F.BF.B.3: Build New Functions from Existing Functions.

FUNCTIONS

F.BF.B.3: Transformations of Graphs of Functions

B. Build new functions from existing functions.

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Overview of Lesson
- activate prior knowledge and review learning objectives (see above)
- explain vocabulary and/or big ideas associated with the lesson
- connect assessment practices with curriculum
- model an assessment problem and solution strategy
- facilitate guided discussion of student activity
- facilitate guided practice of student activity
Selected problem set(s)
- facilitate a summary and share out of student work
Homework – Write the Math Assignment

BIG IDEA

The graph of a function is changed when either $f(x)$ or x is multiplied by a scalar, or when a constant is added to or subtracted from either $f(x)$ or x. A graphing calculator can be used to explore the translations of graph views of functions.

Rules: $f(x) \Leftrightarrow f(x) \pm k$ moves the graph $\updownarrow$ up or down.

$+k$ moves every point on the graph up k units.

$-k$ moves every point on the graph down k units

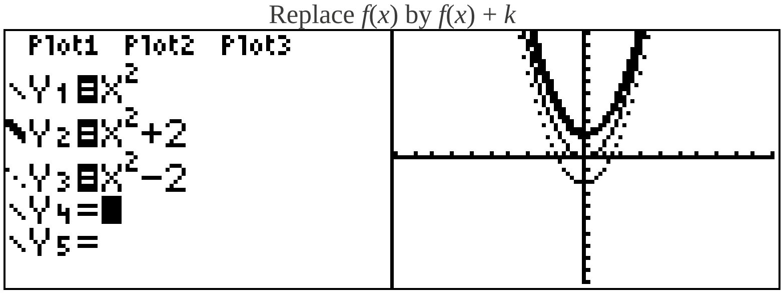
$f(x) \Leftrightarrow f(x \pm k)$ moves the graph $\leftrightarrow$ left or right.

$+k$ moves every point on the graph left k units.

$-k$ moves every point on the graph right k units

Examples:

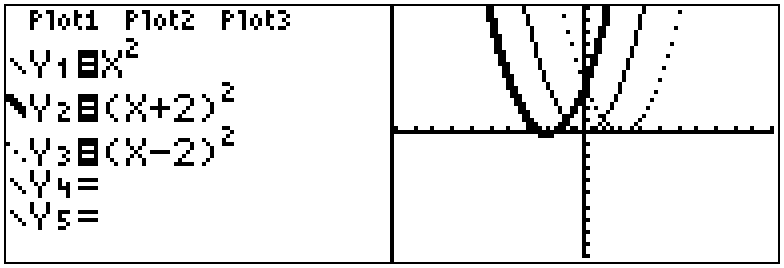
Replace $f(x)$ by $f(x) + k$



The addition or subtraction of a constant **outside the parentheses** moves the graph up or down by the value of the constant.

Replace $f(x)$ by $f(x + k)$

Lesson Plan



The addtition or subtraction of a constant **inside the parentheses** moves the graph left or right by the value of the constant.

Rule: $f(x) \Leftrightarrow f(kx)$ changes the direction and width of a parabola

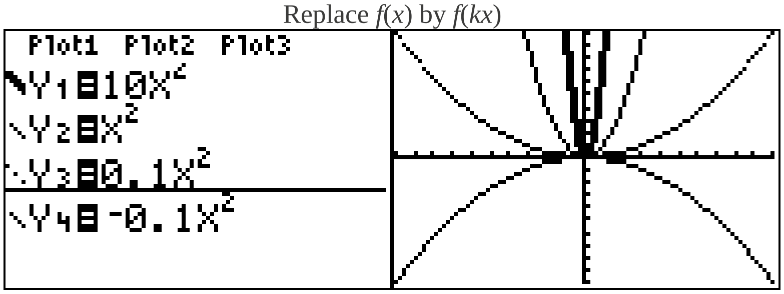
$-k$ inverts the parabola

If k is a fraction less than $|1|$, the parabola will become wider.

If k is a number larger than $|1|$, the parabola will become narrower.

Examples:

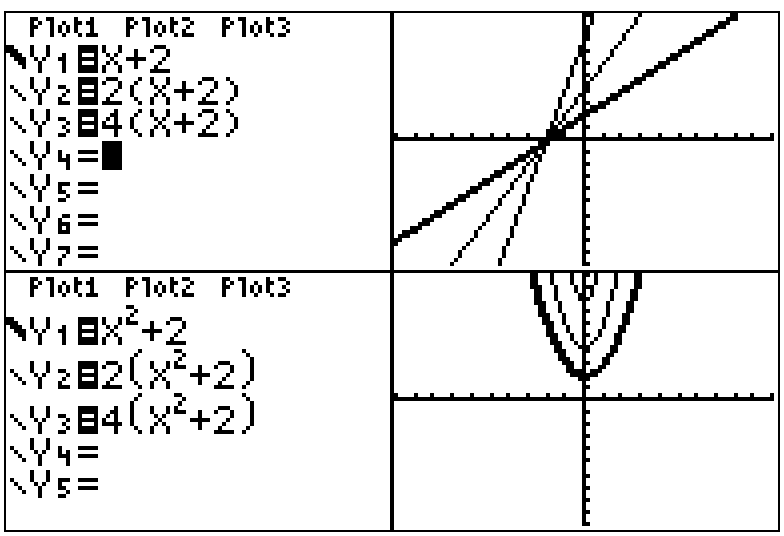
Replace $f(x)$ by $f(kx)$



Changing the value of *a* in a quadratic affects the width and direction of a parabola. The bigger the absolute value of *a*, the narrower the parabola.

$f(x) \Leftrightarrow kf(x)$ changes the y intercept of the graph.

Replace $f(x)$ by $k\,f(x)$

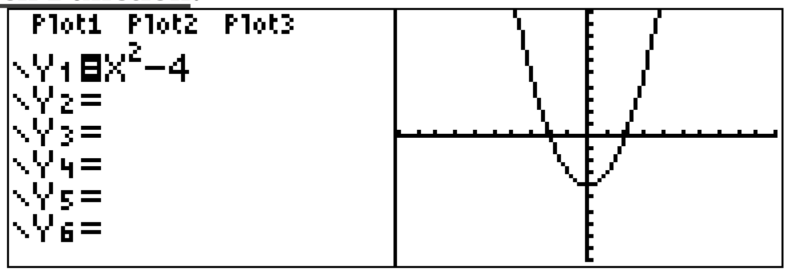


Even and Odd Functions

Even functions: must

1. have exponents that are all even numbers (divisible by 2)
2. reflect in the y-axis.

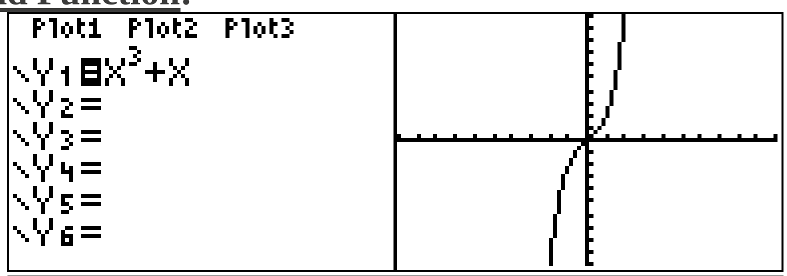
Example of an Even Function:



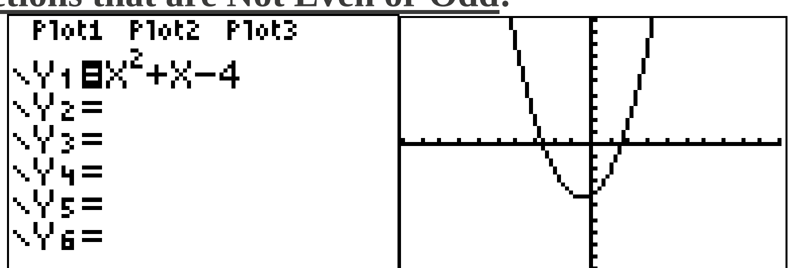
Odd functions: must

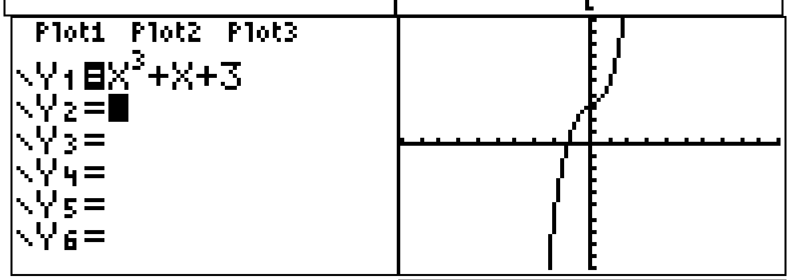
1. have exponents that are all odd numbers
2. reflect in the origin (0,0).

Example of an Odd Function:



Examples of Functions that are Not Even or Odd:





An Algebraic Test to Determine if a Function is Even, Odd, or Neither:

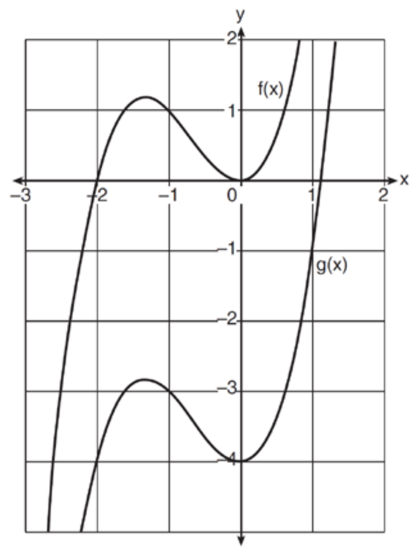
Evaluate the function for $f(-x)$.

Even	Odd	Neither
$f(x) = x^2 + 4$	$f(x) = x^3 + x$	$f(x) = x^3 + x + 3$
$f(-x) = (-x)^2 + 4$	$f(-x) = (-x)^3 + (-x)$	$f(-x) = (-x)^3 + (-x) + 3$
$f(-x) = x^2 + 4$	$f(-x) = -x^3 - x$	$f(-x) = -x^3 - x + 3$
The function is **even** if $f(x)$ has exactly the same terms as $f(-x)$.	The function is **odd** if all the terms of $f(x)$ and $f(-x)$ are additive inverses.	The function is **neither** even or odd if the terms if all the terms are not the same or opposites.

REGENTS PROBLEMS TYPICAL OF THIS STANDARD

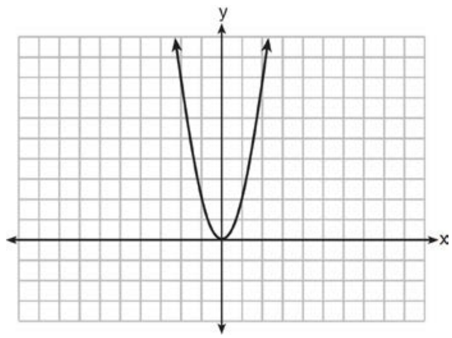
1. How does the graph of $f(x) = 3(x-2)^2 + 1$ compare to the graph of $g(x) = x^2$?
   a. The graph of $f(x)$ is wider than the graph of $g(x)$, and its vertex is moved to the left 2 units and up 1 unit.
   b. The graph of $f(x)$ is narrower than the graph of $g(x)$, and its vertex is moved to the right 2 units and up 1 unit.
   c. The graph of $f(x)$ is narrower than the graph of $g(x)$, and its vertex is moved to the left 2 units and up 1 unit.
   d. The graph of $f(x)$ is wider than the graph of $g(x)$, and its vertex is moved to the right 2 units and up 1 unit.

2. In the diagram below, $f(x) = x^3 + 2x^2$ is graphed. Also graphed is $g(x)$, the result of a translation of $f(x)$.



Determine an equation of $g(x)$. Explain your reasoning.

3. The graph of the equation $y = ax^2$ is shown below.



If a is multiplied by $-\frac{1}{2}$, the graph of the new equation is

a. wider and opens downward
b. wider and opens upward
c. narrower and opens downward
d. narrower and opens upward

4. The vertex of the parabola represented by $f(x) = x^2 - 4x + 3$ has coordinates $(2, -1)$. Find the coordinates of the vertex of the parabola defined by $g(x) = f(x - 2)$. Explain how you arrived at your answer. [The use of the set of axes below is optional.]

5. Graph the function $y = |x - 3|$ on the set of axes below.

Explain how the graph of $y = |x - 3|$ has changed from the related graph $y = |x|$.

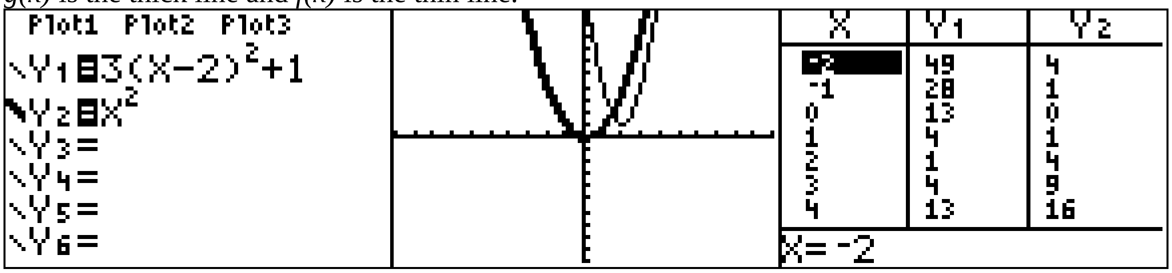
6. On the axes below, graph $f(x) = |3x|$.

If $g(x) = f(x) - 2$, how is the graph of $f(x)$ translated to form the graph of $g(x)$?
If $h(x) = f(x - 4)$, how is the graph of $f(x)$ translated to form the graph of $h(x)$?

F.BF.B.3: Build New Functions from Existing Functions.
Answer Section

1. ANS: B
Strategy: Input both functions in a graphing calculator and compare them.

Let the graph of Y_1 be the graph of $f(x) = 3(x-2)^2 + 1$
Let the graph of Y_2 be the graph of $g(x) = x^2$
Input both functions in a graphing calculator.
g(x) is the thick line and *f(x)* is the thin line.



PTS: 2 REF: 011512ai NAT: F.BF.B.3
TOP: Transformations with Functions and Relations

2. ANS:

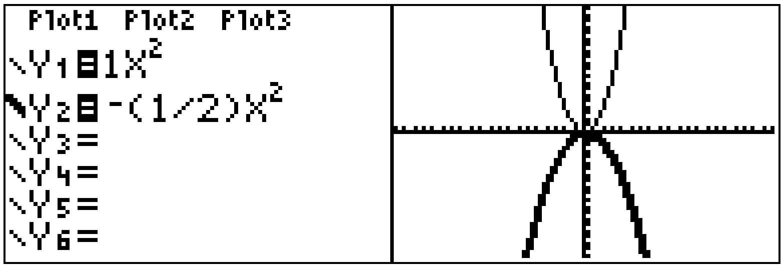
$$g(x) = x^3 + 2x^2 - 4$$

f(x) has a y-intercept of 0.
g(x) has a y-intercept of -4.
Every point on f(x) is a translation down 4 units to create g(x).

PTS: 2 REF: 061632ai NAT: F.BF.B.3 TOP: Graphing Polynomial Functions

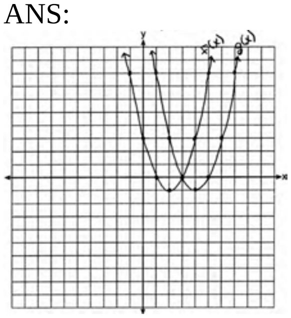
3. ANS: A
Strategy: Use the following general rules for quadratics, then check with a graphiong calculator.
As the value of a approaches 0, the parabola gets wider.
A positive value of a opens upward.
A negative value of a opens downward.

Check with graphing calculator:
Assume $a = 1$, then $y_1 = 1x^2$
If a is multiplied by $-\frac{1}{2}$, then $y_2 = -\frac{1}{2}x^2$.
Input both equations in a graphing calculator, as follows:



PTS: 2 REF: 081417ai NAT: F.BF.B.3
TOP: Transformations with Functions and Relations

4. ANS:



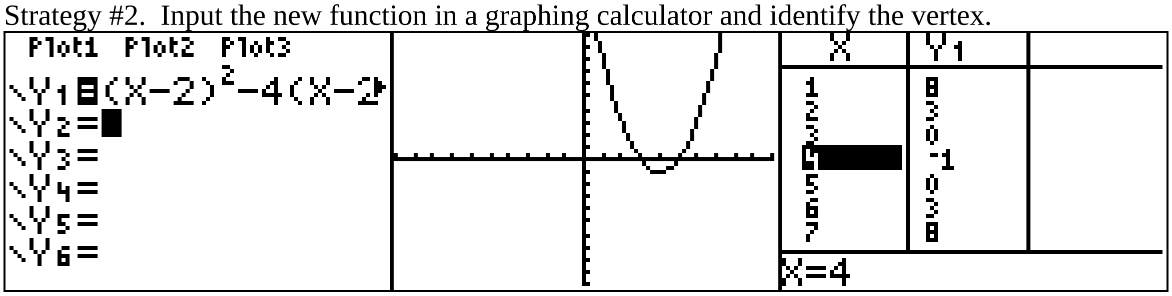
$(4,-1)$. $f(x-2)$ is a horizontal shift two units to the right

Strategy 1: Compose a new function, find the axis of symmetry, solve for g(x) at axis of symmetry, as follows:

$f(x)=x^2-4x+3$ and $g(x)=f(x-2)$ Therefore: $g(x)=(x-2)^2-4(x-2)+3$ $g(x)=x^2-4x+4-4x+8+3$ $g(x)=x^2-8x+15$	$axis\ of\ symmetry=\frac{-b}{2a}=\frac{-(-8)}{2(1)}=\frac{8}{2}=4$ $g(x)=x^2-8x+15$ $g(4)=(4)^2-8(4)+15$ $g(4)=16-32+15$ $g(4)=-1$

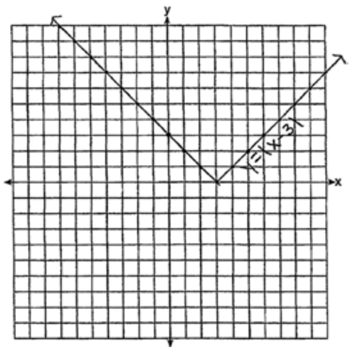
The coordinates of the vertex of *g(x)* are (4,-1)

Strategy #2. Input the new function in a graphing calculator and identify the vertex.



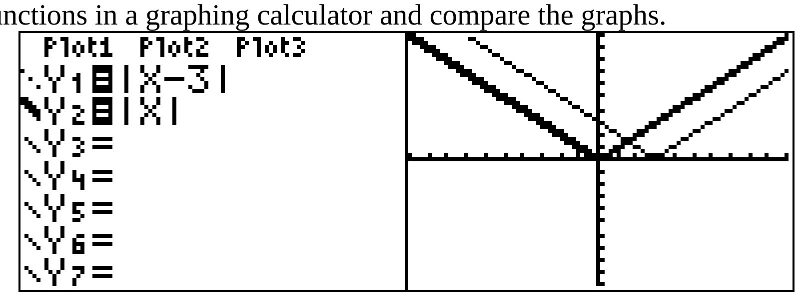
PTS: 2 REF: 061428ai NAT: F.BF.B.3
TOP: Transformations with Functions and Relations

5. ANS:



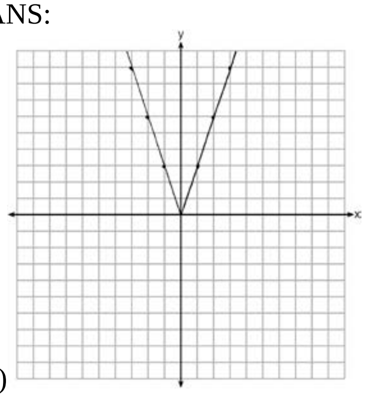
The graph has shifted three units to the right.

Strategy: Input both functions in a graphing calculator and compare the graphs.



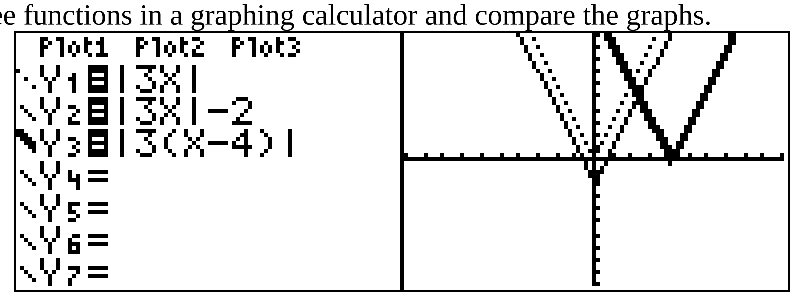
PTS: 2 REF: 061525ai NAT: F.BF.B.3
TOP: Transformations with Functions and Relations

6. ANS:



a)
b) If $g(x) = f(x) - 2$, the graph of $f(x)$ is translated 2 down to form the graph of $g(x)$.
c) If $h(x) = f(x - 4)$, the graph of $f(x)$ translated 4 right to form the graph of $h(x)$.

Strategy: Input the three functions in a graphing calculator and compare the graphs.



PTS: 4 REF: 081433ai NAT: F.BF.B.3
TOP: Transformations with Functions and Relations

Homework - Write the Math Assignment

START Write your name, date, topic of lesson, and class on your paper.

NAME: Mohammed Chen
DATE: December 18, 2015
LESSON: Missing Number in the Average
CLASS: Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
PART 1b. State your understanding of **what the problem is asking**.
PART 1c. **Answer** the problem.
PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
PART 2b. State your understanding of **what the new problem is asking**.
PART 2c. **Answer** the new problem.
PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	**Up to 2** points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	**Up to 2** points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of Polya's four step universal algorithm for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. **The Problem**

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is $360. If the weekly salaries of four of the employees are $340, $340, $345, and $425, what is the salary of the fifth employee?

Part 1b. **What is the problem asking?**

Find the salary of the fifth employee.

Part 1c. **Answer**

The salary of the fifth employee is $350 per week.

Part 1d. **Explanation of Strategy**

The arithmetic mean or average can be represented algebraically as:

$$\overline{X} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

Check: $360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$

Part 2a. **A New Problem**

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b **What is the new problem asking?**

Find Joseph's score on the missing exam.

Part 2c. **Answer to New Problem**

Joseph received a score of 85 on the missing examination.

Part 2d. **Explanation of Strategy**

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.