The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

12B TWELFTH YEAR MATHEMATICS

12B (Solid Geometry)

Friday, June 17, 1966 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet.

1 The perimeter of the base of a regular pyramid is 9 and its slant height is 4. Find the lateral area of the pyramid.

2 A lune having an angle of 10° is drawn on a sphere having a radius of 6 inches. Find in terms of π the number of square inches in the area of the lune.

3 Find the volume of a regular square pyramid with a base edge of 6 and slant height of 5.

4 Find the length of a diagonal of a rectangular solid whose dimensions are 15, 10 and 6.

5 What is the radius of a sphere on which a lune has a perimeter of 16π?

6 A point lies within a dihedral angle of 54° and is equally distant from each face of the angle. If the point is 20 inches from the edge of the angle, find to the *nearest tenth of an inch* the distance from the point to a face of the angle.

7 The lateral area of a cone of revolution is $16\pi\sqrt{10}$ and the slant height is $4\sqrt{10}$. Find the radius of the base.

8 Find in terms of π the volume of a sphere inscribed in a cube whose diagonal is $4\sqrt{3}$.

9 An equilateral triangle whose side is 6 is rotated through 180° about its altitude as an axis. In terms of π and a radical, find the volume of the resulting solid.

10 The area of the base of a pyramid is 48 square inches and the altitude is 12 inches. Find the number of square inches in the area of a section parallel to the base and 3 inches from it.

11 The polar distance of a small circle of a sphere is 60° and the radius of the sphere is 12. Express in radical form the length of the radius of the small circle.

12 The perimeter of an equilateral spherical triangle is 300°. Find the number of spherical degrees in the area of its polar triangle.

13 In $\triangle ABC$, with a right angle at C, $AC = 6$ and $CB = 8$. ED is drawn perpendicular to the plane of triangle ABC at the midpoint D of AB. If ED is 12, what is the length of AE?

14 The base edges and the lateral edges of a regular hexagonal prism are each 4. Find in radical form the volume of the prism.

15 A line segment AB which is $6\sqrt{2}$ inches long makes an angle of 45° with plane P. Find the number of inches in the projection of AB on plane P.

16 What fractional part of the area of a sphere is covered by a trirectangular spherical triangle?

17 The radius of a right circular cylinder is 8 and its altitude is 12. Find the area of the largest section of the cylinder that can be made by a plane containing an element of the cylinder.

18 Corresponding edges of two similar solids are 3 and 5. If the volume of the smaller solid is 81, find the volume of the larger.

19 The base diameter of a cylindrical tank is 8 feet and its height is 20 feet. Express in terms of π the number of cubic feet of water the tank will contain when filled to one-fourth of its depth.

20 The altitudes of two zones drawn on a given sphere are 3 and 7. Find the ratio of the area of the smaller zone to the area of the larger.

21 The lateral area of a frustum of a cone of revolution is 78π square inches. Its slant height is 6 inches and the radius of one base is 8 inches. Find the number of inches in the radius of the other base.

Directions (22–30): Write in the space provided on the separate answer sheet the *number* preceding the expression that best completes *each* statement or answers *each* question.

22 A property of any two great circles of a sphere is that they
(1) are parallel
(2) bisect each other
(3) have the same pole
(4) are perpendicular to each other

23 If two sides of a spherical triangle are 140° and 105°, the third side may be
(1) 30° (3) 115°
(2) 110° (4) 125°

24 The locus of points that are a given distance d from plane M and also $\frac{3}{2}d$ from point P on M is
(1) one point (3) two points
(2) one circle (4) two circles

25 The lateral surface area of a regular hexagonal pyramid is twice the area of the base. The slant height expressed in terms of a base edge, e, is
(1) $\frac{\sqrt{3}}{e}$ (3) $e\sqrt{3}$
(2) $\frac{e}{2}\sqrt{3}$ (4) $2e\sqrt{3}$

26 At least one of the lateral faces of a truncated prism must be a
(1) trapezoid (3) pentagon
(2) triangle (4) hexagon

27 If a regular polyhedron has 8 vertices and 12 edges, it is
(1) an octahedron (3) a tetrahedron
(2) a dodecahedron (4) a hexahedron

28 If an edge of a regular tetrahedron is 6, the length of its altitude is
(1) $4\sqrt{3}$ (3) $2\sqrt{6}$
(2) $\sqrt{33}$ (4) $4\sqrt{2}$

29 If lines m and n are both perpendicular to line l, then
(1) m and n always determine a plane perpendicular to l
(2) m is always parallel to n
(3) m is never parallel to n
(4) m and n may be skew lines

30 The locus of points equidistant from the faces of a trihedral angle is
(1) one point (3) one line
(2) two points (4) one plane

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove *a* or *b* but *not* both: [10]

a If a line is perpendicular to each of two intersecting lines at their point of intersection, it is perpendicular to the plane of the two lines.

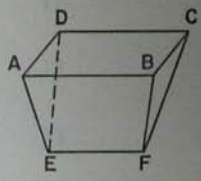
OR

b A spherical angle is measured by the arc of the great circle described from its vertex as a pole and included between its sides produced if necessary.

32 In a square pyramid, V–$ABCD$, Q and R are the midpoints of VC and VD, respectively. A plane through Q and R intersects VB in S and VA in T. Prove that ST is parallel to AB. [10]

33 The radius of a sphere is 10 inches. The sides of a spherical triangle drawn on the sphere are 68°, 52° and 37°. Find to the *nearest square inch* the area of its polar triangle. [10]

34 The accompanying figure represents a wedge. $ABCD$ is a rectangle and EF is a line segment parallel to AB. Using the prismatoid formula find, to the *nearest cubic foot*, the volume of the wedge if $AB = 3$ feet, $AD = 1\frac{1}{2}$ feet, $EF = 2\frac{1}{2}$ feet and the distance from EF to the plane of $ABCD$ is $1\frac{1}{4}$ feet. [10]



35 Line l is perpendicular to plane m at point P.

a Describe fully the locus of points which are

(1) a given distance d from m [2]
(2) a given distance r from l [1]
(3) a given distance s from P [1]

b (1) Name the locus of points satisfying both conditions in part $a(1)$ and part $a(2)$. [2]
(2) Name the locus of points satisfying both conditions in part $a(1)$ and part $a(3)$ if s is greater than d. [2]
(3) Name the locus of points satisfying both conditions in part $a(1)$ and part $a(3)$ if $s = d$. [2]

36 The slant height of a regular triangular pyramid is m and the angle it makes with the base is θ.

a Show that the volume of the pyramid is equal to $m^3\sqrt{3}\cos^2\theta\sin\theta$. [7]

b Find the volume, to the *nearest integer*, when m is 4 and θ is 60°. [3]

*37 Answer *a* or *b* but *not* both:

a (1) Find the distance from the origin to the point (2,2,1). [1]
(2) Write an equation of the sphere whose center is the origin and which passes through the point (2,2,1). [2]
(3) Is the point (0,3,0) on the sphere described in (2)? Answer *yes* or *no*. [1]
(4) Write an equation of the plane through point (2,2,1) parallel to the XZ plane. [2]
(5) Write the coordinates of the midpoint of the line segment joining (2,2,1) and (0,3,0). [2]
(6) Write an equation of the plane through (0,3,0), (1,0,0) and (0,0,—2). [2]

OR

b Find, to the *nearest degree*, the altitude of an equilateral spherical triangle each of whose sides is 70°. [10]

* This question is based on an optional topic in the syllabus.

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ANSWER SHEET

Pupil..... Teacher.....

School.....

Name and author of textbook used.....

All of your answers to part I should be recorded on this answer sheet.

Part I

Answer all questions in this part.

1.....	11.....	21.....
2.....	12.....	22.....
3.....	13.....	23.....
4.....	14.....	24.....
5.....	15.....	25.....
6.....	16.....	26.....
7.....	17.....	27.....
8.....	18.....	28.....
9.....	19.....	29.....
10.....	20.....	30.....

Part I Score:.....

Rater's Initials:

.....

Your answers for part II should be placed on paper supplied by the school.

FOR TEACHERS ONLY

12B

SCORING KEY

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 22–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) 18

(2) 4π

(3) 48

(4) 19

(5) 8

(6) 9.1

(7) 4

(8) $\frac{32\pi}{3}$

(9) $9\pi\sqrt{3}$

(10) 27

(11) $6\sqrt{3}$

(12) 60

(13) 13

(14) $96\sqrt{3}$

(15) 6

(16) $\frac{1}{8}$

(17) 192

(18) 375

(19) 80π

(20) $\frac{3}{7}$

(21) 5

(22) 2

(23) 2

(24) 4

(25) 3

(26) 1

(27) 4

(28) 3

(29) 4

(30) 3

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely mechanical or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent depending on the relative importance of the principle in the solution of the problem.

(33) 354 [10]

(34) 3 [10]

(35) *a* (1) Two planes parallel to m and a distance d from m [2]
(2) A cylindrical surface with axis l and radius r [1]
(3) A spherical surface with center P and radius s [1]

b (1) two circles [2]
(2) two circles [2]
(3) two points [2]

(36) *b* 24 [3]

*(37) *a* (1) 3 [1]
(2) $x^2 + y^2 + z^2 = 9$ [2]
(3) yes [1]
(4) $y = 2$ [2]
(5) $\left(1, \frac{5}{2}, \frac{1}{2}\right)$ [2]
(6) $6x + 2y - 3x = 6$ [2]

b 65 [10]