

A.REI.D.11: Find and Explain Solutions of Systems

SYSTEMS

A.REI.D.11: Find and Explain Solutions to Systems

D. Represent and solve equations and inequalities graphically.

11. Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Overview of Lesson

- activate prior knowledge and review learning objectives (see above)
- explain vocabulary and/or big ideas associated with the lesson
- connect assessment practices with curriculum
- model an assessment problem and solution strategy
- facilitate guided discussion of student activity
- facilitate guided practice of student activity
- [Selected problem set\(s\)](#)
- facilitate a summary and share out of student work
- Homework – Write the Math Assignment

BIG IDEAS

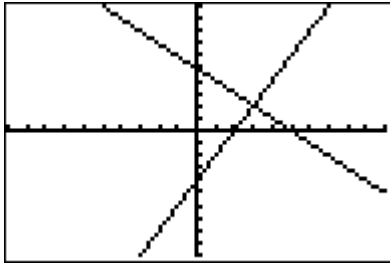
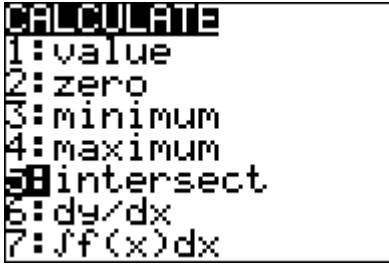
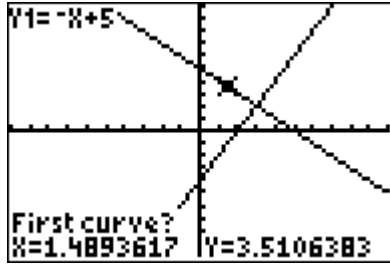
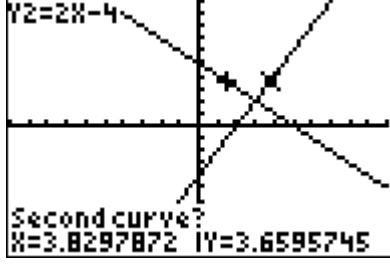
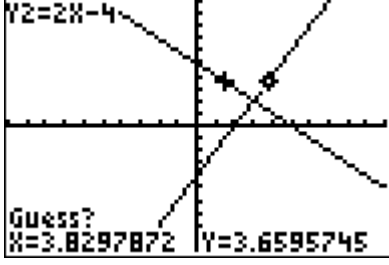
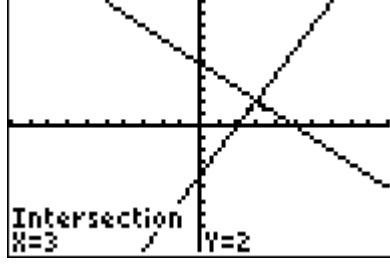
A **solution of a system** of equations makes each equation in the system true. Solutions can be found using three different views of a function.

Example: If $f(x) = -x + 5$ and $g(x) = 2x - 4$, then $f(3) = g(3)$

Graph View	Table View	Function Rule View																					
<p>of a Solution to a System of Equations</p> <p>A solution occurs when the graphs of equations intersect.</p>	<p>of a Solution to System of Equations</p> <table border="1" style="margin: 0 auto; border-collapse: collapse;"> <thead> <tr> <th style="width: 33%;">X</th> <th style="width: 33%;">Y₁</th> <th style="width: 33%;">Y₂</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>5</td> <td>-4</td> </tr> <tr> <td>1</td> <td>4</td> <td>-2</td> </tr> <tr> <td>2</td> <td>3</td> <td>0</td> </tr> <tr> <td>3</td> <td>2</td> <td>2</td> </tr> <tr> <td>4</td> <td>1</td> <td>4</td> </tr> <tr> <td>5</td> <td>0</td> <td>6</td> </tr> </tbody> </table> <p style="text-align: center;">X = 3</p> <p>A solution occurs when one value of x creates the same value of y in all equations.</p> <p>Note:</p> <p>$Y_1 = f(x) = -x + 5$</p> <p>$Y_2 = g(x) = 2x - 4$</p>	X	Y ₁	Y ₂	0	5	-4	1	4	-2	2	3	0	3	2	2	4	1	4	5	0	6	<p>of a Solution to a System of Equations</p> <p style="text-align: center;">$f(x) = -x + 5$</p> <p style="text-align: center;">$g(x) = 2x - 4$</p> <p style="text-align: center;">$f(x) = g(x)$</p> <p style="text-align: center;">$-x + 5 = 2x - 4$</p> <p style="text-align: center;">$9 = 3x$</p> <p style="text-align: center;">$3 = x$</p> <p style="text-align: center;">$f(3) = g(3)$</p> <p>A solution occurs when $f(x) = g(x)$ for a specific value(s) of x.</p>
X	Y ₁	Y ₂																					
0	5	-4																					
1	4	-2																					
2	3	0																					
3	2	2																					
4	1	4																					
5	0	6																					

Lesson Plan

Using the TI-84 family of graphing calculators to calculate the intersection of a graph.

<p style="text-align: center;">1</p>  <p style="text-align: right;">Start</p> <p>in the graph view of the system.</p>	<p style="text-align: center;">2</p>  <p style="text-align: right;">Select</p> <p>the 2nd - Calc - Intersect function. Press enter.</p>	<p style="text-align: center;">3</p>  <p style="text-align: right;">Move</p> <p>the cursor to the left of the intersection. Press enter.</p>
<p style="text-align: center;">4</p>  <p style="text-align: right;">Move</p> <p>the cursor to the right of the intersection. Press enter.</p>	<p style="text-align: center;">5</p>  <p style="text-align: right;">Press</p> <p>enter in response to Guess?</p>	<p style="text-align: center;">6</p>  <p style="text-align: right;">The x</p> <p>and y coordinates of the intersection will appear.</p>

REGENTS PROBLEMS TYPICAL OF THIS STANDARD

- John and Sarah are each saving money for a car. The total amount of money John will save is given by the function $f(x) = 60 + 5x$. The total amount of money Sarah will save is given by the function $g(x) = x^2 + 46$. After how many weeks, x , will they have the same amount of money saved? Explain how you arrived at your answer.
- The graphs of the functions $f(x) = |x - 3| + 1$ and $g(x) = 2x + 1$ are drawn. Which statement about these functions is true?
 - The solution to $f(x) = g(x)$ is 3.
 - The solution to $f(x) = g(x)$ is 1.
 - The graphs intersect when $y = 1$.
 - The graphs intersect when $x = 3$.
- Two functions, $y = |x - 3|$ and $3x + 3y = 27$, are graphed on the same set of axes. Which statement is true about the solution to the system of equations?
 - $(3, 0)$ is the solution to the system because it satisfies the equation $y = |x - 3|$.
 - $(9, 0)$ is the solution to the system because it satisfies the equation $3x + 3y = 27$.
 - $(6, 3)$ is the solution to the system because it satisfies both equations.
 - $(3, 0)$, $(9, 0)$, and $(6, 3)$ are the solutions to the system of equations because they all satisfy at least one of the equations.
- Given the functions $h(x) = \frac{1}{2}x + 3$ and $j(x) = |x|$, which value of x makes $h(x) = j(x)$?
 - 2
 - 2
 - 3
 - 6

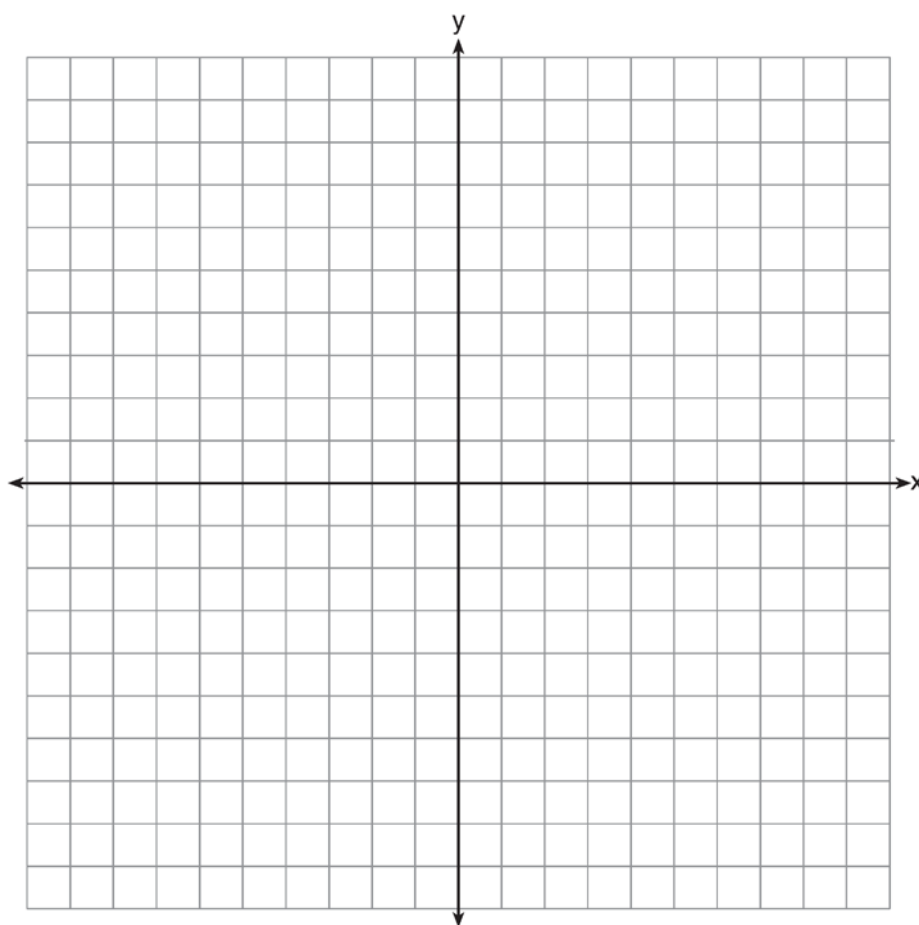
Lesson Plan

5. On the set of axes below, graph

$$g(x) = \frac{1}{2}x + 1$$

and

$$f(x) = \begin{cases} 2x + 1, & x \leq -1 \\ 2 - x^2, & x > -1 \end{cases}$$



How many values of x satisfy the equation $f(x) = g(x)$? Explain your answer, using evidence from your graphs.

A.REI.D.11: Find and Explain Solutions of Systems
Answer Section

1. ANS:

John and Sarah will have the same amount of money saved at 7 weeks. I set the expressions representing their savings equal to each other and solved for the positive value of x by factoring.

Strategy: Set the expressions representing their savings equal to one another and solve for x .

$$f(x) = 60 + 5x \text{ and } g(x) = x^2 + 46$$

$$\text{Let } f(x) = g(x)$$

$$x^2 + 46 = 60 + 5x$$

$$x^2 - 5x - 14 = 0$$

$$(x - 7)(x + 2) = 0$$

$$x = 7$$

DIMS? Does It Make Sense? Yes. After 7 weeks, John and Sarah will each have \$95.00.

John's Savings $f(x) = 60 + 5x$ $f(7) = 60 + 5(7)$ $f(7) = 60 + 35$ $f(7) = 95$	Sarah's Savings $g(x) = x^2 + 46$ $g(7) = (7)^2 + 46$ $g(7) = 49 + 46$ $g(7) = 95$
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PTS: 2

REF: 061527ai

NAT: A.REI.D.11

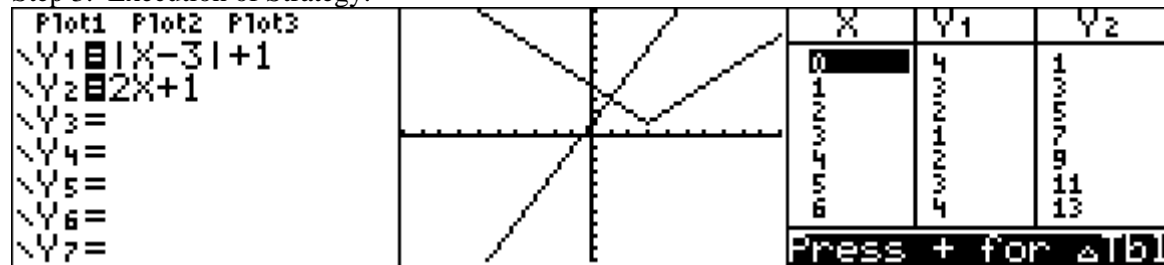
TOP: Quadratic-Linear Systems

2. ANS: B

Step 1. Understand that only of the answer choices is true.

Step 2. Strategy. Input both functions in a graphing calculator and explore the truth of each answer choice.

Step 3. Execution of Strategy.



The graph and table show that the solution for this system of equations is (1,3). This means that $f(1) = 3$ and $g(1) = 3$. Accordingly, when x is 1, $f(x) = g(x)$. The correct answer is choice b).

Step 4. Does it make sense? Yes. All of the other answer choices can be eliminated as wrong. The problem can be checked algebraically as follows:

$$\text{Given: } f(x) = |x - 3| + 1 \text{ and } g(x) = 2x + 1, \text{ find } f(x) = g(x)$$

Lesson Plan

$ x - 3 + 1 = 2x + 1$ $ x - 3 = 2x$ $x - 3 = 2x$ $-3 = x$ <p>This is an extraneous solution.</p> $ -3 - 3 + 1 = 2(-3) + 1$ $ -6 + 1 = -6 + 1$ $6 + 1 = -6 + 1$ $7 \neq -5$	$ x - 3 + 1 = 2x + 1$ $ x - 3 = 2x$ $-x + 3 = 2x$ $3 = 3x$ $1 = x$ <p>This solution checks.</p> $ 1 - 3 + 1 = 2(1) + 1$ $ -2 + 1 = 2 + 1$ $2 + 1 = 2 + 1$ $3 = 3$
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PTS: 2 REF: 061622ai NAT: A.REI.D.11 TOP: Other Systems

3. ANS: C

Strategy: Input both functions in a graphing calculator, then use the table and graph views of the function to select the correct answer.

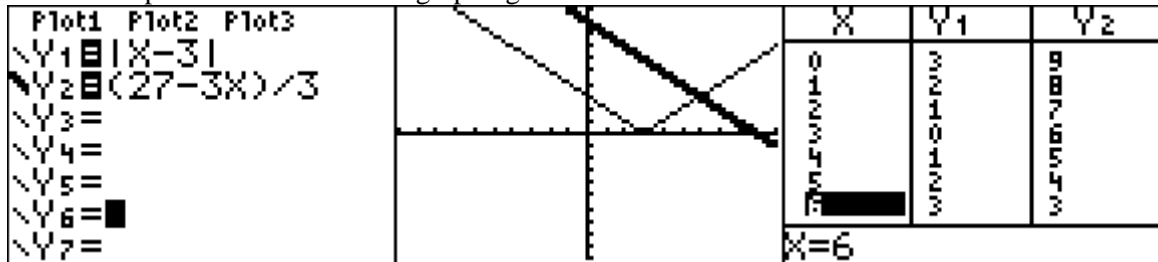
STEP 1. Transpose the second function for input into a graphing calculator.

$$3x + 3y = 27$$

$$3y = 27 - 3x$$

$$y = \frac{27 - 3x}{3}$$

STEP 2. Input both functions in a graphing calculator.

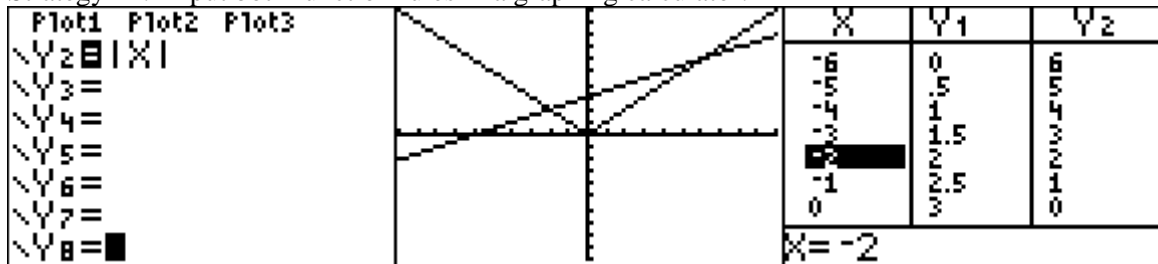


When $x = 6$, the value of y in both equations is 3. $(6, 3)$ is the solution to this system.

PTS: 2 REF: 011518ai NAT: A.REI.D.11 TOP: Nonlinear Systems

4. ANS: A

Strategy #1: Input both function rules in a graphing calculator.



Strategy #2: Set the right expressions of both functions equal to one another. Then solve for the positive and negative values of $|x|$.

Lesson Plan

$$\frac{1}{2}x + 3 = |x|$$

$\frac{1}{2}x + 3 = x$ $x + 6 = 2x$ $6 = x$	$-\left(\frac{1}{2}x + 3\right) = x$ $-\frac{1}{2}x - 3 = x$ $-x - 6 = 2x$ $-6 = 3x$ $-2 = x$
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Check:

$h(x) = \frac{1}{2}x + 3$ $h(-2) = \frac{1}{2}(-2) + 3$ $h(-2) = -1 + 3$ $h(-2) = 2$	$j(x) = x $ $j(-2) = -2 $ $j(x) = 2$
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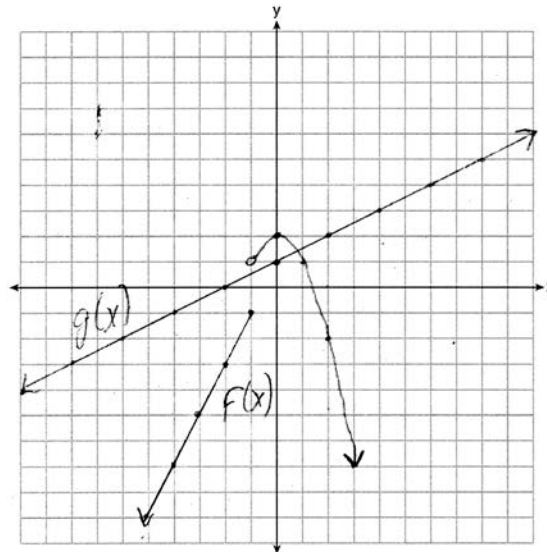
PTS: 2 REF: 011617ai NAT: A.REI.D.11 TOP: Other Systems

5. ANS:

Step 1. Plot $g(x) = \frac{1}{2}x + 1$

Step 2. Plot $f(x) = 2x + 1$ over the interval $x \leq -1$

Step 3. Plot $f(x) = 2 - x^2$ over the interval $x > -1$



Only 1 value of x satisfies the equation $f(x) = g(x)$, because the graphs only intersect once.

PTS: 4 REF: 061636ai NAT: F.IF.C.7 TOP: Other Systems

Homework - Write the Math Assignment

START Write your name, date, topic of lesson, and class on your paper.
 NAME: Mohammed Chen
 DATE: December 18, 2015
 LESSON: Missing Number in the Average
 CLASS: Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
 PART 1b. State your understanding of **what the problem is asking**.
 PART 1c. **Answer** the problem.
 PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
 PART 2b. State your understanding of **what the new problem is asking**.
 PART 2c. **Answer** the new problem.
 PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	Up to 2 points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	Up to 2 points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of [Polya's four step universal algorithm](#) for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. The Problem

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is \$360. If the weekly salaries of four of the employees are \$340, \$340, \$345, and \$425, what is the salary of the fifth employee?

Part 1b. What is the problem asking?

Find the salary of the fifth employee.

Part 1c. Answer

The salary of the fifth employee is \$350 per week.

Part 1d. Explanation of Strategy

The arithmetic mean or average can be represented algebraically as:

$$\bar{X} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

$$\text{Check: } 360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$$

Part 2a. A New Problem

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b. What is the new problem asking?

Find Joseph's score on the missing exam.

Part 2c. Answer to New Problem

Joseph received a score of 85 on the missing examination.

Part 2d. Explanation of Strategy

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.