

S.ID.B.6b: Use Residuals to Assess Fit of a Function

GRAPHS AND STATISTICS

S.ID.B.6b: Use Residuals to Assess Fit of a Function

B. Summarize, represent, and interpret data on two categorical and quantitative variables

6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
 - b. Informally assess the fit of a function by plotting and analyzing residuals. Includes creating residual plots using the capabilities of the calculator (not manually).

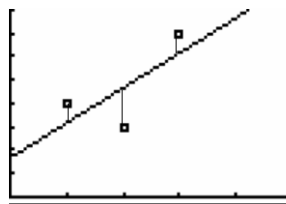
Overview of Lesson

- activate prior knowledge and review learning objectives (see above)
 - explain vocabulary and/or big ideas associated with the lesson
 - connect assessment practices with curriculum
 - model an assessment problem and solution strategy
 - facilitate guided discussion of student activity
 - facilitate guided practice of student activity
- [Selected problem set\(s\)](#)
- facilitate a summary and share out of student work
- Homework – Write the Math Assignment

Vocabulary

A **residual** is the vertical distance between where a regression equation predicts a point will appear on a graph and the actual location of the point on the graph (scatterplot). If there is no difference between where a regression equation places a point and the actual position of the point, the **residual** is zero. A **residual** can also be understood as the difference in predicted and actual y-values (dependent variable values) for a given value of x (the independent variable).

$$\text{residual} = (\text{actual } y \text{ value}) - (\text{predicted } y \text{ value})$$



A **residual plot** is a scatter plot that shows the residuals as points on a vertical axis (y-axis) above corresponding (paired) values of the independent variable on the horizontal axis (x-axis).

Any *pattern* in a residual plot suggests that the regression equation is *not appropriate* for the data.

Big Ideas

Patterns in residual plots are bad.

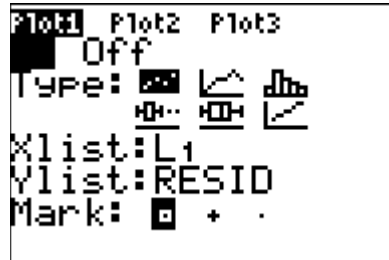
Residual plots without patterns indicate the regression equation is a good fit.

Residual plots with patterns indicate the regression equation is *not* a good fit.

Lesson Plan

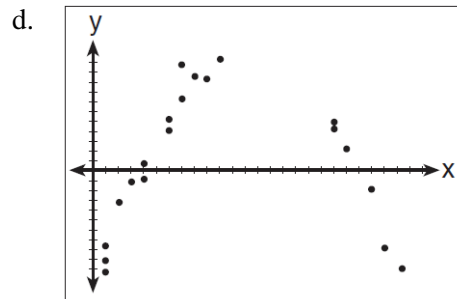
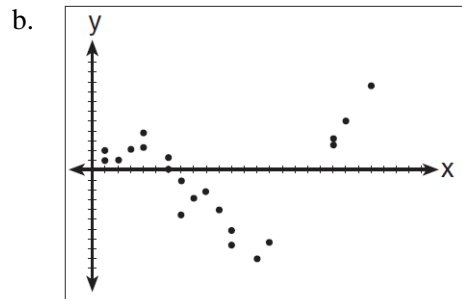
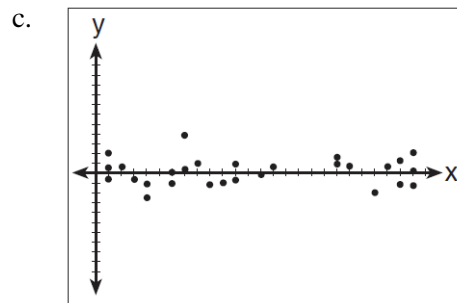
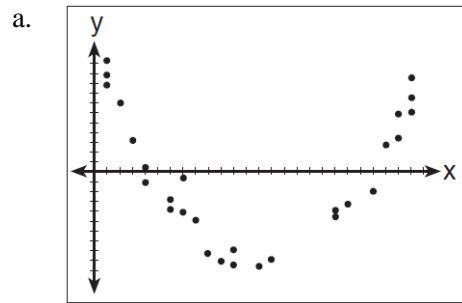
A **residual plot** without a *pattern* and with a near equal distribution of points above and below the x-axis suggests that the regression equation is a *good fit* for the data.

Residuals are automatically stored in graphing calculators when regression equations are calculated. To view a residuals scatterplot in the graphing calculator, you must use 2nd LIST to set the Y list variable to RESID, then use Zoom 9 to plot the residuals.



REGENTS PROBLEMS TYPICAL OF THIS STANDARD

1. After performing analyses on a set of data, Jackie examined the scatter plot of the residual values for each analysis. Which scatter plot indicates the best linear fit for the data?



Lesson Plan

2. The table below represents the residuals for a line of best fit.

x	2	3	3	4	6	7	8	9	9	10
Residual	2	1	-1	-2	-3	-2	-1	2	0	3

Plot these residuals on the set of axes below.



Using the plot, assess the fit of the line for these residuals and justify your answer.

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Answer Section

1. ANS: C

For a residual plot, there should be no observable pattern and about the same number of dots above and below the x axis. Any pattern in a residual plot means that line is **not** a good fit for the data.

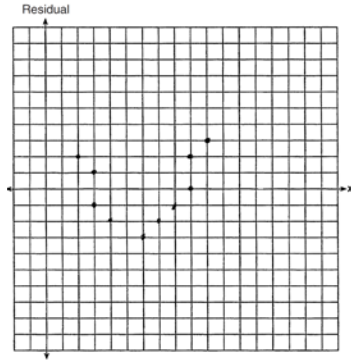
PTS: 2

REF: 011624ai

NAT: S.ID.B.6b

TOP: Correlation Coefficient and Residuals

2. ANS:



The line is a poor fit because the residuals form a pattern.

PTS: 2

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NAT: S.ID.B.6b

TOP: Correlation Coefficient and Residuals

Homework - Write the Math Assignment

START Write your name, date, topic of lesson, and class on your paper.
 NAME: Mohammed Chen
 DATE: December 18, 2015
 LESSON: Missing Number in the Average
 CLASS: Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
 PART 1b. State your understanding of **what the problem is asking**.
 PART 1c. **Answer** the problem.
 PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
 PART 2b. State your understanding of **what the new problem is asking**.
 PART 2c. **Answer** the new problem.
 PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	Up to 2 points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	Up to 2 points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of [Polya's four step universal algorithm](#) for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. The Problem

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is \$360. If the weekly salaries of four of the employees are \$340, \$340, \$345, and \$425, what is the salary of the fifth employee?

Part 1b. What is the problem asking?

Find the salary of the fifth employee.

Part 1c. Answer

The salary of the fifth employee is \$350 per week.

Part 1d. Explanation of Strategy

The arithmetic mean or average can be represented algebraically as:

$$\bar{X} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

$$\text{Check: } 360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$$

Part 2a. A New Problem

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b. What is the new problem asking?

Find Joseph's score on the missing exam.

Part 2c. Answer to New Problem

Joseph received a score of 85 on the missing examination.

Part 2d. Explanation of Strategy

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.