

June 22, 1967

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Write your answers in the spaces provided.

- When $x^{25} - 3$ is divided by $x + 2$, there is an integral remainder. If this remainder is a positive integer, write "positive"; if it is a negative integer, write "negative." 1 _____
- Express the product of $3 + 2i$ and $2i$ in the form $a + bi$. 2 _____
- Solve for a in terms of b and c : $\frac{1}{a} = \frac{1}{b} + \frac{1}{c}$ 3 _____
- How many different groups of three students each may a teacher choose from among 8 students in her class? 4 _____
- Write the expression $[4(\cos 30^\circ + i \sin 30^\circ)]^2$ in the form $r(\cos \theta + i \sin \theta)$. 5 _____
- Express the repeating decimal $0.1212\dots$, in which the digits 1 and 2 are repeated endlessly, as a rational number. 6 _____
- Find the value(s) of x which will satisfy the equation $\sqrt{2x} + 4 = x$. 7 _____
- Find the positive value of b such that the graph of $x^2 + bx + 4 = y$ touches the x -axis in one and only one point. 8 _____
- Find the fourth term of the geometric progression whose first two terms are $\frac{1}{\sqrt{a}}$ and 1, respectively. 9 _____
- Find all the values of x which satisfy the inequality $\frac{7}{6} + \frac{x}{2} < x - \frac{1}{3}$. 10 _____
- How many cars would a dealer have to have on hand in order to show a customer all of the models available from a manufacturer who builds cars in 6 body styles, with a choice of 4 engines, and equipped with either a standard or an automatic transmission? 11 _____
- The graph of $y = f(x)$ crosses the x -axis just once between $x = -0.9$ and $x = -0.8$. If $f(-0.9) > 0$, $f(-0.85) > 0$, and $f(-0.8) < 0$, find to the nearest tenth a negative root of $f(x) = 0$. 12 _____
- Write an equation for the axis of symmetry of the graph of $y = 10x - 2x^2$. 13 _____
- Find the rational fractional root of $6x^3 + x^2 + 5x - 2 = 0$. 14 _____

15. What is the *least* possible degree of a rational integral equation with rational coefficients, if 3 of its roots are -3 , $2 + 3i$, and $4 - \sqrt{7}$? 15_____

16. What is the sum of the roots of the equation $3x^3 - 12x^2 + x = 0$? 16_____

17. If $y = x^4 - 2x^2 - 4$, find the average rate of change of y with respect to x as x varies from $x = -1$ to $x = 0$. 17_____

18. The tens digit of a two-digit number is 4 less than the units digit. If the units digit is represented by u , express the number in terms of u . 18_____

Directions (19-24): Indicate the correct completion for each of the following by writing the number 1, 2, 3, or 4 in the space provided.

19. Which defines a rational integral function of x ?
(1) $x^{\frac{1}{2}} - 1$ (2) $x^2 - 1$ (3) $x^{-\frac{1}{2}} - 1$ (4) $x^{-2} - 1$ 19_____

20. The graphs of $y = ax + b$ and $y = cx + d$ are distinct parallel lines if (1) $a \neq c$ and $b \neq d$ (2) $a \neq c$ and $b = d$
(3) $a = c$ and $b \neq d$ (4) $a = c$ and $b = d$ 20_____

21. Given $\log_3 3^x = 8$. The value of x that satisfies this equation is (1) 16 (2) 2 (3) 3 (4) 4 21_____

22. When drawn on the same set of axes, the points of intersection of the graphs of $x^2 + 4y^2 = 16$ and $xy = 1$ are located in quadrants (1) I and III (2) I and IV (3) II and III (4) II and IV 22_____

23. Given that s varies inversely as the square of t . If a value of t is halved, then the corresponding value of s is multiplied by (1) $\frac{1}{2}$ (2) 2 (3) $\frac{1}{4}$ (4) 4 23_____

24. How many real roots does $x^4 + 6x^2 + 3 = 0$ have? (1) 1 (2) 2 (3) 0 (4) 4 24_____

Part II

*Answer sixteen questions from this part, 25-48. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answers in the space provided.*

25. When drawn on the same set of axes, the graphs of $y = x^3 - x$ and $y = 2x + k$ are tangent to each other at a point whose abscissa is 1. Find the value of k . 25_____

26. Express the complex number $\sqrt{2} + i\sqrt{2}$ in the polar form $r(\cos \theta + i \sin \theta)$. 26_____

27. A root of $x^3 + x^2 - 24 = 0$ lies between 2 and 3. Find this root to the *nearest integer*. 27_____

28. In the equation $x^3 - kx^2 - 5x + 10 = 0$, k is a rational number. If two roots of this equation are $\sqrt{5}$ and 2, find the numerical value of k . 28_____

29. The illumination I , received by a body from a source of light of strength S , varies directly as S and inversely as the square of the distance d between the light source and the body. If k represents the constant of variation, express I in terms of S , d and k . 29_____

30. If $f(x) = 3x^3 - 2x^2 + 1$, find $f(2) - f(3^{-1})$. 30_____

31. In the complex number, plane points P and Q represent the complex numbers i and $-1 + i$, respectively, and O represents the origin. How many degrees are there in angle POQ ? 31_____

*32. For what value of k will the value of the determinant

$$\begin{vmatrix} 4 & k & 3 \\ 2 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix}$$
 be 17? *32_____

33. Write the term of the expansion of $(a + b)^8$ in which the exponent of a is twice the exponent of b . 33_____

34. An object is moving along a straight line. Its distance s , in feet from a fixed point after t seconds, is given by the equation $s = 5t^2 + t^3$. Find the value of t when the acceleration is 19 feet per second per second. 34_____

35. A school committee of 2 members is to be formed by drawing 2 names from a box containing the names of 5 seniors, 4 juniors, and 3 sophomores. What is the probability that the committee will consist of 2 seniors? 35_____

36. Write an equation of the straight line having an x -intercept of 2 and a y -intercept of -4 . 36_____

37. If the number $\frac{2 + 3i}{1 - 3i}$ is expressed in the form $a + bi$ where a and b are real numbers, what is the value of a ? 37_____

38. Find the coordinates of the point of inflection of the curve whose equation is $y = x^3 - 12x + 20$. 38_____

39. The lengths of the sides of a right triangle form an arithmetic progression whose common difference is 3. Find the length of the shorter leg. 39_____

40. If $y = \log_{10} 3$, find the value of 10^{2y} . 40_____

41. Point P in the first quadrant has the coordinates $(2, k)$. The line segment between P and the origin and the line segment between P and the point $(10, 0)$ are perpendicular. Find the value of k . 41_____

*42. Transform the equation $r^2 = 2r \sin \theta - 4r \cos \theta$ from polar to rectangular coordinates. *42_____

Directions (43-48) For each of those chosen, write in the space provided the number preceding the expression that best completes the statement.

43. If $x < 0$, then $\sqrt{x^2}$ is equal to (1) an imaginary number
(2) x (3) a negative number (4) $-x$ 43 _____

44. If Al is a years old and Bob is b years old, how old was Al when Bob was c years old? (1) $a - b + c$ (2) $a - b - c$
(3) $a + b - c$ (4) $b - a + c$ 44 _____

45. If $\log x = 3 + \log 3$, then x equals (1) 3^3 (2) 6
(3) 3,000 (4) 1,003 45 _____

46. The circle whose equation is $(x + 1)^2 + (y + 3)^2 = 9$ is tangent to (1) the y -axis only (2) the x -axis only
(3) both axes (4) neither axis 46 _____

47. If the graphs of $y = 3^x$ and $y = 3^{-x}$ are drawn on the same set of axes, the graphs will intersect at a point which is on the line whose equation is (1) $x = 0$ (2) $y = 0$ (3) $y = x$
(4) $y = -x$ 47 _____

48. The solution of the inequality $x^2 - x - 6 > 0$ is
(1) $-2 < x < 3$ (2) $-3 < x < 2$
(3) $x < -2$ or $x > 3$ (4) $x < -3$ or $x > 2$ 48 _____