M – Functions, Lesson 7, Comparing Functions (r. 2018)

FUNCTIONS
Comparing Functions

CC Standard	NG Standard
F-IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* PARCC: Tasks are limited to linear functions, quadratic functions, square root, cube root, piecewise defined (including step functions and absolute value functions), and exponential functions with domains in the integers.	**AI-F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (Shared standard with Algebra II) Note: Algebra I tasks are limited to the following functions: linear, quadratic, square root, piecewise defined (including step and absolute value), and **exponential functions of the form** $f(x) = a(b)^x$ **where** $a > 0$ **and** $b > 0$ $(b \neq 1)$.

LEARNING OBJECTIVES

Students will be able to:

1) Compare properties of two functions each represented in a different way.

Overview of Lesson

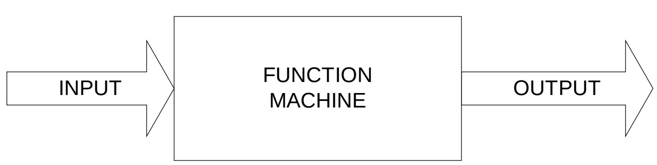
Teacher Centered Introduction	Student Centered Activities
Overview of Lesson **- activate students' prior knowledge** **- vocabulary** **- learning objective(s)** **- big ideas: direct instruction** **- modeling**	**guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work** **- developing essential skills** **- Regents exam questions** **- formative assessment assignment (exit slip, explain the math, or journal entry)**

VOCABULARY

context	graph	vertex
equation	maximum	x-intercept
four views of a function	minimum	y-intercept
function rule	table of values	

BIG IDEAS

Definition of a Function: a function takes the input value of an independent variable and pairs it with one and only one output value of a dependent variable.



Function: A function is a relation that assigns exactly one value of the dependent variable to each value of the independent variable. A function is always a relation.

Example: y=2x

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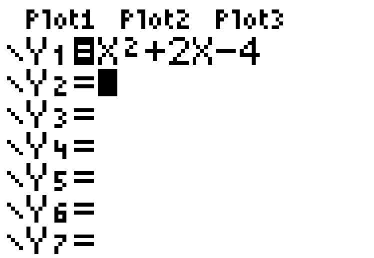
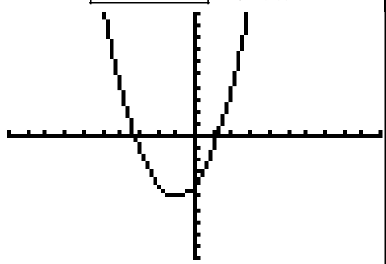
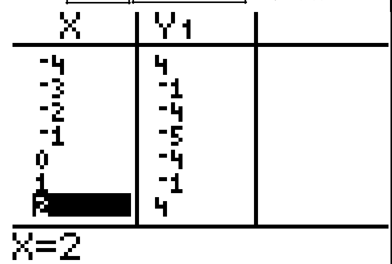
A function can be represented mathematically through four inter-related views. These are:
- #1 a function rule (equation)
- #2 a table of values
- #3 a graph.
- #4 context (words)

The TI-83+ graphing calculator allows you to input the function rule and access the graph and table of values, as shown below:

$\boxed{\text{Y=}}$ Function Rule View	$\boxed{\text{GRAPH}}$ View	$\boxed{2nd}\boxed{\text{TABLE}}$ View
		

Function Rules show the relationship between dependent and independent variables in the form of an equation with two variables.
- § The **independent** variable is the **input** of the function and is typically denoted by the x-variable.
- § The **dependent** variable is the **output** of the function and is typically denoted by the y-variable.

When inputting function rules in a TI 83+ graphing calculator, the y-value (dependent variable) must be isolated as the left expression of the equation.

Tables of Values show the relationship between dependent and independent variables in the form of a table with columns and rows:
- § The **independent** variable is the **input** of the function and is typically shown in the left column of a vertical table or the top row of a horizontal table.
- § The **dependent** variable is the **output** of the function and is typically shown in the right column of a vertical table or the bottom row of a horizontal table.

Graphs show the relationship between dependent and independent variables in the form of line or curve on a coordinate plane:
- § The value of **independent** variable is the **input** of the function and is typically shown on the **x-axis** (horizontal axis) of the coordinate plane.
- § The value of the **dependent** variable is the **output** of the function and is typically shown on the **y-axis** (vertical axis) of the coordinate plane.

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DEVELOPING ESSENTIAL SKILLS

1 The *x*-value of which function's *x*-intercept is larger, *f* or *h*? Justify your answer.

$$f(x) = x - 5$$

x	**h(x)**
−1	6
0	4
1	2
2	0
3	−2

2 Consider the function $p(x) = x^2 - 2x - 4$ and the function *q* represented in the table below.

x	***q*(*x*)**
−2	−8
−1	0
0	0
1	−2
2	0

Determine which function has the *smaller* minimum value for the domain $[-2, 2]$. Justify your answer.

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3 Which function shown below has a greater average rate of change on the interval $[-2,4]$? Justify your answer.

$$g(x) = 4x^3 - 5x^2 + 3$$

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

1 ANS: *f(x)* The graph of *f(x)* crosses the x-axis when x = 5. The graph of *h(x)* crosses the x-axis when x = 2.

2 ANS: *q* has the smaller minimum value for the domain $[-2,2]$. *p*'s minimum is -5 *q*'s minimum is -8.

3 ANS: $g(x)$ has a greater rate of change

$$\frac{f(4)-f(-2)}{4--2} = \frac{80-1.25}{6} = 13.125$$

$$\frac{g(4)-g(-2)}{4--2} = \frac{179--49}{6} = 38$$

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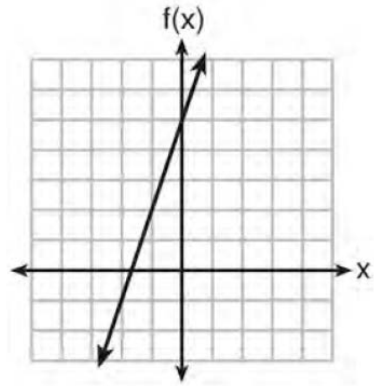
REGENTS EXAM QUESTIONS (through June 2018)

F.IF.C.9: Comparing Functions

464) Which function has the greatest *y*-intercept?

1) $f(x) = 3x$
2) $2x + 3y = 12$
3) the line that has a slope of 2 and passes through (1, -4).
4) 

465) Given the following quadratic functions:

$$g(x) = -x^2 - x + 6$$

and

x	−3	−2	−1	0	1	2	3	4	5
n(x)	−7	0	5	8	9	8	5	0	−7

Which statement about these functions is true?

1) Over the interval $-1 \le x \le 1$, the average rate of change for $n(x)$ is less than that for $g(x)$.
2) The *y*-intercept of $g(x)$ is greater than the *y*-intercept for $n(x)$.
3) The function $g(x)$ has a greater maximum value than $n(x)$.
4) The sum of the roots of $n(x) = 0$ is greater than the sum of the roots of $g(x) = 0$.

466) Let *f* be the function represented by the graph below.

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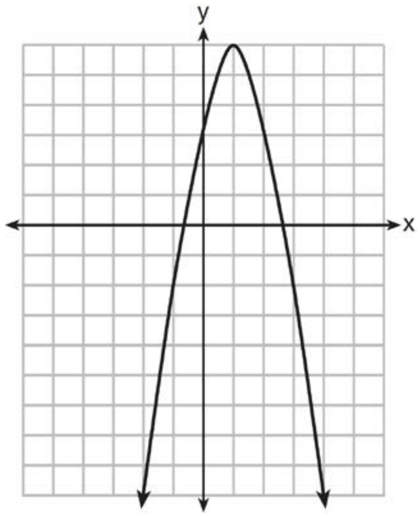


Let g be a function such that $g(x) = -\frac{1}{2}x^2 + 4x + 3$. Determine which function has the larger maximum value. Justify your answer.

467) Which quadratic function has the largest maximum?

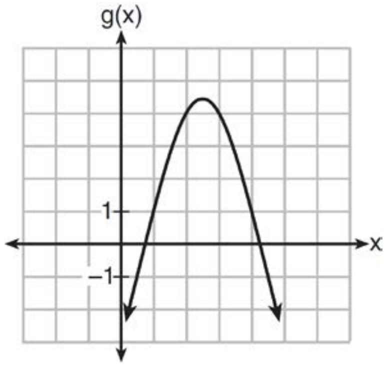
1) $h(x) = (3 - x)(2 + x)$

2)

x	f(x)
−1	−3
0	5
1	9
2	9
3	5
4	−3

3) $k(x) = -5x^2 - 12x + 4$

4)



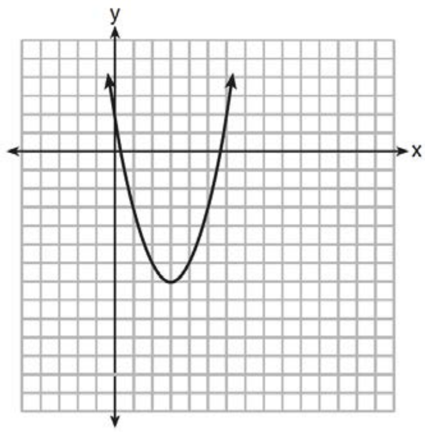
468) Which statement is true about the quadratic functions $g(x)$, shown in the table below, and $f(x) = (x - 3)^2 + 2$?

x	g(x)
0	4
1	−1
2	−4
3	−5
4	−4
5	−1
6	4

1) They have the same vertex.
2) They have the same zeros.
3) They have the same axis of symmetry.
4) They intersect at two points.

469) The graph representing a function is shown below.

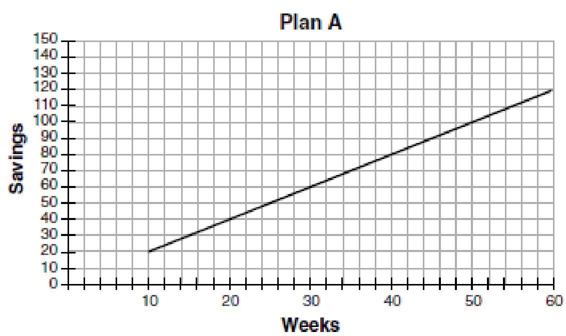
Name: ___________________________________



Which function has a minimum that is *less* than the one shown in the graph?

1) $y = x^2 - 6x + 7$
2) $y = |x + 3| - 6$
3) $y = x^2 - 2x - 10$
4) $y = |x - 8| + 2$

470) Nancy works for a company that offers two types of savings plans. Plan *A* is represented on the graph below.

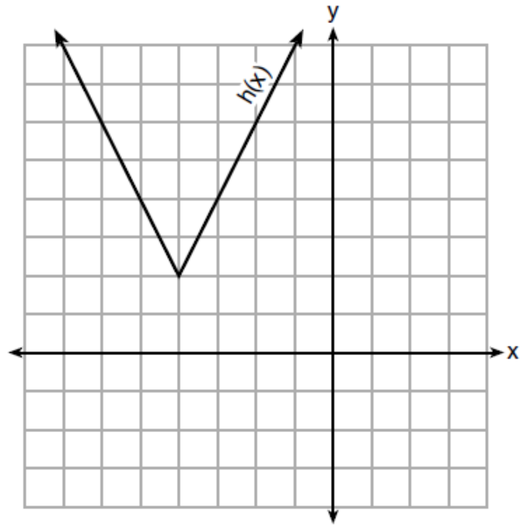


Plan *B* is represented by the function $f(x) = 0.01 + 0.05x^2$, where x is the number of weeks. Nancy wants to have the highest savings possible after a year. Nancy picks Plan *B*. Her decision is

1) correct, because Plan *B* is an exponential function and will increase at a faster rate
2) correct, because Plan *B* is a quadratic function and will increase at a faster rate
3) incorrect, because Plan *A* will have a higher value after 1 year
4) incorrect, because Plan *B* is a quadratic function and will increase at a slower rate

471) The function $h(x)$, which is graphed below, and the function $g(x) = 2|x + 4| - 3$ are given.

Name: ________________________________



Which statements about these functions are true?

I. $g(x)$ has a lower minimum value than $h(x)$.

II. For all values of x, $h(x) < g(x)$.

III. For any value of x, $g(x) \neq h(x)$.

1) I and II, only
2) I and III, only
3) II and III, only
4) I, II, and III

472) Which quadratic function has the largest maximum over the set of real numbers?

1) $f(x) = -x^2 + 2x + 4$

2)

x	k(x)
–1	–1
0	3
1	5
2	5
3	3
4	–1

3) $g(x) = -(x-5)^2 + 5$

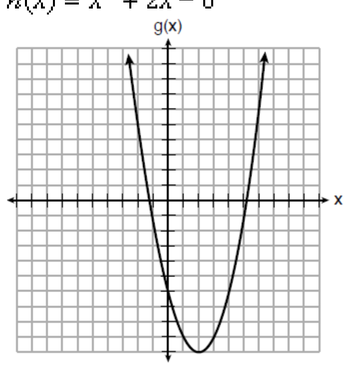
4)

x	h(x)
–2	–9
–1	–3
0	1
1	3
2	3
3	1

473) Which of the quadratic functions below has the *smallest* minimum value?

1) $h(x) = x^2 + 2x - 6$

2)



(2)

3) $k(x) = (x+5)(x+2)$

4)

x	f(x)
–1	–2
0	–5
1	–6
2	–5
3	–2

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SOLUTIONS

464) ANS: 4
Strategy: Find y-intercept for each answer choice, then eliminate wrong answers.
Eliminate $f(x) = 3x$ because $f(0) = 3(0) = 0$.
Eliminate $2x + 3y = 12$ because $2(0) + 3y = 12$

$$3y = 12$$

$$y = 4$$

Eliminate the line that has slope of 2 and passes through (1, -4) because it has a positive slope and it's y-intercept must be less than -4.
Choose the graph because the y-intecept is 5, which is greater than the y-intercepts of the other three choices.

PTS: 2 NAT: F.IF.C.9

465) ANS: 4
Strategy: Each answer choice must be evaluated using a different strategy.
a. Use the slope formula to find the rate of change for

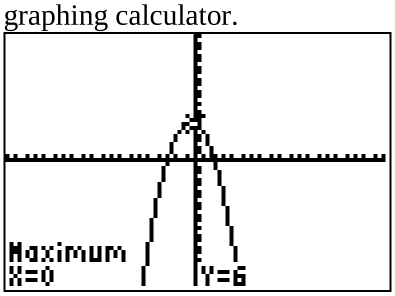
$$m_{g(x)} = \frac{[g(1)] - [g(-1)]}{[1] - [-1]} = \frac{4-6}{2} = \frac{-2}{2} = -1$$

$$m_{n(x)} = \frac{[n(1)] - [n(-1)]}{[1] - [-1]} = \frac{9-5}{2} = \frac{4}{2} = 2$$

Statement a is false. The average rate of change for $n(x)$ is *more* than that for $g(x)$.

b. Compare the y-intercepts for both functions. The y-intercepts occur when $x = 0$.
The y-intercept for g(x) = 6. $g(0) = -0^2 - 0 + 6 = 6$
The y-intercept for n(x) = 8 from the table.
Statement b is false. The *y*-intercept of $g(x)$ is *less* than the *y*-intercept for $n(x)$.

c. Compare the maxima of both functions.
The maxima of $g(x) = -x^2 - x + 6$ is 6. This can be found manually or with a graphing calculator.



The maxima of n(x) = 9, which can be seen in the table.
Statement c is false. The function $g(x)$ has a *smaller* maximum value than $n(x)$.

d. Compare the sum of the roots for both functions.
The sum of the roots for g(x) = $-3 + 2 = -1$ from a graphing calculator.

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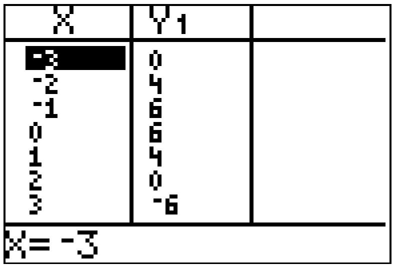


The sum of the roots for n(x) = $-2+4=2$ from the table.

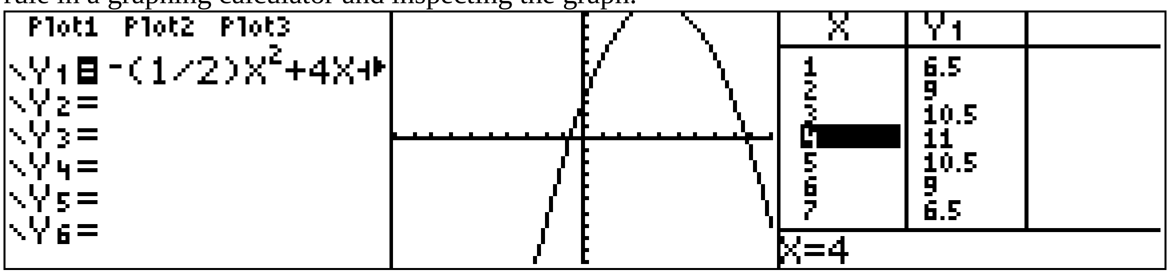
Statement d is true. The sum of the roots of $n(x)=0$ is greater than the sum of the roots of $g(x)=0$.

PTS: 2 NAT: F.IF.C.9 TOP: Graphing Quadratic Functions

466) ANS:

Function g has the larger maximum value. The maximum of function *g* is 11. The maximum of function *f* is 6.

Strategy: Determine the maximum for *f* from the graph. Determine the maximum for g by inputting the function rule in a graphing calculator and inspecting the graph.



The table of values shows the maximum for *g* is 11.

Another way of finding the maximum for g is to use the axis of symmetry formula and the function rule, as follows: $x=\frac{-b}{2a}=\frac{-4}{2\left(-\frac{1}{2}\right)}=\frac{-4}{-1}=4$

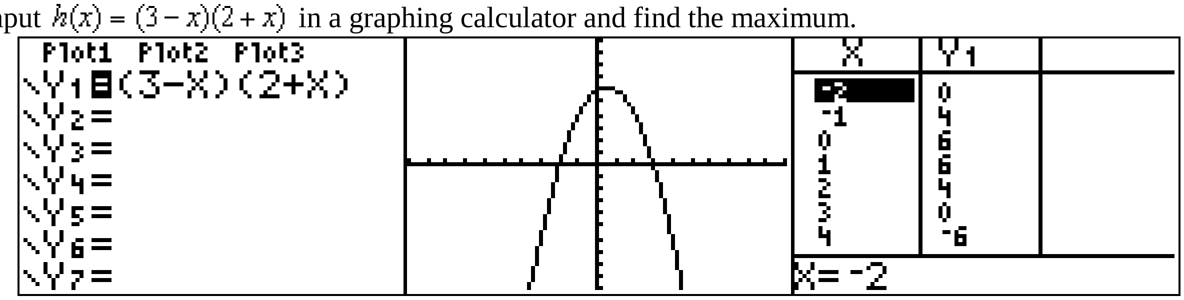
$$y=-\frac{1}{2}(4)^2+4(4)+3=-8+16+3=11$$

PTS: 2 NAT: F.IF.C.9 TOP: Graphing Quadratic Functions

467) ANS: 3

Strategy: Each answer choice needs to be evaluated for the largest maximum using a different strategy..

a) Input $h(x)=(3-x)(2+x)$ in a graphing calculator and find the maximum.



The maximum for answer choice *a* is a little more than 6.

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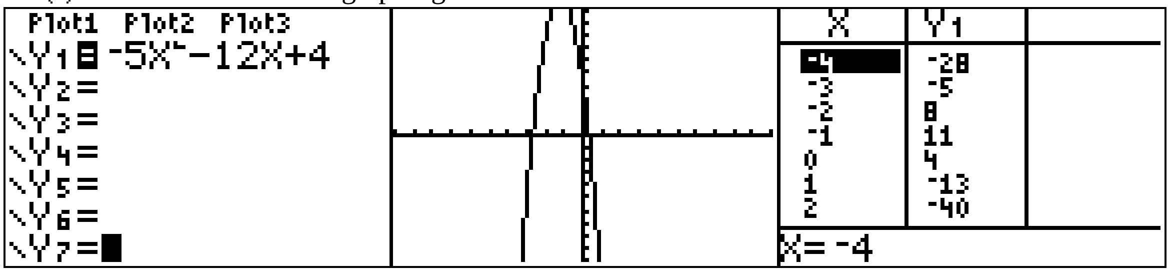
b) The table shows that the maximum is a little more than 9.

c) Input $k(x) = -5x^2 - 12x + 4$ in a graphing calculator and find the maximum.



The table of values shows that the maximum is 11 or more.

d) The graph shows that the maximum is a little more than 4.

Answer choice *c* is the best choice.

PTS: 2 NAT: F.IF.C.9 TOP: Graphing Quadratic Functions

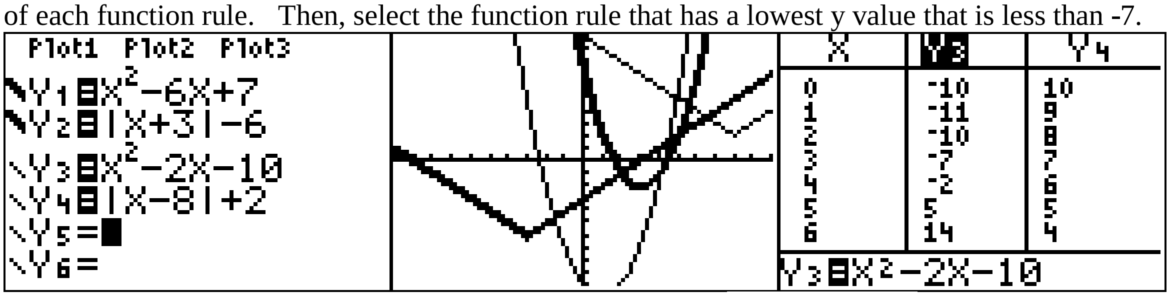
468) ANS: 3

The first function $f(x) = (x-3)^2 + 2$ is in vertex form $y = a(x-h)^2 + k$ and has its vertex at (3,2).
The second function is in table form and has its vertex at (3, -5).
Therefore, the axis of symmetry for both functions is x=3..

PTS: 2 NAT: F.IF.C.9 TOP: Comparing Functions
KEY: AI

469) ANS: 3

Strategy: The graph shows a parabola with a vertex at (3, -7), so the minima is at -7. Identify the lowest y-value of each function rule. Then, select the function rule that has a lowest y value that is less than -7.



The graph view of the four functions shows that the function $y = x^2 - 2x - 10$ has a y-value less than -7.

PTS: 2 NAT: F.IF.C.9 TOP: Comparing Functions

470) ANS: 2

Observe: The function $f(x) = 0.01 + 0.05x^2$ is a second degree equation, so it must be a quadratic function. One year equals 52 weeks.
Strategy:
Step 1.: Solve the Plan *B* function for $x = 52$

$$f(x) = 0.01 + 0.05x^2$$

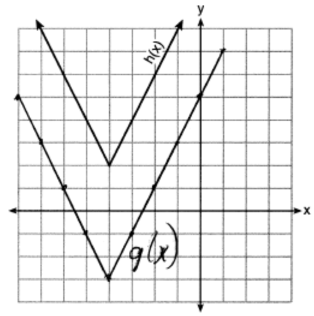
$$f(52) = 0.01 + 0.05(52)^2$$

$$f(52) = 135.21$$

Step 2. Compare the Plan A (52, 105) and Plan B (52, 135.21) coordinates for 52 weeks and observe that B has higher savings.
Step 3. Eliminate wrong answers.
~~a)~~ correct, because Plan *B* is an ~~exponential~~ function and will increase at a faster rate b) correct, because Plan *B* is a quadratic function and will increase at a faster rate
~~c)~~ ~~incorrect~~, because Plan *A* will have a higher value after 1 year
~~d)~~ ~~incorrect~~, because Plan *B* is a quadratic function and will increase at a slower rate

PTS: 2 NAT: F.IF.C.9 TOP: Comparing Functions

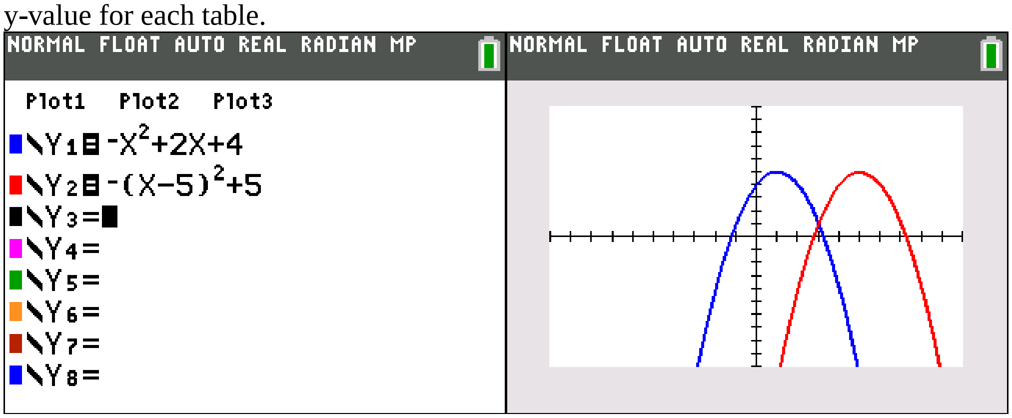
471) ANS: 2
Strategy: Graph $g(x) = 2|x+4| - 3$, then examine the truth value of the answer choices.



I. $g(x)$ has a lower minimum value than $h(x)$. True: $-3 < 2$
II. For all values of *x*, $h(x) < g(x)$. False. $h(x)$ is always > than $g(x)$.
III. For any value of *x*, $g(x) \neq h(x)$. True. $h(x)$ is always 5 more than $g(x)$.

PTS: 2 NAT: F.IF.C.9 TOP: Comparing Functions

472) ANS: 2
Strategy: Find the maximum y-value for each function rule using a graphing calculator. Estimate the maximum y-value for each table.



Both function rules have maximum values of 5.
The maximum value of $k(x)$ is estimated as greater that 5.
The maximum value of $h(x)$ is estimated as less than 5.

PTS: 2 NAT: F.IF.C.9 TOP: Comparing Functions

473) ANS: 2
Strategy: Determine the minimum y-value for each function, then choose the smallest y-value.
STEP 1. Evaluate each answer choice.
Answer Choice 1. The minimum can be found by transforming the function from standard form to vertex form.

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$$h(x) = x^2 + 2x - 6$$

$$x^2 + 2x - 6 = 0$$

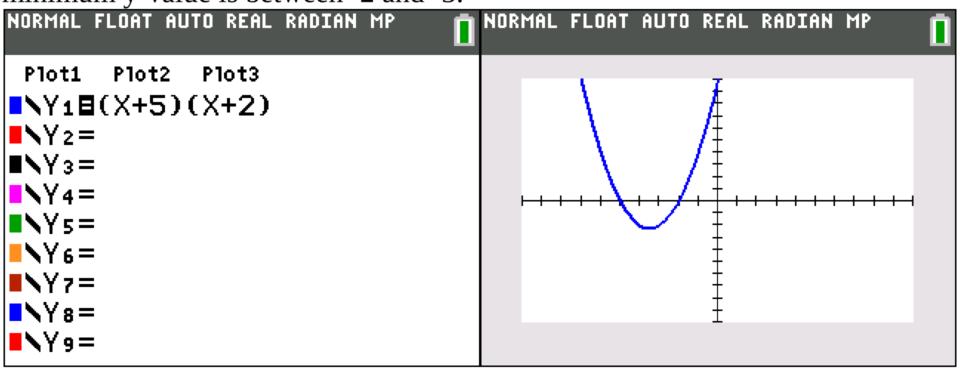
$$x^2 + 2x = 6$$

$$x^2 + 2x + (1)^2 = 6 + (1)^2$$

$$(x + 1)^2 = 7$$

$$(x + 1)^2 - 7 = 0$$

The vertex occurs at (-1, -7), so the minimum y-value is -7.
Answer Choice 2. The minimum can be found by inspection of the graph. The minimum y-value is -10.
Answer Choice 3. The minimum can be found using the graph or table views of the function in a graphing calculator. The minimum y-value is between -2 and -3.



Answer Choice 4. The miinimum can be found by inspection of the table of values. The minim y-value is -6.
STEP 2. Pick the lowest y-value of all the answer choices.

PTS: 2 NAT: F.IF.C.9 TOP: Comparing Functions