Lesson Plan

F.IF.C.7: Graph Root, Piecewise, Step, & Absolute Value Functions

FUNCTIONS

F.IF.C.7: Root, Piecewise, Step, & Absolute Value Functions

C. **Analyze functions using different representations.**

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

Overview of Lesson
- activate prior knowledge and review learning objectives (see above)
- explain vocabulary and/or big ideas associated with the lesson
- connect assessment practices with curriculum
- model an assessment problem and solution strategy
- facilitate guided discussion of student activity
- facilitate guided practice of student activity
    Selected problem set(s)
- facilitate a summary and share out of student work
    Homework – Write the Math Assignment

NOTE: All of the functions in this lesson require special consideration for the domain of the independent variable (the x-axis).

ROOT FUNCTIONS

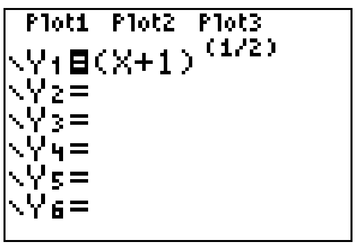
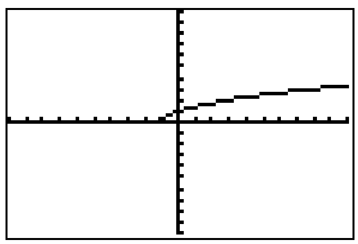
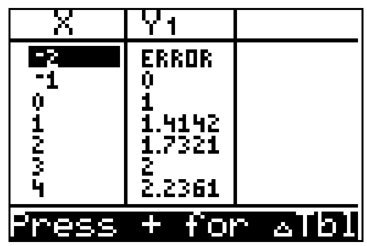
Root functions are associated with equations involving square roots, cube roots, or nth roots. The easiest way to graph a root function is to use the three views of a function that are associated with a graphing calculator.

STEP 1. Input the root function in the y-editor of the calculator. (Note: The use of rational exponents is recommended, i.e. $\sqrt{x} = x^{(1/2)}$, $\sqrt[3]{x} = x^{(1/3)}$, etc.).

STEP 2. Look at the graph of the function.

STEP 3. Use the table of values to transfer coordinate pairs to graph paper.

Example: Graph the root function $f(x) = \sqrt{x+1}$

STEP 1 Input the function rule in the y-editor of your graphing calculator	STEP 2. Look at the graph view of the function.	STEP 3. Select coordinate pairs from the table view to create your own graph.
		

PIECEWISE FUNCTIONS

A **piecewise function** is a function that is defined by two or more *sub* functions, with each sub function applying to a certain interval on the x-axis. Each *sub* function may also be referred to as a *piece* of the overall **piecewise function**, hence the name piecewise.

Example. The following is a piecewise function:

$$f(x) = \begin{cases} 2x+1, & -3 \le x < 3 \\ 4, & 3 \le x \le 7 \end{cases}$$

This example of a piecewise function has two "pieces," or sub functions.

a. Over the interval $-3 \le x < 1$, the sub function is $f(x) = 2x + 1$

b. Over the interval $1 \le x \le 7$, the sub function is $f(x) = 4$.

A table of values for this function might look like this, reflecting two pieces.

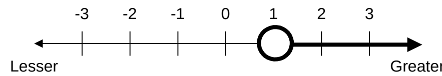
x	$f(x) = 2x + 1$	$f(x) = 4$		x	$f(x) = 2x + 1$	$f(x) = 4$
-3	-5	na		3	na	4
-2	-3	na		4	na	4
-1	-1	na		5	na	4
0	1	na		6	na	4
1	3	4		7	na	4
2	na	4		8	na	na

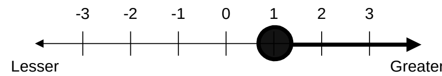
Continuity

Piecewise functions are often discontinuous, which means that the graph will not appear as a single line. In the above table, the piecewise function is discontinuous when $x = 3$. This is because $x = 3$ is not included in the first piece of the piecewise function. Because piecewise functions are often discontinuous, care must be taken to use proper inequalities notation when graphing.

<u>Using Line Segments to Define Pieces</u>

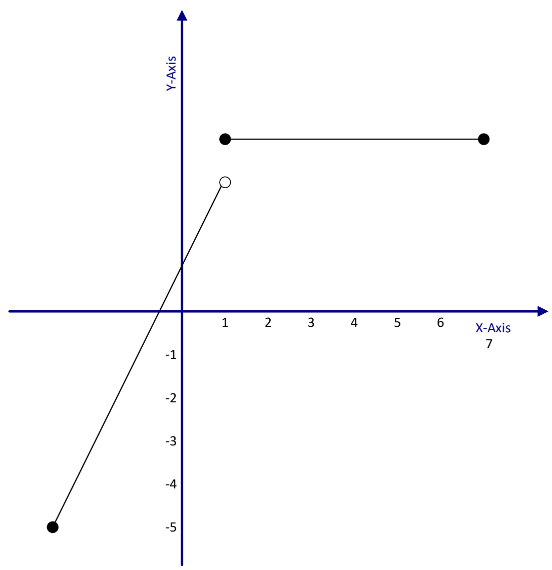
If the circle at the beginning or end of a solution set (graph) is empty, that value *is not included* in the solution set. If the circle is filled in, that value *is included* in the solution set.

The number 1 is not included in the this solution set: 

The number 1 is included in this following solution set: 

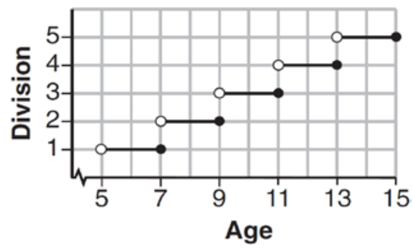
The graph of this piecewise function looks like this:

The graph of $f(x) = \begin{cases} 2x + 1, & -3 \le x < 3 \\ 4, & 3 \le x \le 7 \end{cases}$ appears below:



STEP FUNCTIONS

A step function is typically a piecewise function with many pieces that resemble stair steps.



Each step correspondends to a specific domain. The function rule for the graph above is:

$$f(x) = \begin{cases} 1, & 5 < x \leq 7 \\ 2, & 7 < x \leq 9 \\ 3, & 9 < x \leq 11 \\ 4, & 11 < x \leq 13 \\ 5, & 13 < x \leq 15 \end{cases}$$

ABSOLUTE VALUE FUNCTIONS

Using a Graphing Calculator To Solve and Graph Absolute Value Functions:
Absolute value functions may be solved in a graphing calculator by moving all terms to one side of the inequality and reducing the other side to zero. The inequality is then entered into the graphing calculator's [Y =] feature. Once input, the calculator's [2nd][TABLE] and [GRAPH] features may be accessed and manipulated using the ([2nd][TBL SET]) and graph [WINDOW] features.

Example: *Given*: $|x+1|-3>6$

First, move everything to one side of the inequality, leaving the other side zero.

$|x+1|-3>6$

$|x+1|-9>0$

$or\ \ 0<|x+1|-9$

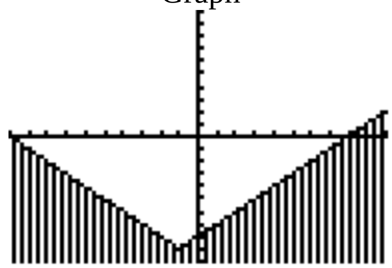
Y= Input

Pay particular attention to setting the inequality sign on the far left of the input screen.

```
Plot1 Plot2 Plot3
▶Y1▮abs(X+1)-9
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

NOTE: The abs entry is found in the graphing calculator's catalog.

Graph



You can see from the graph that the solution boundaries are $-10\ and\ +8$. Test $x=0$ to confirm the answers $x<10\ \ and\ \ x>8$, which are the parts of the graph that lie **above** the x-axis. Typically, you would graph only the x-axis on the Regents Math B Exam.

NOTE: The table of values in the graphing calculator provides an excellent opportunity to reinforce the idea that absolute values cannot have negative values. Any value of x that results in a negative value of y cannot be part of the solution set of an absolute value inequality.

Table of Values

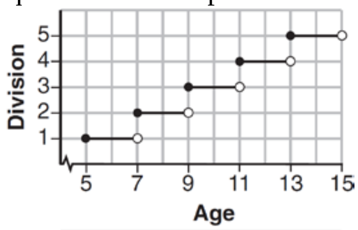
X	Y1	
3	-5	
4	-4	
5	-3	
6	-2	
7	-1	
8	0	
9	1	

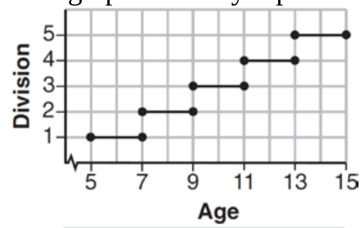
X=4

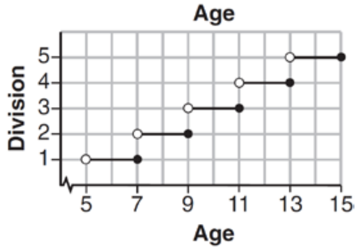
NOTE: The table of values in the graphing calculator provides an excellent opportunity to reinforce the idea that absolute values cannot have negative values.

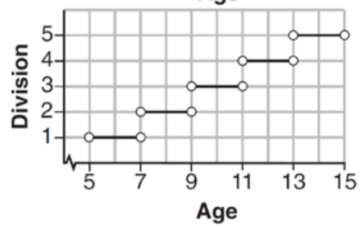
REGENTS PROBLEMS TYPICAL OF THIS STANDARD

1. Morgan can start wrestling at age 5 in Division 1. He remains in that division until his next odd birthday when he is required to move up to the next division level. Which graph correctly represents this information?

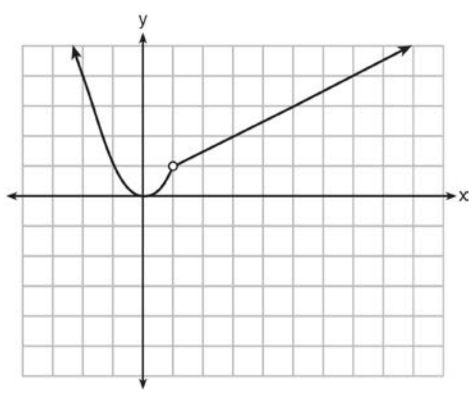
a.


c.


b.


d.


2. A function is graphed on the set of axes below.



Which function is related to the graph?

a. $f(x) = \begin{cases} x^2, & x < 1 \\ x - 2, & x > 1 \end{cases}$

c. $f(x) = \begin{cases} x^2, & x < 1 \\ 2x - 7, & x > 1 \end{cases}$

b. $f(x) = \begin{cases} x^2, & x < 1 \\ \frac{1}{2}x + \frac{1}{2}, & x > 1 \end{cases}$

d. $f(x) = \begin{cases} x^2, & x < 1 \\ \frac{3}{2}x - \frac{9}{2}, & x > 1 \end{cases}$

3. Graph the following function on the set of axes below.

$$f(x) = \begin{cases} |x|, & -3 \leq x < 1 \\ 4, & 1 \leq x \leq 8 \end{cases}$$

4. Draw the graph of $y = \sqrt{x} - 1$ on the set of axes below.

F.IF.C.7: Graph Root, Piecewise, Step, & Absolute Value Functions
Answer Section

1. ANS: A
Strategy: Focus on whether the line segments should begin and end with closed or open circles. A closed circle is included. An open circle is not included.

PTS: 2 REF: 061507ai NAT: F.IF.C.7 TOP: Graphing Step Functions
KEY: bimodalgraph

2. ANS: B
Strategy: Since $f(x) = x^2,\ x < 1$ is included in every answer choice, concentrate on the linear functions for $x > 1$.

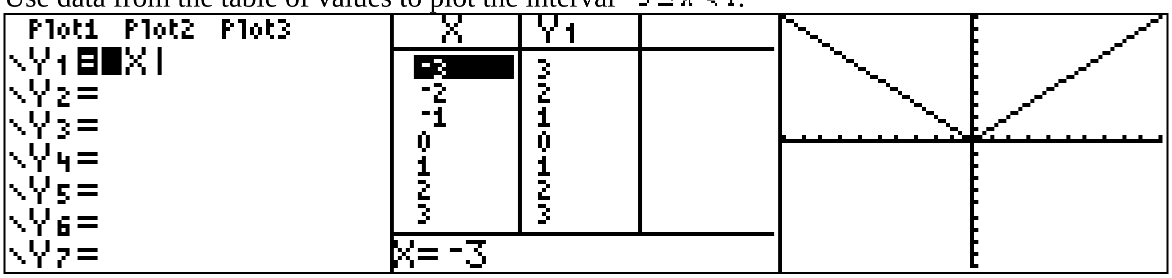
The linear equation has a slope of $\frac{rise}{run} = \frac{1}{2}$. The only linear function that has a slope of $\frac{1}{2}$ is $f(x) = \frac{1}{2}x + \frac{1}{2}$, which is answer choice b.

PTS: 2 REF: 081422ai NAT: F.IF.C.7 TOP: Graphing Piecewise-Defined Functions

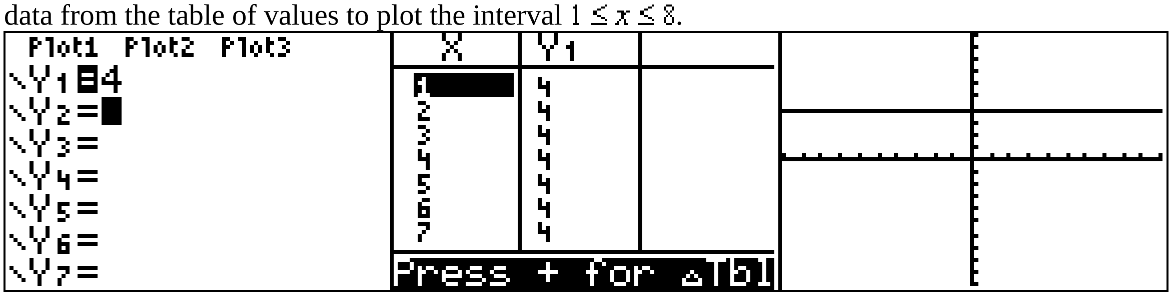
3. ANS:

Strategy: Use a graphing calculator and graph the function in sections, paying careful attention to open and closed circles at the end of each function segment.

STEP 1. Graph $f(x) = |x|$ over the interval $-3 \le x < 1$. Use a closed dot for $(-3, 3)$ and an open dot for $(1, 1)$. Use data from the table of values to plot the interval $-3 \le x < 1$.



STEP 2: Graph $f(x) = 4$ over the interval $1 \le x \le 8$. Use a closed dot for $(1, 4)$ and a closed dot for $(8, 4)$. Use data from the table of values to plot the interval $1 \le x \le 8$.



Do not connect the two graph segments.

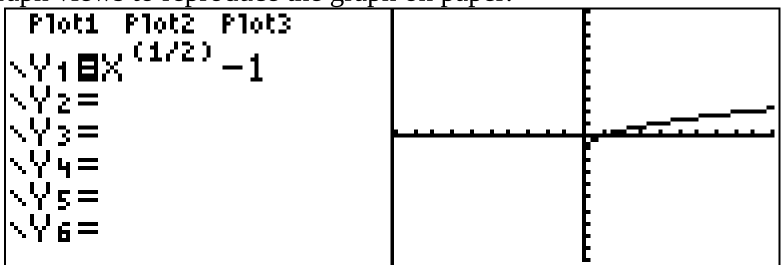
PTS: 2 REF: 011530ai NAT: F.IF.C.7 TOP: Graphing Piecewise-Defined Functions

4. ANS:

Lesson Plan

Strategy: Input the function in a graphing calculator, then use the graph and table views to construct the graph on paper.

STEP 1: Use exponential notation to input the function into the graphing calculator, where $\sqrt{x} - 1 = x^{(1/2)} - 1$. Then use the table and graph views to reproduce the graph on paper.



X	Y1
-6	ERROR
-5	ERROR
-4	ERROR
-3	ERROR
-2	ERROR
-1	ERROR
0	-1

X=0

X	Y1
1	0
2	.41421
3	.73205
4	1
5	1.2361
6	1.4495
7	1.6458

X=7

X	Y1
8	1.8284
9	2
10	2.1623
11	2.3166
12	2.4641
13	2.6056
14	2.7417

X=14

Note: Do nopt plot coordinates with errors. Focus on plotting coordinates with integer values and estimate the graph between the points with integer values when drawing the graph.

STEP 2: Limit the domain of the function to $-6 \le x \le 10$. Used closed dots to show the ends of the function at coordinates (-6, -2) and for (10, 2).

PTS: 2 REF: 061425ai NAT: F.IF.C.7 TOP: Graphing Root Functions

Homework - Write the Math Assignment

START	Write your name, date, topic of lesson, and class on your paper.	
	NAME:	Mohammed Chen
	DATE:	December 18, 2015
	LESSON:	Missing Number in the Average
	CLASS:	Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
PART 1b. State your understanding of **what the problem is asking**.
PART 1c. **Answer** the problem.
PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
PART 2b. State your understanding of **what the new problem is asking**.
PART 2c. **Answer** the new problem.
PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	**Up to 2** points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	**Up to 2** points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of Polya's four step universal algorithm for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. **The Problem**

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is \$360. If the weekly salaries of four of the employees are \$340, \$340, \$345, and \$425, what is the salary of the fifth employee?

Part 1b. **What is the problem asking?**

Find the salary of the fifth employee.

Part 1c. **Answer**

The salary of the fifth employee is \$350 per week.

Part 1d. **Explanation of Strategy**

The arithmetic mean or average can be represented algebraically as:

$$\overline{X} = \frac{x_1 + x_2 + ... + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

Check: $360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$

Part 2a. **A New Problem**

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b **What is the new problem asking?**

Find Joseph's score on the missing exam.

Part 2c. **Answer to New Problem**

Joseph received a score of 85 on the missing examination.

Part 2d. **Explanation of Strategy**

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.