F.IF.B.4: Identify and Interpret Key Features of Graphs

EQUATIONS AND INEQUALITIES

F.IF.B.4: Identify and Interpret Key Features of Graphs

B. Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity (linear, exponential and quadratic).

Overview of Lesson

- activate prior knowledge and review learning objectives (see above)
- explain vocabulary and/or big ideas associated with the lesson
- connect assessment practices with curriculum
- model an assessment problem and solution strategy
- facilitate guided discussion of student activity
- facilitate guided practice of student activity

  Selected problem set(s)

- facilitate a summary and share out of student work

  Homework – Write the Math Assignment

Vocabulary

***x*-intercept** The point at which the graph of a relation intercepts the *x*-axis. The ordered pair for this point has a value of $y = 0$.

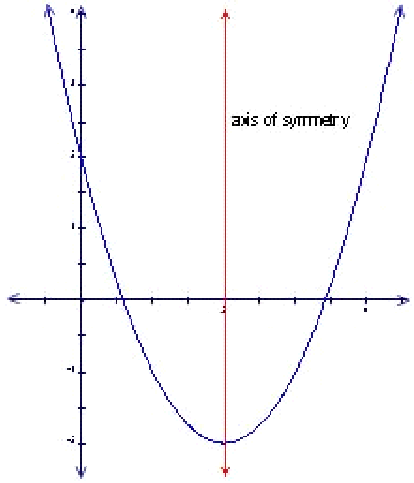
  Example: The equation y = 8 + 2x has an *x*-intercept of -4.

***y*-intercept** The point at which a graph of a relation intercepts the *y*-axis. The ordered pair for this point has a value of $x = 0$.

  Example: The equation y = 8 + 2x has a y-intercept of 8.

axis of symmetry ***(G)*** A line that divides a plane figure into two congruent reflected halves; Any line through a figure such that a point on one side of the line is the same distance to the axis as its corresponding point on the other side.

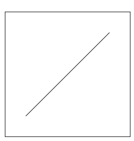
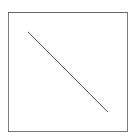
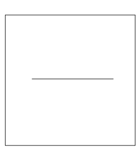
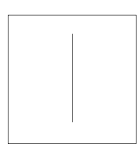
Example:



This is a graph of the parabola
$y = x^2 - 4x + 2$ together with its axis of symmetry $x = 2$.

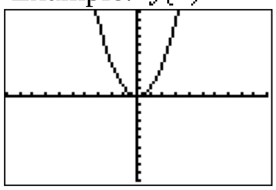
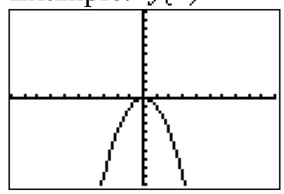
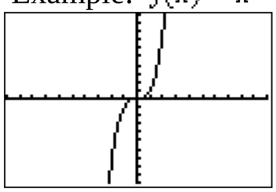
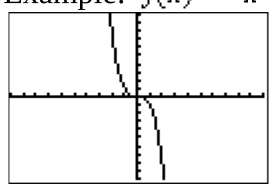
period (of a function) *(A2T) The horizontal distance after which the graph of a function starts repeating itself.* The smallest value of k in a function f for which there exists some constant k such that $f(t) = f(t + k)$ for every number t in the domain of f.

Slope

 **Positive Slope** Goes up from left to right.	 **Negative Slope**. Goes down from left to right.
 **Zero Slope**. A horizontal line has a slope of zero.	 **Undefined Slope**. A vertical line has an undefined slope.

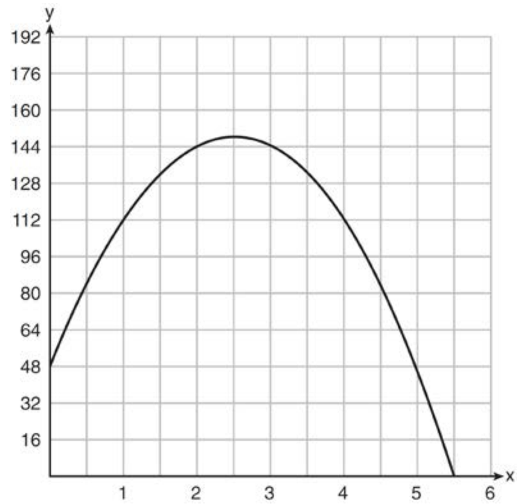
End Behaviors

The **end behaviors** of a graph refers to the directions (behaviors) of the graph of $f(x)$ as x approaches infinity in either direction. To determine the end behavior of the graph of any polynomial function, you need to know the degree of the polynomial and whether the leading coefficient is positive or negative. The table below shows the four possible sets of end behaviors of a polynomial function.

	Leading Coefficient is **Positive**	Leading Coefficient is **Negative**
Degree of function is **Even**	Example: $f(x) = x^2$  **End Behaviors** Left tail increases Right tail increases	Example: $f(x) = -x^2$  **End Behaviors** Left tail decreases Right tail decreases
Degree of function is **Odd**	Example: $f(x) = x^3$  **End Behaviors** Left tail decreases Right tail increases	Example: $f(x) = -x^3$  **End Behaviors** Left tail increases Right tail decreases

REGENTS PROBLEMS TYPICAL OF THIS STANDARD

1. A ball is thrown into the air from the edge of a 48-foot-high cliff so that it eventually lands on the ground. The graph below shows the height, y, of the ball from the ground after x seconds.



For which interval is the ball's height always *decreasing*?

a. $0 \le x \le 2.5$

b. $0 < x < 5.5$

c. $2.5 < x < 5.5$

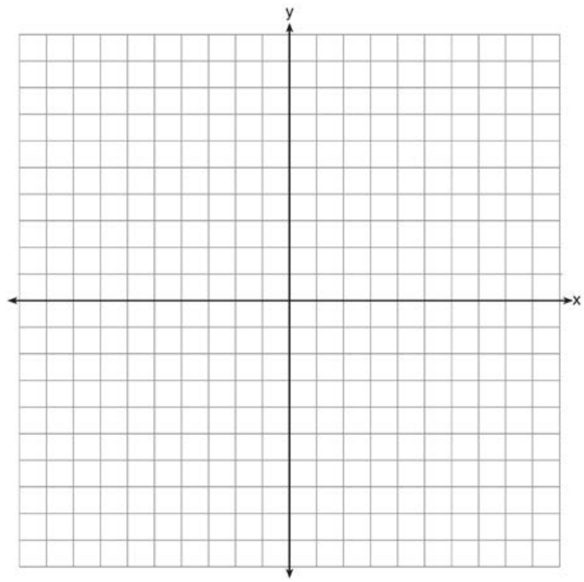
d. $x \ge 2$

2. A football player attempts to kick a football over a goal post. The path of the football can be modeled by the function $h(x) = -\frac{1}{225}x^2 + \frac{2}{3}x$, where *x* is the horizontal distance from the kick, and $h(x)$ is the height of the football above the ground, when both are measured in feet. On the set of axes below, graph the function $y = h(x)$ over the interval $0 \le x \le 150$.

Determine the vertex of $y = h(x)$. Interpret the meaning of this vertex in the context of the problem. The goal post is 10 feet high and 45 yards away from the kick. Will the ball be high enough to pass over the goal post? Justify your answer.

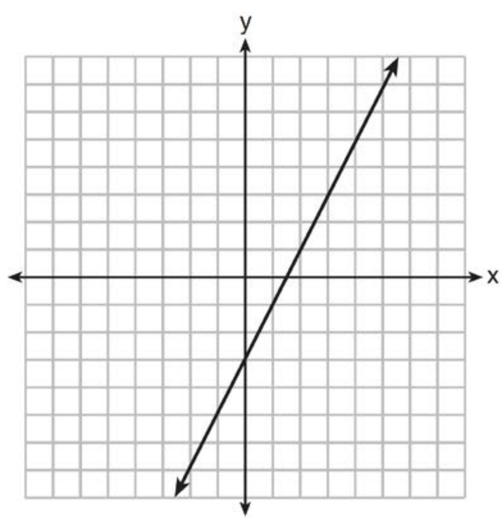
3. A toy rocket is launched from the ground straight upward. The height of the rocket above the ground, in feet, is given by the equation $h(t) = -16t^2 + 64t$, where *t* is the time in seconds. Determine the domain for this function in the given context. Explain your reasoning.

4. On the set of axes below, draw the graph of $y = x^2 - 4x - 1$.



State the equation of the axis of symmetry.

5. Which function has the same *y*-intercept as the graph below?



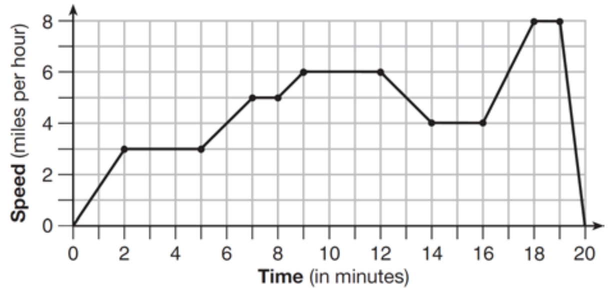
a. $y = \frac{12 - 6x}{4}$

b. $27 + 3y = 6x$

c. $6y + x = 18$

d. $y + 3 = 6x$

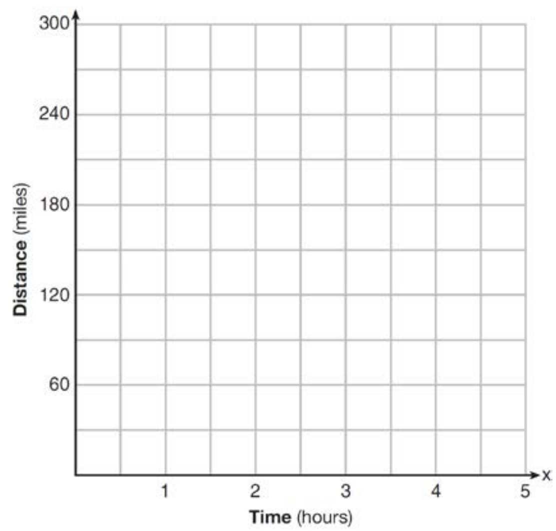
6. The graph below represents a jogger's speed during her 20-minute jog around her neighborhood.



Which statement best describes what the jogger was doing during the $9 - 12$ minute interval of her jog?

a. She was standing still.
b. She was increasing her speed.
c. She was decreasing her speed
d. She was jogging at a constant rate.

7. A driver leaves home for a business trip and drives at a constant speed of 60 miles per hour for 2 hours. Her car gets a flat tire, and she spends 30 minutes changing the tire. She resumes driving and drives at 30 miles per hour for the remaining one hour until she reaches her destination. On the set of axes below, draw a graph that models the driver's distance from home.



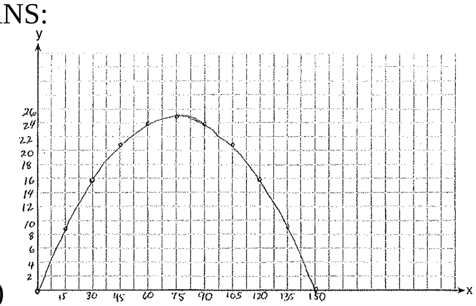
F.IF.B.4: Identify and Interpret Key Features of Graphs
Answer Section

1. ANS: C
Strategy: Identify the domain of x that corresponds to a negative slope (decreasing height) in the function, then eliminate wrong answers.

STEP 1. The axis of symmetry for the parabola is $x = 2.5$ and the graph has a negative slope after $x = 2.5$ all the way to $x = 5.5$, meaning that the height of the ball is decreasing over this interval.

STEP 2. Eliminate wrong answers.
Answer choice a can be eliminated because the the slope of the graph increases over the interval $0 \le x \le 2.5$.
Answer choice b can be eliminated because the the slope of the graph both increases and decreases over the interval $0 \le x \le 2.5$.
Answer choice c is the correct choice, because it shows the domain of x where the graph has a negative slope.
Answer choice d can be eliminated because the the slope of the graph increases from $x \ge 2$ until $x = 2.5$.

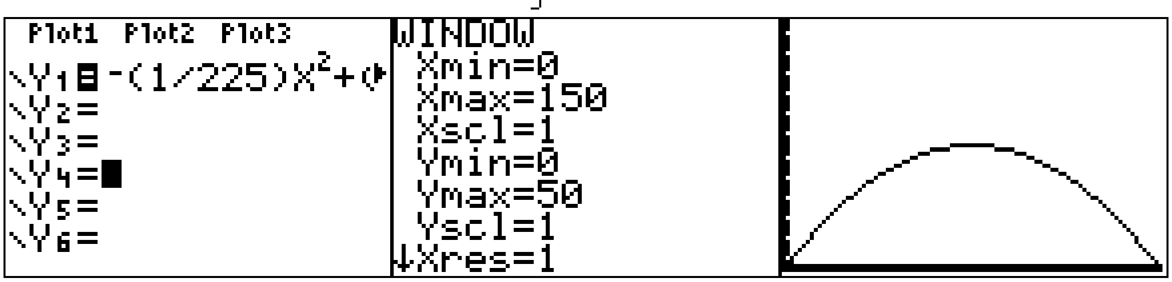
PTS: 2 REF: 061409ai NAT: F.IF.B.4 TOP: Graphing Quadratic Functions

2. ANS:
a) 

b) The vertex is at $(75, 25)$. This means that the ball will reach it highest (25 feet) when the horizontal distance is 75 feet.
c) No, the ball will not clear the goal post because it will be less than 10 feet high.

Strategy: Input the equation into a graphing calculator and use the table and graph views to complete the graph on paper, then find the vertex and determine if the ball will pass over the goal post.

STEP 1. Input $h(x) = -\frac{1}{225}x^2 + \frac{2}{3}x$ into a graphing calculator. Set the window to reflect the interval $0 \le x \le 150$ and estimate the height to be approximately $\frac{1}{3}$ the domain of x.



X	Y1
0	0
1	.66222
2	1.3156
3	1.96
4	2.5956
5	3.2222
6	3.84

Press + for ΔTbl

X	Y1
13	7.9156
14	8.4622
15	9
16	9.5289
17	10.049
18	10.56
19	11.062

X=19

Observe that the table of values has integer solutions at 15 unit intervals, so change the ΔTbl to 15.

TABLE SETUP
TblStart=0
ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask

TABLE SETUP
TblStart=0
ΔTbl=15
Indpnt: Auto Ask
Depend: Auto Ask

The change in ΔTbl results in a table of values that is easier to graph on paper.

X	Y1
0	0
15	9
30	16
45	21
60	24
75	25
90	24

Press + for ΔTbl

X	Y1
0	0
15	9
30	16
45	21
60	24
75	25
90	24

Press + for ΔTbl

Use the graph view and the table of values to complete the graph on paper.

STEP 2. Use the table of values to find the vertex. The vertex is located at $(75, 25)$.

X	Y1
30	16
45	21
60	24
75	25
90	24
105	21
120	16

X=75

STEP 3. Convert 45 yards to 135 feet and determine if the the ball will be 10 feet or higher when $x = 135$.

X	Y1
90	24
105	21
120	16
135	9
150	0
165	-11
180	-24

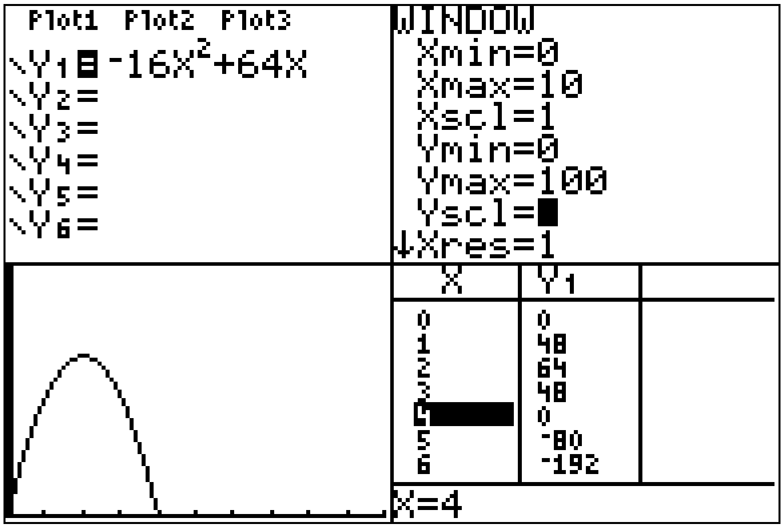
X=135

or $y = -\frac{1}{225}(135)^2 + \frac{2}{3}(135) = -81 + 90 = 9$

The ball will be 9 feet above the ground and will not go over the 10 feet high goal post.

PTS: 6 REF: 061537ai NAT: F.IF.B.4 TOP: Graphing Quadratic Functions

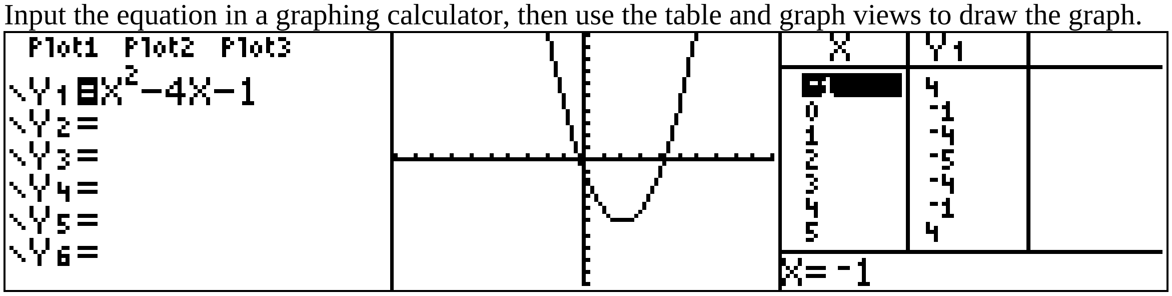
3. ANS:
The rocket launches at $t = 0$ and lands at $t = 4$, so the domain of the function is $0 \le x \le 4$.
Strategy: Input the function into a graphing calculator and determine the flight of the rocket using the graph and table views of the function.



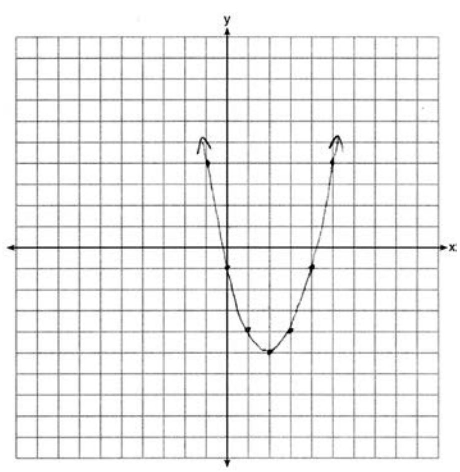
The toy rocket is in the air between 0 and 4 seconds, so the domain of the function is $0 \le x \le 4$.

PTS: 2 REF: 081531ai NAT: F.IF.B.4 TOP: Graphing Quadratic Functions

4. ANS:
Input the equation in a graphing calculator, then use the table and graph views to draw the graph.



The axis of symmetry is $x = 2$



The equation for the axis of symmetry can also be found using the formula $x = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2$

Lesson Plan

PTS: 2 REF: 061627ai NAT: F.IF.B.4 TOP: Graphing Quadratic Functions
NOT: NYSED classifies this as A.REI.D

5. ANS: D

Strategy: Identify the y-intercept in the graph, then test each answer choice to see if it has the same y-intercept.

STEP 1. Identify the y-intercept in the graph.
The y-intercept is can be defined as the y-value of the coordinate where the graph intercepts (passes through) the y-axis. The graph shows that the function passes through the y-axis at the point $(0,-3)$, so the value of the y-intercept is -3.

STEP 2. Test the other equations to see if the point $(0,-3)$works.

a	$y=\frac{12-6x}{4}$ Does not work $-3=\frac{12-6(0)}{4}$ $-3=\frac{12}{4}$ $-3\neq 3$	c	$6y+x=18$ Does not work $6(-3)+(0)=18$ $-18\neq 18$
b	$27+3y=6x$ Does not work $27+3(-3)=6(0)$ $27-9=0$ $18\neq 0$	d	$y+3=6x$ $(0,-3)$works! $(-3)+3=6(0)$ $0=0$

PTS: 2 REF: 011509ai NAT: F.IF.B.4 TOP: Graphing Linear Functions

6. ANS: D

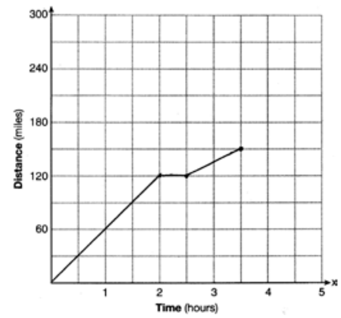
Strategy: Pay close attention to the labels on the x-axis and the y-axis, then eliminate wrong answers. NOTE: A horizontal line (no slope) means that speed is not changing.

Answer a can be eliminated because she would have a speed of 0 if she were standing still. She was only standing still at the start and end of her jog.
Answer b can be eliminated because the speed does not change during the $9-12$ minute interval of her jog.
Answer c can be eliminated because the speed does not change during the $9-12$ minute interval of her jog.
Answer d is the correct choice because a horizontal line (no slope) means that speed is not changing.

PTS: 2 REF: 061502ai NAT: F.IF.B.4 TOP: Relating Graphs to Events

7. ANS:

Lesson Plan



Strategy - Use the speed of the car as the rate of change to complete the graph.

STEP 1. Plot 2 hours at 60 miles per hour slope, based on the language “... a constant speed of 60 miles per hour for 2 hours.”

STEP 2. Plot $\frac{1}{2}$ hour at 0 slope based on the language “...she spends 30 minutes changing the tire.”

STEP 3. Plot 1 hour at 30 miles per hour slope based on the language “...drives at 30 miles per hour for the remaining one hour...”

PTS: 2 REF: 081528ai NAT: F.IF.B.4 TOP: Relating Graphs to Events

Homework - Write the Math Assignment

START Write your name, date, topic of lesson, and class on your paper.

NAME: Mohammed Chen
DATE: December 18, 2015
LESSON: Missing Number in the Average
CLASS: Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
PART 1b. State your understanding of **what the problem is asking**.
PART 1c. **Answer** the problem.
PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
PART 2b. State your understanding of **what the new problem is asking**.
PART 2c. **Answer** the new problem.
PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	**Up to 2** points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	**Up to 2** points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of Polya's four step universal algorithm for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. <u>The Problem</u>

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is $360. If the weekly salaries of four of the employees are $340, $340, $345, and $425, what is the salary of the fifth employee?

Part 1b. <u>What is the problem asking?</u>

Find the salary of the fifth employee.

Part 1c. <u>Answer</u>

The salary of the fifth employee is $350 per week.

Part 1d. <u>Explanation of Strategy</u>

The arithmetic mean or average can be represented algebraically as:

$$\overline{X} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

Check: $360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$

Part 2a. <u>A New Problem</u>

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b <u>What is the new problem asking?</u>

Find Joseph's score on the missing exam.

Part 2c. <u>Answer to New Problem</u>

Joseph received a score of 85 on the missing examination.

Part 2d. <u>Explanation of Strategy</u>

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.