A.APR.B.3: Solving Polynomial Equations

1 When factoring to reveal the roots of the equation $x^3 + 2x^2 - 9x - 18 = 0$, which equations can be used?

I. $x^2(x+2) - 9(x+2) = 0$

II. $x(x^2 - 9) + 2(x^2 - 9) = 0$

III. $(x-2)(x^2 - 9) = 0$

1) I and II, only
2) I and III, only
3) II and III, only
4) I, II, and III

2 What are the zeros of $P(m) = (m^2 - 4)(m^2 + 1)$?

1) 2 and -2, only
2) $2, -2$, and -4
3) -4, i, and $-i$
4) $2, -2$, i, and $-i$

3 Which values of x are solutions of the equation $x^3 + x^2 - 2x = 0$?

1) $0, 1, 2$
2) $0, 1, -2$
3) $0, -1, 2$
4) $0, -1, -2$

4 The zeros of the function $f(x) = x^3 - 9x^2$ are

1) 9, only
2) 0 and 9
3) 0 and 3, only
4) -3, 0, and 3

5 The zeros of the function $f(x) = 2x^3 + 12x - 10x^2$ are

1) $\{2, 3\}$
2) $\{-1, 6\}$
3) $\{0, 2, 3\}$
4) $\{0, -1, 6\}$

6 How many negative solutions to the equation $2x^3 - 4x^2 + 3x - 1 = 0$ exist?

1) 1
2) 2
3) 3
4) 0

7 Given $c(m) = m^3 - 2m^2 + 4m - 8$, the solution of $c(m) = 0$ is

1) ± 2
2) 2, only
3) $2i, 2$
4) $\pm 2i, 2$

8 The zeros for $f(x) = x^4 - 4x^3 - 9x^2 + 36x$ are

1) $\{0, \pm 3, 4\}$
2) $\{0, 3, 4\}$
3) $\{0, \pm 3, -4\}$
4) $\{0, 3, -4\}$

9 What is the solution set of the equation $3x^5 - 48x = 0$?

1) $\{0, \pm 2\}$
2) $\{0, \pm 2, 3\}$
3) $\{0, \pm 2, \pm 2i\}$
4) $\{\pm 2, \pm 2i\}$

10 What are the zeros of $s(x) = x^4 - 9x^2 + 3x^3 - 27x - 10x^2 + 90$?

1) $\{-3, -2, 5\}$
2) $\{-2, 3, 5\}$
3) $\{-3, -2, 3, 5\}$
4) $\{-5, -3, 2, 3\}$

11 Evan graphed a cubic function, $f(x) = ax^3 + bx^2 + cx + d$, and determined the roots of $f(x)$ to be ± 1 and 2. What is the value of b, if $a = 1$?
1) 1
2) 2
3) −1
4) −2

12 Given 3 is a root of $f(x) = x^4 - x^3 - 21x^2 + 45x$, what are the other unique roots of $f(x)$?
1) −5, only
2) −5 and 0
3) −3, 1 and 5
4) −5, −3 and 0

13 Which statement regarding polynomials and their zeros is true?
1) $f(x) = (x^2 - 1)(x + a)$ has zeros of 1 and $-a$, only.
2) $f(x) = x^3 - ax^2 + 16x - 16a$ has zeros of 4 and a, only.
3) $f(x) = (x^2 + 25)(x + a)$ has zeros of ± 5 and $-a$.
4) $f(x) = x^3 - ax^2 - 9x + 9a$ has zeros of ± 3 and a.

14 Determine algebraically the zeros of $f(x) = 3x^3 + 21x^2 + 36x$.

15 Solve the equation $2x^3 - x^2 - 8x + 4 = 0$ algebraically for all values of x.

16 Algebraically determine the zeros of the function below.

$$r(x) = 3x^3 + 12x^2 - 3x - 12$$

17 Solve the equation $8x^3 + 4x^2 - 18x - 9 = 0$ algebraically for all values of x.

18 Solve $x^3 + 5x^2 = 4x + 20$ algebraically.

19 Solve algebraically for all values of x: $x^4 + 4x^3 + 4x^2 = -16x$

20 Solve, giving 4 roots, $x^4 - 13x^2 + 36 = 0$

21 Solve: $x^4 - 3x^2 = 4$ (Find 4 roots.)

22 Solve: $x^4 + 4x^2 = 32$. Find 4 roots.

23 Find *four* roots of the following equation: $2x^4 + 5x^2 = 207$

24 Given $z(x) = 6x^3 + bx^2 - 52x + 15$, $z(2) = 35$, and $z(-5) = 0$, algebraically determine all the zeros of $z(x)$.

25 Denise is designing a storage box in the shape of a cube. Each side of the box has a length of 10 inches. She needs more room and decides to construct a larger box in the shape of a cube with a volume of 2,000 cubic inches. By how many inches, to the *nearest tenth,* should she *increase* the length of each side of the original box?

A.APR.B.3: Solving Polynomial Equations
Answer Section

1 ANS: 1

$x^3 + 2x^2 - 9x - 18 = 0 \quad x^3 - 9x + 2x^2 - 18 = 0 \quad x^3 - 9x + 2x^2 - 18 = 0$

$x^2(x+2) - 9(x+2) = 0 \quad x(x^2 - 9) + 2(x^2 - 9) = 0 \quad x(x^2 - 9) + 2(x^2 - 9) = 0$

$(x+2)(x^2 - 9) = 0$

REF: 011903aii

2 ANS: 4 REF: 081708aii

3 ANS: 2

$x^3 + x^2 - 2x = 0$

$x(x^2 + x - 2) = 0$

$x(x+2)(x-1) = 0$

$x = 0, -2, 1$

REF: 011103a2

4 ANS: 2

$f(x) = x^3 - 9x^2 = x^2(x-9) = 0$

$x = 0, 9$

REF: 012009ai

5 ANS: 3

$2x^3 + 12x - 10x^2 = 0$

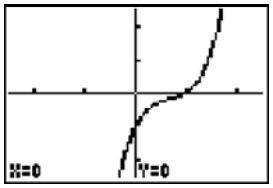
$2x(x^2 - 5x + 6) = 0$

$2x(x-3)(x-2) = 0$

$x = 0, 2, 3$

REF: 081719ai

6 ANS: 4



REF: 061222a2

7 ANS: 4

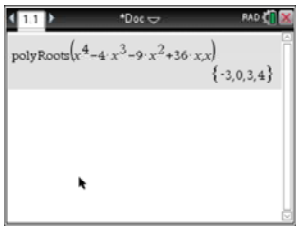
$$m^3 - 2m^2 + 4m - 8 = 0$$

$$m^2(m-2) + 4(m-2) = 0$$

$$\left(m^2 + 4\right)(m-2) = 0$$

REF: 081821aii

8 ANS: 1



$$x^4 - 4x^3 - 9x^2 + 36x = 0$$

$$x^3(x-4) - 9x(x-4) = 0$$

$$(x^3 - 9x)(x-4) = 0$$

$$x(x^2 - 9)(x-4) = 0$$

$$x(x+3)(x-3)(x-4) = 0$$

$$x = 0, \pm 3, 4$$

REF: 061606aii

9 ANS: 3

$$3x^5 - 48x = 0$$

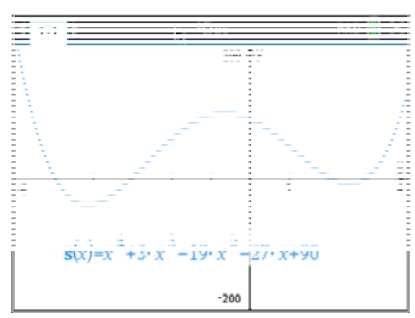
$$3x(x^4 - 16) = 0$$

$$3x(x^2 + 4)(x^2 - 4) = 0$$

$$3x(x^2 + 4)(x+2)(x-2) = 0$$

REF: 011216a2

10 ANS: 4



$$s(x) = x^4 - 9x^2 + 3x^3 - 27x - 10x^2 + 90 = x^4 + 3x^3 - 19x^2 - 27x + 90$$

REF: 062303aii

11 ANS: 4

$f(x) = (x+1)(x-1)(x-2) = (x^2-1)(x-2) = x^3 - 2x^2 - x + 2$

REF: 081921aii

12 ANS: 2

3	1	-1	-21	45	0
		3	6	-45	0
	1	2	-15	0	0

$x^3 + 2x^2 - 15x = 0$

$x(x^2 + 2x - 15) = 0$

$x(x+5)(x-3) = 0$

$x = 0, -5, 3$

REF: 012403aii

13 ANS: 4

1) -1 is also a zero. 2) $x^2(x-a) + 16(x-a) = (x^2+16)(x-a)$ a is the only zero. 3) $-a$ is the only zero. 4) $x^2(x-a) - 9(x-a) = (x^2-9)(x-a)$.

REF: 012019aii

14 ANS:

$3x^3 + 21x^2 + 36x = 0$

$3x(x^2 + 7x + 12) = 0$

$3x(x+4)(x+3) = 0$

$x = 0, -4, -3$

REF: 011930ai

15 ANS:

$x^2(2x-1) - 4(2x-1) = 0$

$(x^2-4)(2x-1) = 0$

$(x+2)(x-2)(2x-1) = 0$

$x = \pm 2, \frac{1}{2}$

REF: 081537a2

16 ANS:

$$3(x^3+4x^2-x-4)=0$$
$$(x^2(x+4)-(x+4))=0$$
$$(x^2-1)(x+4)=0$$
$$x=\pm 1,-4$$

REF: 012325aii

17 ANS:

$\pm\frac{3}{2},-\frac{1}{2}.$

$$8x^3+4x^2-18x-9=0$$
$$4x^2(2x+1)-9(2x+1)=0$$
$$(4x^2-9)(2x+1)=0$$
$$4x^2-9=0 \text{ or } 2x+1=0$$
$$(2x+3)(2x-3)=0 \qquad x=-\frac{1}{2}$$
$$x=\pm\frac{3}{2}$$

REF: fall0937a2

18 ANS:

$$x^3+5x^2-4x-20=0$$
$$x^2(x+5)-4(x+5)=0$$
$$(x^2-4)(x+5)=0$$
$$(x+2)(x-2)(x+5)=0$$
$$x=\pm 2,-5$$

REF: 061437a2

19 ANS:

$$x^4+4x^3+4x^2+16x=0$$
$$x(x^3+4x^2+4x+16)=0$$
$$x(x^2(x+4)+4(x+4))=0$$
$$x(x^2+4)(x+4)=0$$
$$x=0,\pm 2i,-4$$

REF: 061339a2

20 ANS:

$\pm 2, \pm 3$

REF: 089604al

21 ANS:
$\pm 2, \pm i$

REF: 069407al

22 ANS:
$\pm 2, \pm 2i\sqrt{2}$

REF: 089409al

23 ANS:
$\pm 3, \pm \dfrac{i\sqrt{46}}{2}$

REF: 089812al

24 ANS:

$0 = 6(-5)^3 + b(-5)^2 - 52(-5) + 15 \quad z(x) = 6x^3 + 19x^2 - 52x + 15$

$0 = -750 + 25b + 260 + 15$

$475 = 25b$

$19 = b$

-5	6	19	-52	15
		-30	55	15
	6	-11	3	0

$6x^2 - 11x + 3 = 0$

$(2x-3)(3x-1) = 0$

$x = \dfrac{3}{2}, \dfrac{1}{3}, -5$

REF: fall1515aii

25 ANS:

$V = s^3$

2.6. $2000 = (s+10)^3$. Increase the length of each side of the box by 2.6 inches (12.6 – 10).

$\sqrt[3]{2000} = s + 10$

$s \approx 12.6$

REF: 060724b