

**F.IF.8b: Use Properties of Exponents to Interpret Expressions**

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8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as  $y = (1.02)^t$ ,  $y = (0.97)^t$ ,  $y = (1.01)^{12t}$ ,  $y = (1.2)^{\frac{t}{10}}$  and classify them as representing exponential growth or decay.

**Overview of Lesson**

- activate prior knowledge
  - present vocabulary and/or big ideas associated with the lesson
  - connect assessment practices with curriculum
  - model an assessment problem and solution strategy
  - facilitate guided practice of student activity
  - facilitate a summary and share out of student work
- Optional: Provide additional problem sets**  
**Optional HW - Write the math assignment.**

**Vocabulary**

An **exponential function** is a function that contains a variable for an exponent.

Example:  $y = 2^x$

Exponential functions are useful for modeling real world events such as:

- |                     |   |
|---------------------|---|
| ▪ population growth | ▪ compound interest on investments                |
| ▪ bacteria growth   | ▪ concentrations of medicine in the body          |
| ▪ radioactive decay | ▪ concentrations of pollutants in the environment |

**Exponential growth** is modeled by the formula:  $A = P(1 + r)^t$

**Exponential decay** is modeled by the formula:  $A = P(1 - r)^t$

In both formulas:

- $A$  represents the amount after growth or decay.
- $P$  represents the initial (starting) amount.
- $r$  represents the rate of growth or decay for each time cycle.
- $t$  represents the number of time cycles.

**Sample Problem:**

On January 1, 1999, the price of gasoline was \$1.39 per gallon. If the price of gasoline increased by 0.5% per month, what was the cost of one gallon of gasoline, to the *nearest cent*, on January 1 one year later?

**Solution:**

This is exponential growth. The price is increasing.

$A$  represents the amount after twelve months.

$P$  represents the initial (starting) amount, which is \$1.39.

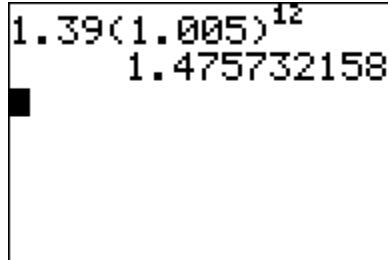
## Lesson Plan

$r$  represents the rate of growth or decay for each month, which is 0.5% per month.  
 $t$  represents the number of time cycles, which is 12.

Write the exponential growth equation:

$$A = \$1.39(1 + 0.5\%)^{12}$$

$$A = 1.39(1.005)^{12}$$



1.39(1.005)<sup>12</sup>  
1.475732158

The cost of gasoline one year later would be \$1.48.

## REGENTS PROBLEMS

- The value in dollars,  $v(x)$ , of a certain car after  $x$  years is represented by the equation  $v(x) = 25,000(0.86)^x$ . To the *nearest dollar*, how much more is the car worth after 2 years than after 3 years?
  - 2589
  - 6510
  - 15,901
  - 18,490
- Some banks charge a fee on savings accounts that are left inactive for an extended period of time. The equation  $y = 5000(0.98)^x$  represents the value,  $y$ , of one account that was left inactive for a period of  $x$  years. What is the  $y$ -intercept of this equation and what does it represent?
  - 0.98, the percent of money in the account initially
  - 0.98, the percent of money in the account after  $x$  years
  - 5000, the amount of money in the account initially
  - 5000, the amount of money in the account after  $x$  years

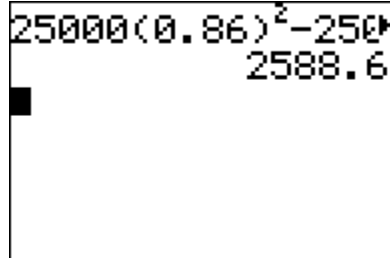
## Lesson Plan

### F.IF.8b: Use Properties of Exponents to Interpret Expressions Answer Section

1. ANS: A

Strategy #1

Input  $25,000(0.86)^2 - 25,000(0.86)^3$  into a graphing calculator and press enter.



$$25,000(0.86)^2 - 25,000(0.86)^3 = 18490 - 15901.40 = 2588.60$$

Strategy #2: Input the function rule in a graphing calculator and obtain the value of the car after 2 years and 3 years from the table of values. Then, compute the difference.

STEP 1: Input the function rule and obtain data from the table of values.

| Plot1 Plot2 Plot3             | X | Y1    |
|-------------------------------|---|-------|
| Y1 = 25000(0.86) <sup>x</sup> | 0 | 25000 |
| Y2 =                          | 1 | 21500 |
| Y3 =                          | 2 | 18490 |
| Y4 =                          | 3 | 15901 |
| Y5 =                          | 4 | 13675 |
| Y6 =                          | 5 | 11761 |
|                               | 6 | 10114 |

Press + for Δ|b|

STEP 2: Compare the value of the car after 2 years and after 3 years.

The car is worth \$18,490 after 2 years.

The car is worth \$15,901 after 3 years.

The difference is  $18490 - 15901 = 2589$

$$25,000(0.86)^2 - 25,000(0.86)^3 = 18490 - 15901.40 = 2588.60$$

PTS: 2

REF: 011508a1

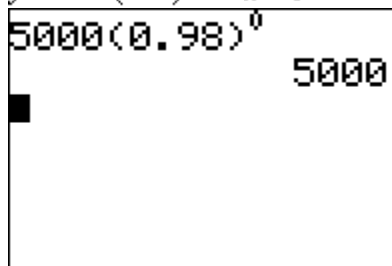
NAT: F.IF.8b

TOP: Evaluating Exponential Expressions

2. ANS: C

Strategy 1: The y-intercept of a function occurs when the value of  $x$  is 0. The strategy is to evaluate the function

$$y = 5000(0.98)^x \text{ for } x = 0$$



This represents the amount of money in the account before exponential decay begins.

Strategy 2. Input the equation in a graphing calculator and view the table of values.

## Lesson Plan

| Plot1 Plot2 Plot3  | X                               | Y1   |  |
|--|---------------------------------|--|--|
| $Y_1 = 5000(0.98)^x$<br>$Y_2 =$<br>$Y_3 =$<br>$Y_4 =$<br>$Y_5 =$ | 0<br>1<br>2<br>3<br>4<br>5<br>6 | 5000<br>4900<br>4802<br>4706<br>4611.8<br>4519.6<br>4429.2 |  |
| Press + for $\Delta$ tbl   |                                 |  |  |

The table of values clearly shows the initial value of the account and its exponential decay.

PTS: 2

REF: 011515a1

NAT: F.IF.8b

TOP: Modeling Exponential Equations

## Lesson Plan

### **Standard “Writing the Math” Assignment**

- START Write your name, date, topic of lesson, and class on your paper.
- PART 1. Copy **the problem** from the lesson and underline/highlight key words.
- PART 2. State your understanding of **what the problem is asking**.
- PART 3. **Answer** the problem.
- PART 4. **Explanation of strategy**.
- PART 5. Create **a new problem** that addresses the same mathematical understandings.
- PART 6. State the **answer to your new problem**.

### **Clearly label each of the six parts.**

### **Grading Rubric**

Each homework writing assignment is graded using a four-point rubric, as follows:

|  |  |
|--|--|
| Part 1. Statement of the problem.  | $\frac{1}{2}$ point is awarded for correctly restating the original problem.   |
| Part 2. Statement of what the problem is really asking.                                    | $\frac{1}{2}$ point is awarded for correctly identifying what the problem is asking you to find or do.                 |
| Part 3. Solution to the problem.   | $\frac{1}{2}$ point is awarded for a correct solution to the problem.  |
| Part 4. Written analysis of the mathematics and solution strategy involved in the problem. | Up to 1 point is awarded for an explanation of the mathematics and solution strategy involved in the problem.          |
| Part 5. An alternative version of the problem.   | Up to 1 point is awarded for creating a new problem that involves similar mathematics and a similar solution strategy. |
| Part 6. Solution to the alternative version of the problem.                                | $\frac{1}{2}$ point is awarded for correctly solving the new problem that you have created.                            |

This assignment/activity is designed to incorporate elements of [Polya’s four step universal algorithm](#) for problem solving with the idea that writing is thinking.

### **Rationale for Assignment**

Each New York Regents Algebra I (Common Core) examination contains 13 open response problems. An analysis of the first three Algebra I examinations revealed that approximately 51% (20 out of 39) of these open response problems instructed students to: 1) *describe*; 2) *state*; 2) *explain*; 3) *justify* or otherwise write about their answers. It is theorized that students can benefit from explicit instruction and writing routines that are applicable to solving these problems.

## Lesson Plan

### EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Student's Name \_\_\_\_\_

Topic: \_\_\_\_\_

Date: \_\_\_\_\_

Class: \_\_\_\_\_

**Part 1. The Problem**

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is \$360. If the weekly salaries of four of the employees are \$340, \$340, \$345, and \$425, what is the salary of the fifth employee?

**Part 2. What is the problem asking?**

Find the salary of the fifth employee.

**Part 3. Answer**

The salary of the fifth employee is \$350 per week.

**Part 4. Explanation of Strategy**

The arithmetic mean or average can be represented algebraically as:

$$\bar{X} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so  $n = 5$ . The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

Check:  $360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$

**Part 5. A New Problem**

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

**Part 6. Answer to New Problem**

Joseph received a score of 85 on the missing examination.

#### EVALUATION

|   |                  |   |                  |
|---|------------------|---|------------------|
| <i>Name and Proper Heading</i>              |                  | <i>Part 4. Explanation of strategy.</i> | <b>1 Point</b>   |
| <i>Part 1. Problem copied correctly?</i>    | <b>1/2 Point</b> | <i>Part 5. New problem.</i>             | <b>1 Point</b>   |
| <i>Part 2. Understand the problem?</i>      | <b>1/2 Point</b> | <i>Part 6. Solution to new problem.</i> | <b>1/2 Point</b> |
| <i>Part 3. Correct and complete answer?</i> | <b>1/2 Point</b> | <b>TOTAL</b>                            | <b>4 Points</b>  |