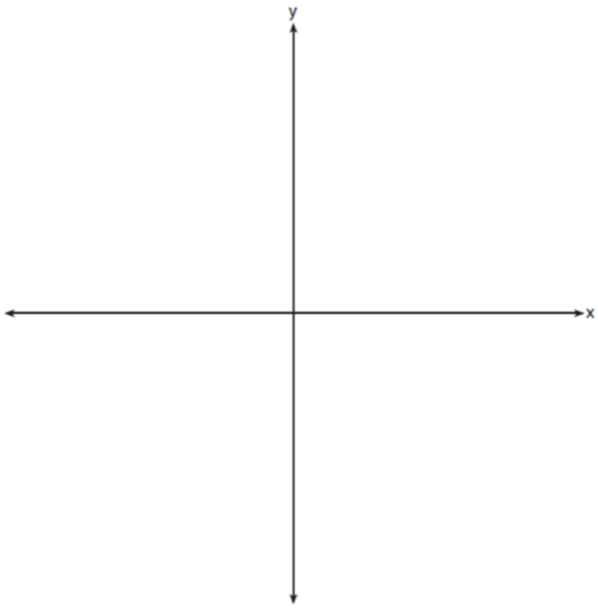


3 A radio wave has an amplitude of 3 and a wavelength (period) of π meters. On the accompanying grid, using the interval 0 to 2π, draw a possible sine curve for this wave that passes through the origin.



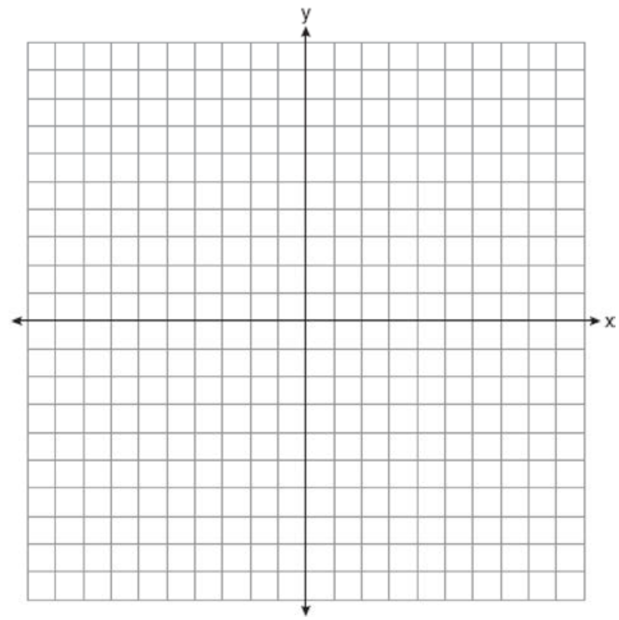
Name: ________________________

4 a) On the axes below, sketch *at least one* cycle of a sine curve with an amplitude of 2, a midline at $y=-\frac{3}{2}$, and a period of 2π.

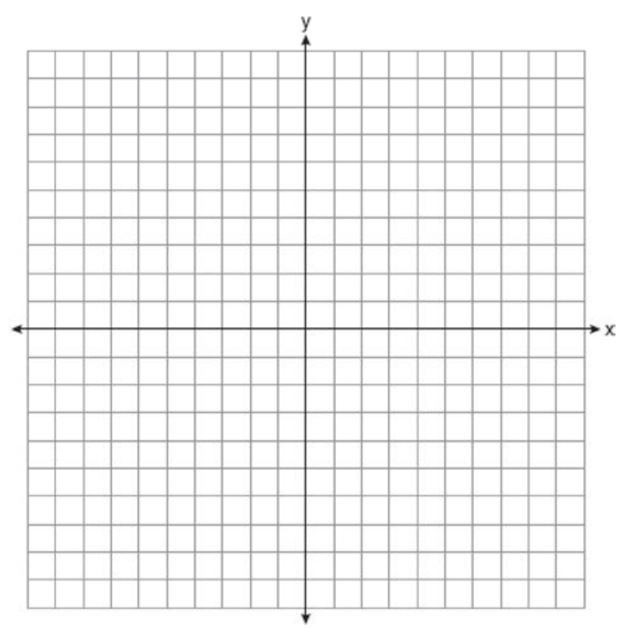


b) Explain any differences between a sketch of $y=2\sin\left(x-\frac{\pi}{3}\right)-\frac{3}{2}$ and the sketch from part a.

5 Sketch the graph of $y=3\sin 2x$ in the interval $-\pi \le x \le \pi$.

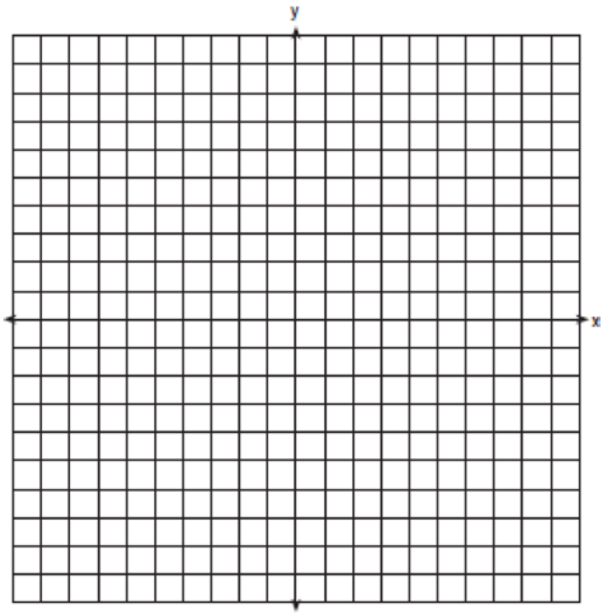


6 Sketch and label the function $y=2\sin\frac{1}{2}x$ in the interval $-2\pi \le x \le 2\pi$.

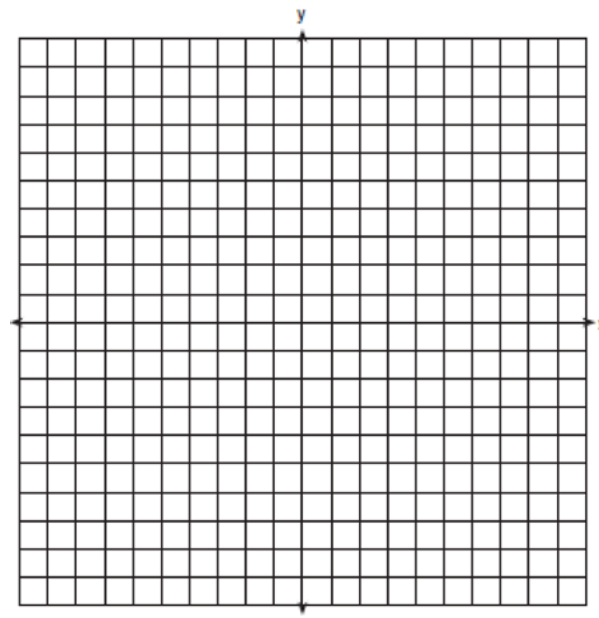


Name: ________________________

7 On the accompanying set of axes, graph the equations $y = 4\cos x$ and $y = 2$ in the domain $-\pi \le x \le \pi$. Express, in terms of π, the interval for which $4\cos x \ge 2$.

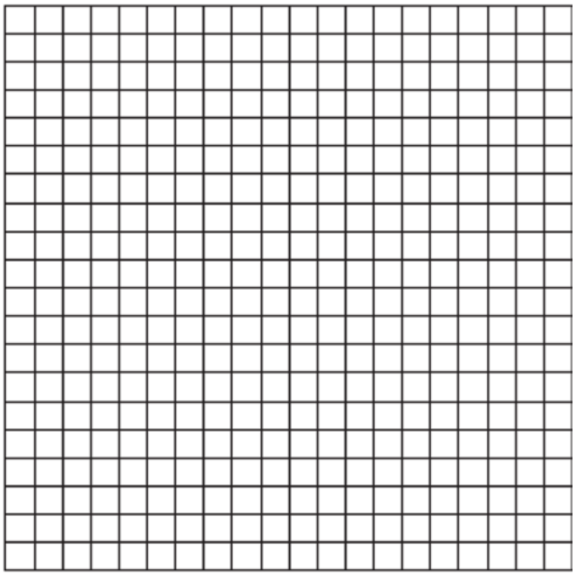


8 *a* On the accompanying set of axes, sketch the graph of the equations $y = 2\cos x$ in the interval $-\pi \le x \le \pi$.
b On the same set of axes, reflect the graph drawn in part *a* in the *x*-axis and label it *b*.
c Write an equation of the graph drawn in part *b*.
d Using the equation from part *c*, find the value of y when $x = \dfrac{\pi}{6}$.



Name: ________________________

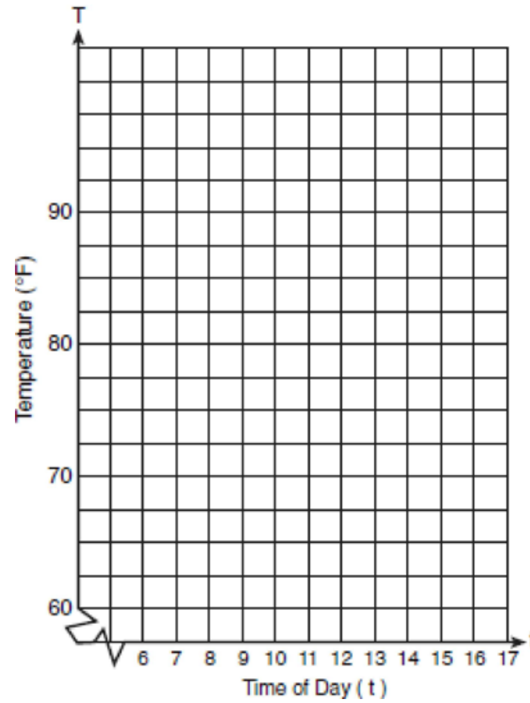
9 On the same set of axes, sketch and label the graphs of $y = 2\cos\frac{1}{2}x$ and $y = -1$ for the values of x in the interval $0 \le x \le 2\pi$. State the number of values of x in the interval $0 \le x \le 2\pi$ that satisfy the equation $2\cos\frac{1}{2}x = -1$.



10 A building's temperature, T, varies with time of day, t, during the course of 1 day, as follows:

$$T = 8\cos t + 78$$

The air-conditioning operates when $T \ge 80$°F. Graph this function for $6 \le t < 17$ and determine, to the *nearest tenth of an hour*, the amount of time in 1 day that the air-conditioning is on in the building.



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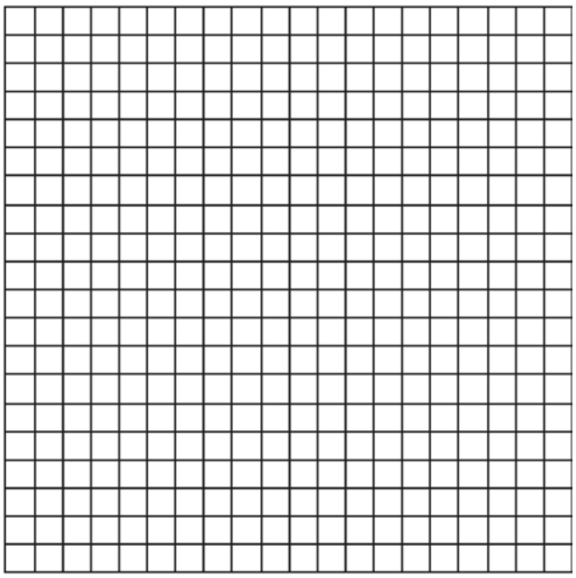
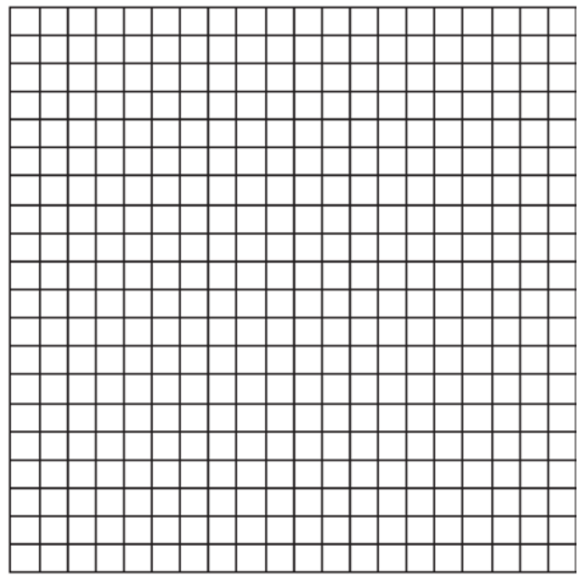
11 The tide at a boat dock can be modeled by the equation $y = -2\cos\left(\frac{\pi}{6}t\right) + 8$, where t is the number of hours past noon and y is the height of the tide, in feet. For how many hours between $t = 0$ and $t = 12$ is the tide at least 7 feet? [The use of the grid is optional.]



12 The average annual snowfall in a certain region is modeled by the function $S(t) = 20 + 10\cos\left(\frac{\pi}{5}t\right)$, where S represents the annual snowfall, in inches, and t represents the number of years since 1970. What is the minimum annual snowfall, in inches, for this region? In which years between 1970 and 2000 did the minimum amount of snow fall? [The use of the grid is optional.]



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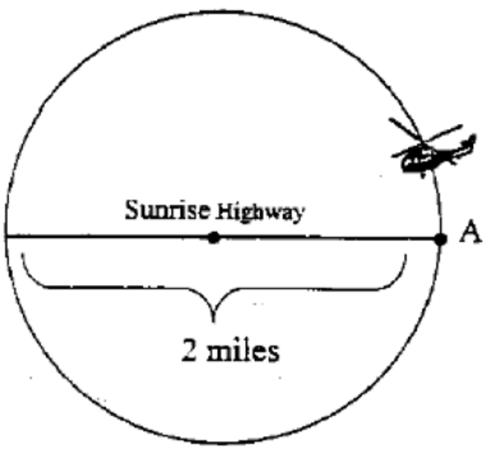
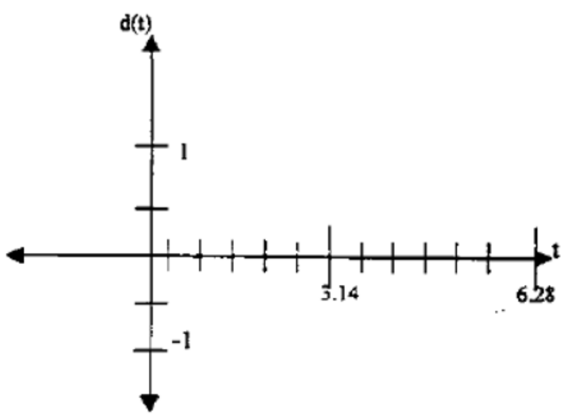
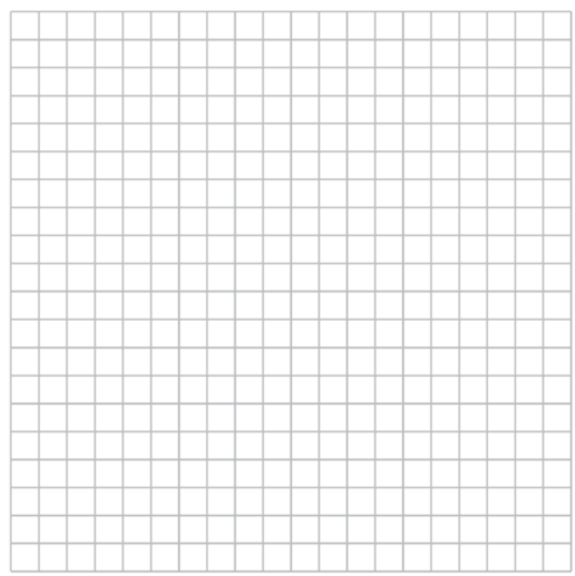
13 A helicopter, starting at point A on Sunrise Highway, circles a 2-mile section of the highway in a counterclockwise direction. If the helicopter is traveling at a constant speed and it takes approximately 6.28 minutes to make one complete revolution to return to point A, sketch a possible graph of distance (dependent variable) from the helicopter to the highway, versus time (independent variable). If the helicopter is north of the highway, distance (d) is positive; if the helicopter is south of the highway, distance (d) is negative. (Disregard the height of the helicopter.) State the equation of this graph.





14 The ocean tides near Carter Beach follow a repeating pattern over time, with the amount of time between each low and high tide remaining relatively constant. On a certain day, low tide occurred at 8:30 a.m. and high tide occurred at 3:00 p.m. At high tide, the water level was 12 inches above the average local sea level; at low tide it was 12 inches below the average local sea level. Assume that high tide and low tide are the maximum and minimum water levels each day, respectively. Write a cosine function of the form $f(t) = A\cos(Bt)$, where A and B are real numbers, that models the water level, $f(t)$, in inches above or below the average Carter Beach sea level, as a function of the time measured in t hours since 8:30 a.m. On the grid below, graph one cycle of this function.

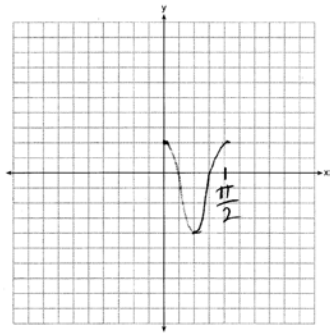


People who fish in Carter Beach know that a certain species of fish is most plentiful when the water level is increasing. Explain whether you would recommend fishing for this species at 7:30 p.m. or 10:30 p.m. using evidence from the given context.

F.IF.C.7: Graphing Trigonometric Functions 4
Answer Section

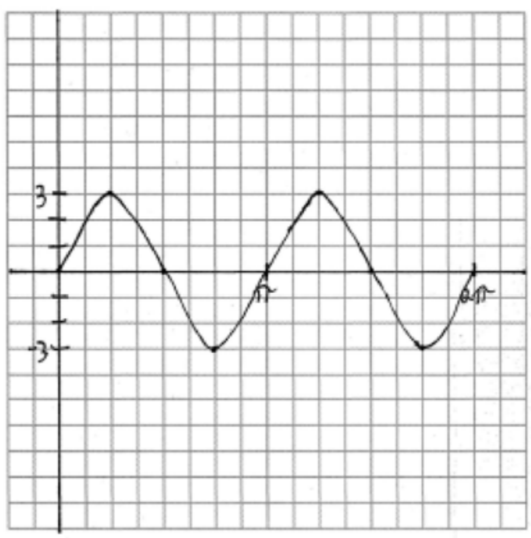
1 ANS: 2 REF: 069522siii

2 ANS:



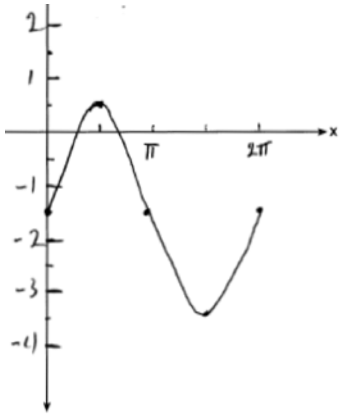
REF: 061628aii

3 ANS:



REF: 060832b

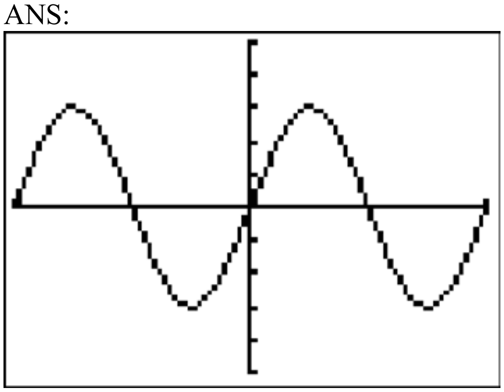
4 ANS:



Part a sketch is shifted $\frac{\pi}{3}$ units right.

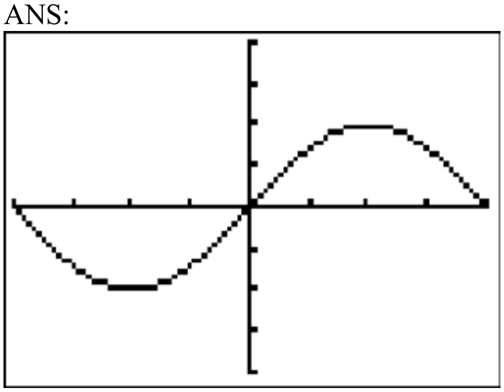
REF: 081735aii

5 ANS:



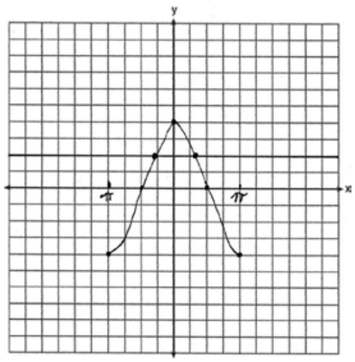
REF: 069040siii

6 ANS:



REF: 019536siii

7 ANS:



$-\frac{\pi}{3} \le x \le \frac{\pi}{3}.$

$$4\cos x \ge 2$$

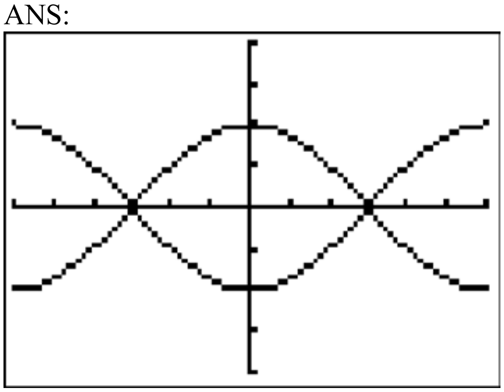
$$\cos x \ge \frac{2}{4}$$

$$x \ge \cos^{-1}\left(\frac{1}{2}\right)$$

$$-\frac{\pi}{3} \le x \le \frac{\pi}{3}$$

REF: 080532b

8 ANS:



$y = -2\cos x,\ -\sqrt{3}$

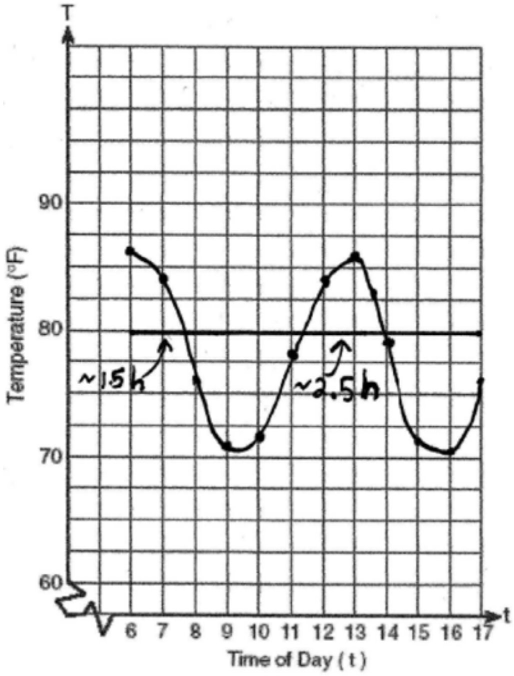
REF: 069637siii

9 ANS:

1

REF: 018436siii

10 ANS:



4.2.

$$8\cos t + 78 \geq 80$$
$$8\cos t \geq 2$$
$$\cos t \geq \frac{2}{8}$$
$$t \geq \cos^{-1}\frac{1}{4}$$

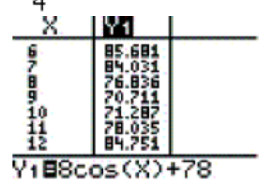
. $\cos^{-1}\frac{1}{4} \approx 1.3$

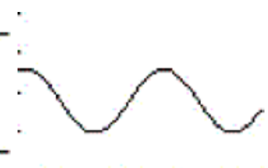
$1.3 + 2\pi \approx 7.6$. 7.6-6 = 1.6 hours.

$1.3 + 4\pi \approx 13.9$

$4\pi - 1.3 \approx 11.3$

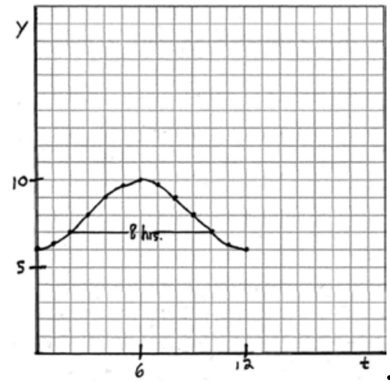
13.9-11.3 = 2.6 hours. 1.6+2.6 = 4.2 hours.





REF: 010329b

11 ANS:

8. 

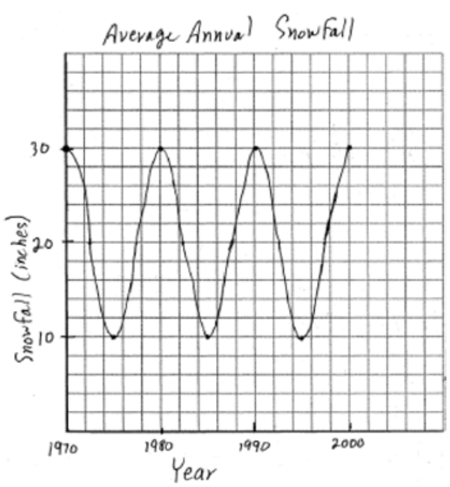
$$-2\cos\left(\frac{\pi}{6}t\right) + 8 \ge 7$$
$$-2\cos\left(\frac{\pi}{6}t\right) \ge -1$$
$$\cos\left(\frac{\pi}{6}t\right) \le \frac{1}{2}$$
$$\frac{\pi}{6}t \le \cos^{-1}\left(\frac{1}{2}\right)$$
$$\frac{\pi}{3} \le \frac{\pi}{6}t \le \frac{5\pi}{3}$$
$$2 \le t \le 10$$

REF: 080433b

12 ANS:

10, 1975, 1985, 1995. The minimum of the cosine function is -1. $20 + 10(-1) = 10$.

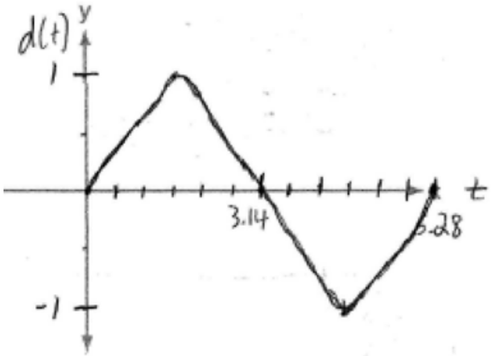
$$20 + 10\cos\frac{\pi}{5}t = 10$$
$$10\cos\frac{\pi}{5}t = -10$$
$$\cos\frac{\pi}{5}t = -1$$
$$\frac{\pi}{5}t = \cos^{-1} -1$$
$$\cos^{-1} -1 = \pi, 3\pi, 5\pi$$



$\frac{\pi}{5}t = \pi$, $t = 5$ (1975). $\frac{\pi}{5}t = 3\pi$, $t = 15$ (1985). $\frac{\pi}{5}t = 5\pi$, $t = 25$ (1995).

REF: 060731b

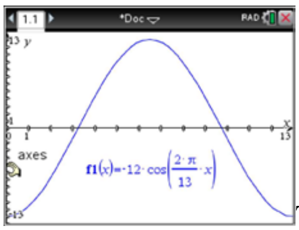
13 ANS:



d(t) = sin(t)

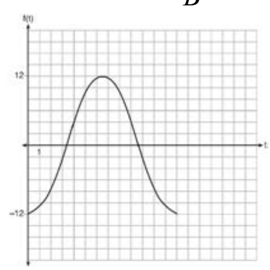
REF: fall9931b

14 ANS:



The amplitude, 12, can be interpreted from the situation, since the water level has a minimum of −12 and a maximum of 12. The value of A is −12 since at 8:30 it is low tide. The period of the function is 13 hours, and is expressed in the function through the parameter B. By experimentation with technology or using the relation $P = \frac{2\pi}{B}$ (where P is the period), it is determined that $B = \frac{2\pi}{13}$.

$f(t) = -12\cos\left(\frac{2\pi}{13}t\right)$



In order to answer the question about when to fish, the student must interpret the function and determine which choice, 7:30 pm or 10:30 pm, is on an increasing interval. Since the function is increasing from $t = 13$ to $t = 19.5$ (which corresponds to 9:30 pm to 4:00 am), 10:30 is the appropriate choice.

REF: spr1514aii