Calculus Practice: Differential Equations 1a

For each problem, find the particular solution of the differential equation that satisfies the initial condition.

1) $\dfrac{dy}{dx} = 2x\sqrt{y},\ y > 0,\ y(3) = \dfrac{9}{4}$

A) $2\sqrt{y} = x^2 - 6$
$y = \left(\dfrac{x^2}{2} - 3\right)^2,\ x > \sqrt{6}$

B) $e^y = 4e^x - 1$
$y = \ln\left(4e^x - 1\right),\ x > \ln\dfrac{1}{4}$

C) $2\sqrt{y} = x^2 - 6$
$y = \left(\dfrac{x^2}{2} - 3\right)^2,\ x < -\sqrt{6}$

D) $2\sqrt{y} = \dfrac{x^2}{2} - 4$
$y = \left(\dfrac{x^2}{4} - 2\right)^2,\ x > 2\sqrt{2}$

2) $\dfrac{dy}{dx} = \dfrac{1}{\cos y},\ y(-1) = 0$

A) $\sin y = x + 1$
$y = \sin^{-1}\left(x + 1\right),\ -2 < x < 0$

B) $\sin y = x - 2$
$y = \sin^{-1}\left(x - 2\right),\ 1 < x < 3$

C) $\sin y = x$
$y = \sin^{-1} x,\ -1 < x < 1$

D) $\tan y = x + 3$
$y = \tan^{-1}\left(x + 3\right)$

3) $\dfrac{dy}{dx} = \dfrac{2y^2}{x},\ y(1) = 1$

A) $\dfrac{y^3}{3} = x + \dfrac{x^3}{3}$
$y = \sqrt[3]{x^3 + 3x},\ x > 0$

B) $-\dfrac{1}{2y} = \ln|x| - \dfrac{1}{2}$
$y = -\dfrac{1}{2\ln|x| - 1},\ 0 < x < \sqrt{e}$

C) $-\dfrac{1}{y} = 4x - 3$
$y = -\dfrac{1}{4x - 3},\ x > \dfrac{3}{4}$

D) $-\dfrac{1}{y} = \ln|x| - 1$
$y = -\dfrac{1}{\ln|x| - 1},\ 0 < x < e$

4) $\dfrac{dy}{dx} = \dfrac{1 + x^2}{y^2},\ y(1) = \sqrt[3]{4}$

A) $\dfrac{y^3}{3} = 2x + \dfrac{x^3}{3}$
$y = \sqrt[3]{x^3 + 6x},\ x < 0$

B) $\dfrac{y^3}{3} = x + \dfrac{x^3}{3}$
$y = \sqrt[3]{x^3 + 3x},\ x > 0$

C) $\dfrac{y^3}{3} = -2x + \dfrac{x^3}{3}$
$y = \sqrt[3]{x^3 - 6x},\ -\sqrt{6} < x < 0$

D) $-\dfrac{1}{y} = \ln|x| - 1$
$y = -\dfrac{1}{\ln|x| - 1},\ 0 < x < e$

5) $\frac{dy}{dx} = -\frac{2yx}{\ln\ y},\ y(-1) = \frac{1}{e}$

A) $\ln\ \left|y - 1\right| = -\frac{1}{x}$

$y = e^{-\frac{1}{x}} + 1,\ x > 0$

B) $\frac{(\ln\ y)^2}{2} = -\frac{x^2}{2} + 1$

$y = e^{-\sqrt{-x^2 + 2}},\ -\sqrt{2} < x < \sqrt{2}$

C) $\frac{y^2}{2} = -\frac{x^2}{2} + \frac{3}{2}$

$y = -\sqrt{-x^2 + 3},\ -\sqrt{3} < x < \sqrt{3}$

D) $\frac{(\ln\ y)^2}{2} = -x^2 + \frac{3}{2}$

$y = e^{-\sqrt{-2x^2 + 3}},\ -\frac{\sqrt{6}}{2} < x < \frac{\sqrt{6}}{2}$

6) $\frac{dy}{dx} = \frac{y + 1}{x},\ y(-2) = -3$

A) $\ln\ \left|y + 2\right| = \ln\ \left|x\right|$

$y = x - 2,\ x > 0$

B) $\ln\ \left|y + 1\right| = \ln\ \left|x\right|$

$y = -x - 1,\ x < 0$

C) $\ln\ \left|y + 1\right| = \ln\ \left|x\right|$

$y = x - 1,\ x < 0$

D) $\ln\ \left|y + 1\right| = \ln\ \left|x\right|$

$y = -x - 1,\ x > 0$

7) $\frac{dy}{dx} = -\frac{2x}{y},\ y(-1) = -1$

A) $\ln\ \left|y + 2\right| = -\frac{1}{x} + \frac{2\ln\ 3 + 1}{2}$

$y = 3e^{-\frac{1}{x}} - 2,\ x > 0$

B) $\frac{y^2}{2} = -x^2 + \frac{3}{2}$

$y = -\sqrt{-2x^2 + 3},\ -\frac{\sqrt{6}}{2} < x < \frac{\sqrt{6}}{2}$

C) $\frac{y^2}{2} = -\frac{x^2}{2} + \frac{3}{2}$

$y = -\sqrt{-x^2 + 3},\ -\sqrt{3} < x < \sqrt{3}$

D) $\frac{y^2}{2} = -\frac{x^2}{2} + 1$

$y = -\sqrt{-x^2 + 2},\ -\sqrt{2} < x < \sqrt{2}$

8) $\frac{dy}{dx} = \frac{3y}{x^2},\ y(1) = -\frac{3}{e^3}$

A) $\frac{\ln\ \left|y\right|}{4} = -\frac{1}{x} + \frac{4 + \ln\ 2}{4}$

$y = 2e^{-\frac{4}{x}},\ x > 0$

B) $\ln\ \left|y\right| = -\frac{1}{x}$

$y = -e^{-\frac{1}{x}},\ x > 0$

C) $\frac{\ln\ \left|y\right|}{3} = -\frac{1}{x} + \frac{3 + \ln\ 3}{3}$

$y = -3e^{-\frac{3}{x}},\ x > 0$

D) $\ln\ \left|y + 1\right| = -\frac{1}{x} + \ln\ 2 + 1$

$y = -2e^{-\frac{1}{x}} - 1,\ x > 0$

Calculus Practice: Differential Equations 1a

For each problem, find the particular solution of the differential equation that satisfies the initial condition.

1) $\frac{dy}{dx} = 2x\sqrt{y},\ y > 0,\ y(3) = \frac{9}{4}$

*A) $2\sqrt{y} = x^2 - 6$
$y = \left(\frac{x^2}{2} - 3\right)^2,\ x > \sqrt{6}$

B) $e^y = 4e^x - 1$
$y = \ln\left(4e^x - 1\right),\ x > \ln\frac{1}{4}$

C) $2\sqrt{y} = x^2 - 6$
$y = \left(\frac{x^2}{2} - 3\right)^2,\ x < -\sqrt{6}$

D) $2\sqrt{y} = \frac{x^2}{2} - 4$
$y = \left(\frac{x^2}{4} - 2\right)^2,\ x > 2\sqrt{2}$

2) $\frac{dy}{dx} = \frac{1}{\cos y},\ y(-1) = 0$

*A) $\sin y = x + 1$
$y = \sin^{-1}\left(x + 1\right),\ -2 < x < 0$

B) $\sin y = x - 2$
$y = \sin^{-1}\left(x - 2\right),\ 1 < x < 3$

C) $\sin y = x$
$y = \sin^{-1} x,\ -1 < x < 1$

D) $\tan y = x + 3$
$y = \tan^{-1}\left(x + 3\right)$

3) $\frac{dy}{dx} = \frac{2y^2}{x},\ y(1) = 1$

A) $\frac{y^3}{3} = x + \frac{x^3}{3}$
$y = \sqrt[3]{x^3 + 3x},\ x > 0$

*B) $-\frac{1}{2y} = \ln|x| - \frac{1}{2}$
$y = -\frac{1}{2\ln|x| - 1},\ 0 < x < \sqrt{e}$

C) $-\frac{1}{y} = 4x - 3$
$y = -\frac{1}{4x - 3},\ x > \frac{3}{4}$

D) $-\frac{1}{y} = \ln|x| - 1$
$y = -\frac{1}{\ln|x| - 1},\ 0 < x < e$

4) $\frac{dy}{dx} = \frac{1 + x^2}{y^2},\ y(1) = \sqrt[3]{4}$

A) $\frac{y^3}{3} = 2x + \frac{x^3}{3}$
$y = \sqrt[3]{x^3 + 6x},\ x < 0$

*B) $\frac{y^3}{3} = x + \frac{x^3}{3}$
$y = \sqrt[3]{x^3 + 3x},\ x > 0$

C) $\frac{y^3}{3} = -2x + \frac{x^3}{3}$
$y = \sqrt[3]{x^3 - 6x},\ -\sqrt{6} < x < 0$

D) $-\frac{1}{y} = \ln|x| - 1$
$y = -\frac{1}{\ln|x| - 1},\ 0 < x < e$

5) $\frac{dy}{dx} = -\frac{2yx}{\ln y}, y(-1) = \frac{1}{e}$

A) $\ln |y - 1| = -\frac{1}{x}$
$y = e^{-\frac{1}{x}} + 1, x > 0$

B) $\frac{(\ln y)^2}{2} = -\frac{x^2}{2} + 1$
$y = e^{-\sqrt{-x^2 + 2}}, -\sqrt{2} < x < \sqrt{2}$

C) $\frac{y^2}{2} = -\frac{x^2}{2} + \frac{3}{2}$
$y = -\sqrt{-x^2 + 3}, -\sqrt{3} < x < \sqrt{3}$

*D) $\frac{(\ln y)^2}{2} = -x^2 + \frac{3}{2}$
$y = e^{-\sqrt{-2x^2 + 3}}, -\frac{\sqrt{6}}{2} < x < \frac{\sqrt{6}}{2}$

6) $\frac{dy}{dx} = \frac{y + 1}{x}, y(-2) = -3$

A) $\ln |y + 2| = \ln |x|$
$y = x - 2, x > 0$

B) $\ln |y + 1| = \ln |x|$
$y = -x - 1, x < 0$

*C) $\ln |y + 1| = \ln |x|$
$y = x - 1, x < 0$

D) $\ln |y + 1| = \ln |x|$
$y = -x - 1, x > 0$

7) $\frac{dy}{dx} = -\frac{2x}{y}, y(-1) = -1$

A) $\ln |y + 2| = -\frac{1}{x} + \frac{2\ln 3 + 1}{2}$
$y = 3e^{-\frac{1}{x}} - 2, x > 0$

*B) $\frac{y^2}{2} = -x^2 + \frac{3}{2}$
$y = -\sqrt{-2x^2 + 3}, -\frac{\sqrt{6}}{2} < x < \frac{\sqrt{6}}{2}$

C) $\frac{y^2}{2} = -\frac{x^2}{2} + \frac{3}{2}$
$y = -\sqrt{-x^2 + 3}, -\sqrt{3} < x < \sqrt{3}$

D) $\frac{y^2}{2} = -\frac{x^2}{2} + 1$
$y = -\sqrt{-x^2 + 2}, -\sqrt{2} < x < \sqrt{2}$

8) $\frac{dy}{dx} = \frac{3y}{x^2}, y(1) = -\frac{3}{e^3}$

A) $\frac{\ln |y|}{4} = -\frac{1}{x} + \frac{4 + \ln 2}{4}$
$y = 2e^{-\frac{4}{x}}, x > 0$

B) $\ln |y| = -\frac{1}{x}$
$y = -e^{-\frac{1}{x}}, x > 0$

*C) $\frac{\ln |y|}{3} = -\frac{1}{x} + \frac{3 + \ln 3}{3}$
$y = -3e^{-\frac{3}{x}}, x > 0$

D) $\ln |y + 1| = -\frac{1}{x} + \ln 2 + 1$
$y = -2e^{-\frac{1}{x}} - 1, x > 0$