

Calculus Practice: Differentiating Products and Quotients of Functions 4a

Differentiate each function with respect to x .

$$1) f(x) = \frac{3x^2}{4x^{\frac{2}{3}} - 5}$$

$$\begin{aligned} \text{A) } f'(x) &= 3x^2 \cdot \frac{8}{3}x^{-\frac{1}{3}} \\ &= 8x^{\frac{5}{3}} \\ &\quad \left(4x^{\frac{2}{3}} - 5\right) \cdot 6x - 3x^2 \cdot \frac{8}{3}x^{-\frac{1}{3}} \end{aligned}$$

$$\begin{aligned} \text{B) } f'(x) &= \frac{\left(4x^{\frac{2}{3}} - 5\right) \cdot 6x - 3x^2 \cdot \frac{8}{3}x^{-\frac{1}{3}}}{(3x^2)^2} \\ &= \frac{16x^{\frac{2}{3}} - 30}{9x^3} \end{aligned}$$

$$\begin{aligned} \text{C) } f'(x) &= \frac{\left(4x^{\frac{2}{3}} - 5\right) \cdot 6x - 3x^2 \cdot \frac{8}{3}x^{-\frac{1}{3}}}{\left(4x^{\frac{2}{3}} - 5\right)^2} \\ &= \frac{16x^{\frac{5}{3}} - 30x}{16x^{\frac{4}{3}} - 40x^{\frac{2}{3}} + 25} \end{aligned}$$

$$\begin{aligned} \text{D) } f'(x) &= \frac{\left(4x^{\frac{2}{3}} - 5\right) \cdot 6x - 3x^2 \cdot \frac{8}{3}x^{-\frac{1}{3}}}{4x^{\frac{2}{3}} - 5} \\ &= \frac{16x^{\frac{5}{3}} - 30x}{4x^{\frac{2}{3}} - 5} \end{aligned}$$

$$2) f(x) = \frac{5}{\sqrt[3]{x+2}}$$

$$\begin{aligned} \text{A) } f'(x) &= -\frac{5 \cdot \frac{1}{3}x^{-\frac{2}{3}}}{5^2} \\ &= -\frac{1}{15x^{\frac{2}{3}}} \end{aligned}$$

$$\begin{aligned} \text{B) } f'(x) &= -\frac{\frac{1}{3}x^{-\frac{2}{3}}}{\left(x^{\frac{1}{3}} + 2\right)^2} \\ &= -\frac{1}{3x^{\frac{4}{3}} + 12x + 12x^{\frac{2}{3}}} \end{aligned}$$

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$$3) y = \frac{x^2 - 4}{3 + \frac{1}{x^4}}$$

$$\begin{aligned} \text{A) } \frac{dy}{dx} &= \frac{(3 + x^{-4}) \cdot 2x - (x^2 - 4) \cdot -4x^{-5}}{(x^2 - 4)^2} \\ &= \frac{6x^6 + 6x^2 - 16}{x^9 - 8x^7 + 16x^5} \end{aligned}$$

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$$4) y = \frac{4x^4 - 4x^3 + 2x^2}{3 + 2x^{-3}}$$

$$\begin{aligned} \text{A) } \frac{dy}{dx} &= \frac{(3 + 2x^{-3})(16x^3 - 12x^2 + 4x) - (4x^4 - 4x^3 + 2x^2) \cdot -6x^{-4}}{(4x^4 - 4x^3 + 2x^2)^2} \\ &= \frac{12x^5 - 9x^4 + 3x^3 + 14x^2 - 12x + 5}{4x^{10} - 8x^9 + 8x^8 - 4x^7 + x^6} \end{aligned}$$

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$$\begin{aligned} \text{D) } \frac{dy}{dx} &= (4x^4 - 4x^3 + 2x^2) \cdot -6x^{-4} \\ &= -24 + \frac{24}{x} - \frac{12}{x^2} \end{aligned}$$

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