F.BF.B.4: Inverse of Functions 2b

1. What is the inverse of \( f(x) = \frac{x}{x+2} \), where \( x \neq -2 \)?

2. The inverse of the function \( f(x) = \frac{x + 1}{x - 2} \) is

3. What is the inverse of \( f(x) = x^3 - 2 \)?

4. What is the inverse of the function \( y = \log_3 x \)?

5. What is the inverse of the function \( f(x) = \log_4 x \)?

6. The inverse of a function is a logarithmic function in the form \( y = \log_b x \). Which equation represents the original function?

7. If \( f(x) = a^x \) where \( a > 1 \), then the inverse of the function is

8. Which equation defines a function whose inverse is not a function?
   1) \( y = |x| \)
   2) \( y = -x \)
   3) \( y = 3x + 2 \)
   4) \( y = 2^x \)

9. Which two functions are inverse functions of each other?
   1) \( f(x) = \sin x \) and \( g(x) = \cos(x) \)
   2) \( f(x) = 3 + 8x \) and \( g(x) = 3 - 8x \)
   3) \( f(x) = e^x \) and \( g(x) = \ln x \)
   4) \( f(x) = 2x - 4 \) and \( g(x) = -\frac{1}{2} x + 4 \)

10. If \( f(x) = x^2 - 6 \), find \( f^{-1}(x) \).

11. Given: \( f(x) = x^2 \) and \( g(x) = 2^x \)
    a) The inverse of \( g \) is a function, but the inverse of \( f \) is not a function. Explain why this statement is true.
    b) Find \( g^{-1}(f(3)) \) to the nearest tenth.

12. For the function \( f(x) = (x - 3)^3 + 1 \), find \( f^{-1}(x) \).
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Answer Section

1  ANS:
\[ f^{-1}(x) = \frac{-2x}{x-1} \]
\[ x = \frac{y}{y+2} \]
\[ xy + 2x = y \]
\[ xy - y = -2x \]
\[ y(x - 1) = -2x \]
\[ y = \frac{-2x}{x-1} \]

REF: 081924aii

2  ANS:
\[ f^{-1}(x) = \frac{2x + 1}{x-1} \]
\[ x = \frac{y + 1}{y-2} \]
\[ xy - 2x = y + 1 \]
\[ xy - y = 2x + 1 \]
\[ y(x - 1) = 2x + 1 \]
\[ y = \frac{2x + 1}{x-1} \]

REF: 081714aii

3  ANS:
\[ f^{-1}(x) = \sqrt[3]{x+2} \]
\[ y = x^3 - 2 \]
\[ x = y^3 - 2 \]
\[ x + 2 = y^3 \]
\[ \sqrt[3]{x+2} = y \]

REF: 061815aii

4  ANS:
\[ y = 3^x \]

REF: 011708aii
5 ANS:
$$f^{-1}(x) = 4^x$$
REF: 061521a2

6 ANS:
$$y = b^x$$
$$y = \log_b x$$
$$x = b^y$$
$$y = b^x$$
REF: 060115b

7 ANS:
$$f^{-1}(x) = \log_a x$$
REF: 011917aii

8 ANS: 1 
REF: 068932siii

9 ANS: 3 
REF: 081027a2

10 ANS:
$$y = x^2 - 6. \quad f^{-1}(x) \text{ is not a function.}$$
$$x = y^2 - 6$$
$$x + 6 = y^2$$
$$\pm \sqrt{x+6} = y$$
REF: 061132a2

11 ANS:
$$g(x) = y = 2^x$$
$$g^{-1}(x) = x = 2^y$$
$$\log x = \log 2^y$$
$$\log x = y \log 2$$
$$y = \frac{\log x}{\log 2}, \text{ which is a function because for every value of } x, \text{ there is a unique } y.$$
12 ANS:

\[ x = (y - 3)^3 + 1 \]

\[ x - 1 = (y - 3)^3 \]

\[ 3\sqrt{x - 1} = y - 3 \]

\[ 3\sqrt{x - 1} + 3 = y \]

\[ f^{-1}(x) = 3\sqrt{x - 1} + 3 \]

REF: fall1509a1i