

Calculus Practice: Differentiating Products and Quotients of Functions 2a

Differentiate each function with respect to x .

1) $f(x) = \left(3x^{\frac{5}{4}} + 4\right)x^4$

A) $f'(x) = \left(3x^{\frac{5}{4}} + 4\right) \cdot 4x^3 + x^4 \cdot \frac{15}{4}x^{\frac{1}{4}}$

$$= \frac{63x^{\frac{17}{4}}}{4} + 16x^3$$

B) $f'(x) = \left(3x^{\frac{5}{4}} + 4\right)x^4 + \frac{15}{4}x^{\frac{1}{4}} \cdot 4x^3$

$$= 3x^{\frac{21}{4}} + 4x^4 + 15x^{\frac{13}{4}}$$

C) $f'(x) = x^4 + 4x^3$

D) $f'(x) = 3x^{\frac{5}{4}} + 4 + \frac{15}{4}x^{\frac{1}{4}}$

$$= 3x^{\frac{5}{4}} + \frac{15x^{\frac{1}{4}}}{4} + 4$$

3) $f(x) = \left(2 - \frac{2}{x^3}\right) \cdot 3x^5$

A) $f'(x) = 2 - 2x^{-3} + 6x^{-4}$
$$= 2 - \frac{2}{x^3} + \frac{6}{x^4}$$

B) $f'(x) = \left(2 - 2x^{-3}\right) \cdot 15x^4 + 3x^5 \cdot 6x^{-4}$
$$= 30x^4 - 12x$$

C) $f'(x) = \left(2 - 2x^{-3}\right) \cdot 3x^5 + 6x^{-4} \cdot 15x^4$
$$= 6x^5 - 6x^2 + 90$$

D) $f'(x) = 3x^5 + 15x^4$

2) $f(x) = \left(-4x^{\frac{2}{3}} + 3\right) \cdot 3x^5$

A) $f'(x) = \left(-4x^{\frac{2}{3}} + 3\right) \cdot 3x^5 - \frac{8}{3}x^{-\frac{1}{3}} \cdot 15x^4$

$$= -12x^{\frac{17}{3}} + 9x^5 - 40x^{\frac{11}{3}}$$

B) $f'(x) = -4x^{\frac{2}{3}} + 3 - \frac{8}{3}x^{-\frac{1}{3}}$

$$= -4x^{\frac{2}{3}} + 3 - \frac{8}{3x^{\frac{1}{3}}}$$

C) $f'(x) = \left(-4x^{\frac{2}{3}} + 3\right) \cdot 15x^4 + 3x^5 \cdot -\frac{8}{3}x^{-\frac{1}{3}}$

$$= -68x^{\frac{14}{3}} + 45x^4$$

D) $f'(x) = 3x^5 \cdot -\frac{8}{3}x^{-\frac{1}{3}}$

$$= -8x^{\frac{14}{3}}$$

4) $f(x) = \left(4\sqrt[4]{x} - 2\right) \cdot 5x^5$

A) $f'(x) = 5x^5x^{-\frac{3}{4}}$
$$= 5x^{\frac{17}{4}}$$

B) $f'(x) = \left(4x^{\frac{1}{4}} - 2\right) \cdot 5x^5 + x^{-\frac{3}{4}} \cdot 25x^4$

$$= 20x^{\frac{21}{4}} - 10x^5 + 25x^{\frac{13}{4}}$$

C) $f'(x) = \left(4x^{\frac{1}{4}} - 2\right) \cdot 25x^4 + 5x^5x^{-\frac{3}{4}}$

$$= 105x^{\frac{17}{4}} - 50x^4$$

D) $f'(x) = 5x^5 + 25x^4$

$$5) f(x) = \left(2x^{\frac{4}{5}} + 4\right)(4x^4 - 3)$$

$$\begin{aligned} \text{A) } f'(x) &= (4x^4 - 3) \cdot \frac{8}{5}x^{-\frac{1}{5}} \\ &= \frac{32x^4 - 24}{5x^{\frac{1}{5}}} \end{aligned}$$

$$\text{B) } f'(x) = 4x^4 - 3 + 16x^3$$

$$= 4x^4 + 16x^3 - 3$$

$$\begin{aligned} \text{C) } f'(x) &= \left(2x^{\frac{4}{5}} + 4\right) \cdot 16x^3 + (4x^4 - 3) \cdot \frac{8}{5}x^{-\frac{1}{5}} \\ &= \frac{192x^{\frac{19}{5}}}{5} + 64x^3 - \frac{24}{5x^{\frac{1}{5}}} \end{aligned}$$

$$\begin{aligned} \text{D) } f'(x) &= \left(2x^{\frac{4}{5}} + 4\right)(4x^4 - 3) + \frac{8}{5}x^{-\frac{1}{5}} \cdot 16x^3 \\ &= 8x^{\frac{24}{5}} + 16x^4 + \frac{128x^{\frac{14}{5}}}{5} - 6x^{\frac{4}{5}} - 12 \end{aligned}$$

$$7) y = \frac{5}{x^3}(4x^4 - 4)$$

$$\begin{aligned} \text{A) } \frac{dy}{dx} &= 5x^{-3} \cdot 16x^3 \\ &= 80 \end{aligned}$$

$$\begin{aligned} \text{B) } \frac{dy}{dx} &= 4x^4 - 4 + 16x^3 \\ &= 4x^4 + 16x^3 - 4 \end{aligned}$$

$$\begin{aligned} \text{C) } \frac{dy}{dx} &= 5x^{-3}(4x^4 - 4) - 15x^{-4} \cdot 16x^3 \\ &= 20x - \frac{240}{x} - \frac{20}{x^3} \end{aligned}$$

$$\begin{aligned} \text{D) } \frac{dy}{dx} &= 5x^{-3} \cdot 16x^3 + (4x^4 - 4) \cdot -15x^{-4} \\ &= 20 + \frac{60}{x^4} \end{aligned}$$

$$6) f(x) = (\sqrt[4]{x} + 5)(-4x^2 + 4)$$

$$\begin{aligned} \text{A) } f'(x) &= \left(x^{\frac{1}{4}} + 5\right) \cdot -8x + (-4x^2 + 4) \cdot \frac{1}{4}x^{-\frac{3}{4}} \\ &= -9x^{\frac{5}{4}} - 40x + \frac{1}{x^{\frac{3}{4}}} \end{aligned}$$

$$\begin{aligned} \text{B) } f'(x) &= \left(x^{\frac{1}{4}} + 5\right)(-4x^2 + 4) + \frac{1}{4}x^{-\frac{3}{4}} \cdot -8x \\ &= -4x^{\frac{9}{4}} - 20x^2 + 2x^{\frac{1}{4}} + 20 \end{aligned}$$

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$$8) y = \frac{2}{x^4}(4x^4 + 5)$$

$$\begin{aligned} \text{A) } \frac{dy}{dx} &= (4x^4 + 5) \cdot -8x^{-5} \\ &= -\frac{32}{x} - \frac{40}{x^5} \end{aligned}$$

$$\begin{aligned} \text{B) } \frac{dy}{dx} &= 2x^{-4} \cdot 16x^3 \\ &= \frac{32}{x} \end{aligned}$$

$$\begin{aligned} \text{C) } \frac{dy}{dx} &= 2x^{-4}(4x^4 + 5) - 8x^{-5} \cdot 16x^3 \\ &= 8 - \frac{128}{x^2} + \frac{10}{x^4} \end{aligned}$$

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