

A.APR.C.4: Polynomial Identities

- 1 Emmeline is working on one side of a polynomial identity proof used to form Pythagorean triples. Her work is shown below:

$$(5x)^2 + (5x^2 - 5)^2$$

Step 1: $25x^2 + (5x^2 - 5)^2$

Step 2: $25x^2 + 25x^2 + 25$

Step 3: $50x^2 + 25$

Step 4: $75x^2$

What statement is true regarding Emmeline's work?

- 1) Emmeline's work is entirely correct. 3) There are mistakes in step 2 and step 4.
2) There is a mistake in step 2, only. 4) There is a mistake in step 4, only.
- 2 The expression $(x + a)(x + b)$ can *not* be written as
- 1) $a(x + b) + x(x + b)$ 3) $x^2 + (a + b)x + ab$
2) $x^2 + abx + ab$ 4) $x(x + a) + b(x + a)$
- 3 Which equation does *not* represent an identity?
- 1) $x^2 - y^2 = (x + y)(x - y)$ 3) $(x + y)^2 = x^2 + 2xy + y^2$
2) $(x - y)^2 = (x - y)(x - y)$ 4) $(x + y)^3 = x^3 + 3xy + y^3$
- 4 Which statement(s) are true for all real numbers?
- I $(x - y)^2 = x^2 + y^2$
II $(x + y)^3 = x^3 + 3xy + y^3$
- 1) I, only 3) I and II
2) II, only 4) neither I nor II
- 5 For which equations will the value $s = 4$ make the statement an identity?
- I $(2x - 3)^2 = 4x^2 - 3sx + 9$
II $(x - 2)^3 = (x - 2)(x^2 + sx + s)$
- 1) I, only 3) I and II
2) II, only 4) neither I nor II
- 6 Mr. Farison gave his class the three mathematical rules shown below to either prove or disprove. Which rules can be proved for all real numbers?
- I $(m + p)^2 = m^2 + 2mp + p^2$
II $(x + y)^3 = x^3 + 3xy + y^3$
III $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$
- 1) I, only 3) II and III
2) I and II 4) I and III
- 7 Which equation represents a polynomial identity?
- 1) $x^3 + y^3 = (x + y)^3$ 3) $x^3 + y^3 = (x + y)(x^2 - xy - y^2)$
2) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ 4) $x^3 + y^3 = (x - y)(x^2 + xy + y^2)$

8 The expression $(x^2 + y^2)^2$ is *not* equivalent to

1) $(x^2 - y^2)^2 + (2xy)^2$

3) $x^2(x^2 + 2y^2) + (y^2)^2$

2) $(x + y)^4 + 2(xy)^2$

4) $(2x^2 + y^2)^2 - (3x^4 + 2x^2y^2)$

9 Which equation is true for all real values of x ?

1) $x^4 + x = (x + 1)(x^3 - x^2 + x)$

3) $x^4 + x = (x^2 + x)^2$

2) $x^4 + x = (x + 1)(x^3 + x)$

4) $x^4 + x = (x - 1)(x^3 + x^2 + x)$

10 How many equations below are identities?

• $x^2 + y^2 = (x^2 - y^2) + (2xy)^2$

• $x^3 + y^3 = (x - y) + (x^2 - xy + y^2)$

• $x^4 + y^4 = (x - y)(x - y)(x^2 + y^2)$

1) 1

3) 3

2) 2

4) 0

11 Given the following polynomials

$$x = (a + b + c)^2$$

$$y = a^2 + b^2 + c^2$$

$$z = ab + bc + ac$$

Which identity is true?

1) $x = y - z$

3) $x = y - 2z$

2) $x = y + z$

4) $x = y + 2z$

12 Given the polynomial identity $x^6 + y^6 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$, which equation must also be true for all values of x and y ?

1) $x^6 + y^6 = x^2(x^4 - x^2y^2 + y^4) + y^2(x^4 - x^2y^2 + y^4)$

2) $x^6 + y^6 = (x^2 + y^2)(x^2 - y^2)(x^2 - y^2)$

3) $(x^3 + y^3)^2 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$

4) $(x^6 + y^6) - (x^2 + y^2) = x^4 - x^2y^2 + y^4$

13 Algebraically prove that the difference of the squares of any two consecutive integers is an odd integer.

14 Verify the following Pythagorean identity for all values of x and y :

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$$

15 Erin and Christa were working on cubing binomials for math homework. Erin believed they could save time with a shortcut. She wrote down the rule below for Christa to follow.

$$(a + b)^3 = a^3 + b^3$$

Does Erin's shortcut always work? Justify your result algebraically.

16 Algebraically determine the values of h and k to correctly complete the identity stated below.

$$2x^3 - 10x^2 + 11x - 7 = (x - 4)(2x^2 + hx + 3) + k$$

A.APR.C.4: Polynomial Identities Answer Section

1 ANS: 3 REF: 012003aai

2 ANS: 2 REF: 011806aai

3 ANS: 4

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

REF: 012417aai

4 ANS: 4

$$(x-y)^2 = x^2 - 2xy + y^2 \quad (x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

REF: 061902aai

5 ANS: 1

$$(2x-3)^2 = 4x^2 - 12x + 9 \quad (x-2)^3 = (x-2)(x-2)^2 = (x-2)(x^2 - 4x + 4)$$

$$s = 4$$

$$s = -4 \text{ and } 4$$

REF: 062405aai

6 ANS: 4

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 \neq x^3 + 3xy + y^3$$

REF: 081620aai

7 ANS: 2 REF: 012311aai

8 ANS: 2

$$1) x^4 - 2x^2y^2 + y^4 + 4x^2y^2; \quad 3) x^4 + 2x^2y^2 + y^4; \quad 4) 4x^4 + 4x^2y^2 + y^4 - 3x^4 - 2x^2y^2$$

REF: 012522aai

9 ANS: 1

$$x^4 + x$$

$$x(x^3 + 1)$$

$$x(x+1)(x^2 - x + 1)$$

$$(x+1)(x^3 - x^2 + x)$$

REF: 082404aai

10 ANS: 4

$$(x^2 - y^2) + (2xy)^2 = x^2 + 4x^2y^2 - y^2$$

$$(x-y) + (x^2 - xy + y^2) = x^2 + x - y - xy + y^2$$

$$(x-y)(x-y)(x^2 + y^2) = (x^2 - 2xy + y^2)(x^2 + y^2) = x^4 - 2x^3y + x^2y^2 + x^2y^2 - 2xy^3 + y^4$$

REF: 062322aai

11 ANS: 4

$$(a+b+c)^2 = a^2 + ab + ac + ab + b^2 + bc + ac + ab + c^2$$

$$x = a^2 + b^2 + c^2 + 2(ab + bc + ac)$$

$$x = y + 2z$$

REF: 061822aii

12 ANS: 1

$$2) (x^4 - x^2y^2 + y^4) \neq (x^2 - y^2)(x^2 - y^2); 3) x^6 + y^6 \neq (x^3 + y^3)^2; 4) \frac{x^6 + y^6}{x^2 + y^2} \neq x^6 + y^6 - (x^2 + y^2)$$

REF: 082219aii

13 ANS:

Let x equal the first integer and $x + 1$ equal the next. $(x + 1)^2 - x^2 = x^2 + 2x + 1 - x^2 = 2x + 1$. $2x + 1$ is an odd integer.

REF: fall1511aii

14 ANS:

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$$

$$x^4 + 2x^2y^2 + y^4 = x^4 - 2x^2y^2 + y^4 + 4x^2y^2$$

$$x^4 + 2x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4$$

REF: 081727aii

15 ANS:

$$(a+b)^3 = a^3 + b^3 \quad \text{No. Erin's shortcut only works if } a = 0, b = 0 \text{ or } a = -b.$$

$$a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3$$

$$3ab^2 + 3a^2b = 0$$

$$3ab(b+a) = 0$$

$$a = 0, b = 0, a = -b$$

REF: 011927aii

16 ANS:

$$2x^3 - 10x^2 + 11x - 7 = 2x^3 + hx^2 + 3x - 8x^2 - 4hx - 12 + k \quad h = -2$$

$$-2x^2 + 8x + 5 = hx^2 - 4hx + k \quad k = 5$$

REF: 011733aii