The University of the State of New York

THE STATE EDUCATION DEPARTMENT

EXAMINATION IN EXPERIMENTAL TWELFTH YEAR MATHEMATICS

June 1965

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer twenty-five of the thirty questions in this part. Each correct answer will receive two credits. No partial credit will be allowed.

Directions (1–11): For *each question chosen*, in the space provided on the separate answer sheet, write the *numeral* preceding the expression that best completes the statement or answers the question.

1 If p represents the statement "Sue goes to Europe", and q represents the statement "She has won a trip around the world", select the symbolic statement equivalent to the statement "In order that Sue go to Europe, it is necessary that she win a trip around the world."

(1) $q \rightarrow p$ (3) $\sim p \rightarrow \sim q$
(2) $q \rightarrow \sim p$ (4) $p \rightarrow q$
(5) $q \leftrightarrow p$

2 Which is *not* a tautology?

(1) $[(p \vee q) \wedge \sim p] \rightarrow q$
(2) $(p \rightarrow q) \rightarrow (\sim q \rightarrow \sim p)$
(3) $(p \wedge q) \rightarrow (p \vee q)$
(4) $p \vee \sim p$
(5) $[(p \vee q) \wedge \sim q] \rightarrow \sim p$

3 The statement $\exists_x \forall_y\, [y + x = 3y]$ is a statement about real numbers. The negation of this statement is

(1) $\forall_x \exists_y\, [y + x \neq 3y]$
(2) $\exists_x \exists_y\, [y + x \neq 3y]$
(3) $\forall_x \forall_y\, [y + x \neq 3y]$
(4) $\forall_x \exists_y\, [y + x = 3y]$
(5) $\exists_x \exists_y\, [y + x = 3y]$

4 The inverse of $\sim p \rightarrow \sim q$ is equivalent to

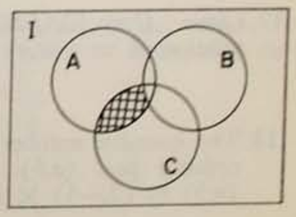
(1) $p \rightarrow \sim q$ (3) $q \rightarrow \sim p$
(2) $q \rightarrow p$ (4) $p \rightarrow q$
(5) $\sim q \rightarrow p$

5 If the functions f and g are defined specifically as $f(x) = x^2 - 2$ and $g(x) = 2x + 1$, then the composite function $f[g(x)]$ is

(1) $x^2 + 2x - 1$
(2) $4x^2 + 4x - 1$
(3) $4x^2 - 2$
(4) $2x^2 - 3$
(5) $4x^2 + 4x + 1$

6 In the Venn diagram, A, B and C are interiors of the circles lying within the rectangle I. The shaded area is represented by

(1) $A \cap (B \cap C)'$
(2) $(A \cup C) \cap B'$
(3) $(A \cap C) \cap B$
(4) $(B \cap C) \cap A'$
(5) $(A \cap C) \cap B'$



7 $(A' \cup B) \cap B =$

(1) $B \cap A'$
(2) $(A \cap B) \cup A'$
(3) $(A' \cap B) \cup A$
(4) $(A \cap B') \cup (A \cap B)$
(5) $(A' \cap B) \cup B$

8 If x and y are elements in the set of real numbers, which is *not* a function?

(1) $f = \{(x,y) \mid y = x^2 + 1\}$
(2) $f = \{(x,y) \mid y = 9 - x^2\}$
(3) $f = \{(x,y) \mid y \geq x + 1\}$
(4) $f = \{(x,y) \mid y = 2x^3\}$
(5) $f = \{(x,y) \mid y = x^2 - |x|\}$

9 If the function f is defined as $f(x) = \dfrac{x^2 - 1}{x + 1}$, which statement concerning the function is correct?

(1) The function is defined for all values of x.
(2) The function is defined for all values of x other than -1.
(3) The function approaches the limit 0 as x approaches -1.
(4) The range of the function is the set of all real positive numbers.
(5) The domain of the function is the set of all real numbers.

10 A tangent is drawn at a relative maximum point on the graph of a polynomial function. Which statement concerning this tangent is correct?

(1) The slope of the tangent is decreasing.
(2) The slope of the tangent is increasing.
(3) The slope of the tangent has increased and then decreased.
(4) The slope of the tangent is defined.
(5) none of these

11 The intersection of a set of points described by the conditions $x^2 + y^2 = 9$ and $x^2 = 4y + 12$ is

(1) the empty or null set
(2) a set consisting of one point
(3) a set consisting of two points
(4) a set consisting of three points
(5) a set consisting of four points

12 Given: $U = \{2,4,6,8\}$. Graph the relation $A = \{(x,y) \mid y < x\}$

13 The complex number $a + bi$ is represented by the ordered pair (a,b). Find the value of (a,b) if $(a,b) = (2,-1) \times (3,4)$.

14 If $f(x) = 3x^2 - 2x + 5$, find $\lim\limits_{h \to 0} \dfrac{f(x + h) - f(x)}{h}$.

15 Express $3(\cos 150° + i \sin 150°)$ in $a + bi$ form.

16 A coin is tossed three times. Find the probability of the event represented by the composite statement $\sim p \wedge q$ if

p: exactly two heads show
q: at least two heads show

17 Find the coordinates of the point of inflection of the graph of the function $y = x^3 - 3x^2 - 3x + 2$.

18 Write an equation of a polynomial in x of lowest degree with real coefficients, two of whose roots are $2 - i$ and 2.

19 Write an equation of the line tangent to $y = x^2 - 6x + 4$ at the point $(1,-1)$.

20 Find $\log_2 8\sqrt{8}$.

21 Find the remainder when $x^4 - 3x^2 + 7$ is divided by $x + 1$.

22 Find the smallest positive integer which satisfies the congruence $3x + 4 \equiv x + 7 \pmod{7}$.

23 A pair of dice, one colored white and the other colored red, is thrown. The number on the top face of the white die is w, and the number on the top face of the red die is r. $A = \{(w,r) \mid w \geq 5\}$ and $B = \{(w,r) \mid r \leq 2\}$; what is the probability of an event belonging to $A \cap B$?

24 Solve the inequality $x^2 - x \leq 2$.

25 A pyramid is cut by a plane parallel to its base at a distance from the base equal to two-thirds the length of the altitude. The area of the base is 18. Find the area of the section determined by the pyramid and the cutting plane.

26 Express in polar form the root of $x^3 - 8 = 0$ which lies in the second quadrant.

27 The distance between two points in space, $P_1\,(x,-1,-1)$ and $P_2\,(3,-3,1)$ is 3. Find a value of x.

28 Find the coordinates of the foci of $x^2 + 2y^2 = 2$.

29 The focus of a parabola is the point $(0,2)$ and its directrix is the line $y = -2$. Write an equation of the parabola.

30 A collection of objects, group $G = \{p,q,r,s\}$ is given with the binary operation $*$ defined by the table.

*	p	q	r	s
p	p	q	r	s
q	q	p	s	r
r	r	s	p	q
s	s	r	q	p

Referring to the table, find the value of $(q * r) * (r * s)$.

Answers to the following questions should be placed on paper supplied by the school.

Part II

Answer five questions from this part.

31 Find to the *nearest tenth* the smallest positive real root of the equation $x^3 - 5x^2 + 6 = 0$. [10]

32 Find the equation of the circle which passes through the three points (1,1), (1,—1), (—2,1). [10]

33 Using the definition of the derivative, find the derivative of the function
$f(x) = (2x - 3) \times (x + 4)$. [10]

34 Assuming that n is a positive integer, prove by mathematical induction that

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}. \quad [10]$$

35 Graph the solution set of
$A = \{(x,y) \mid 4x^2 + 9y^2 < 36 \text{ and } y - 2x > 1\}$.
Indicate the solution set by shading and, in words, describe the boundaries included or not included. [10]

36 A coin marked head and tail and a fair tetrahedron whose faces are marked 1, 2, 3 and 4 are tossed and rolled in that order.

a Set up the sample space for this experiment. [2]

b What is the probability that the coin shows "head" and the tetrahedron shows less than 4 on the bottom face? [2]

c What is the probability that the tetrahedron shows 2 on the bottom face? [2]

d What is the probability that the coin shows "tail" and the bottom face of the tetrahedron does *not* show 1? [2]

e What is the probability that the coin shows "head" and the bottom face of the tetrahedron shows a 2 or a 4? [2]

37 The function $f(x) = \dfrac{1}{\sqrt{x+1}}$ is defined for $\{x \mid x$ is a real number and $x > -1\}$.

a What is the range of $f(x)$? [2]

b Write an expression for the inverse of $f(x)$. [3]

c Is the inverse of $f(x)$ a function? [1]

d Sketch the function $f(x)$ on a set of coordinate axes. [2]

e Sketch the inverse of the function on the same set of coordinate axes. [2]

38 A right circular cylinder with maximum volume is inscribed in a sphere with radius r. Show that the altitude of the cylinder $= \dfrac{2r}{\sqrt{3}}$. [10]

Tear Here

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June 1965

ANSWER SHEET

Pupil .... Teacher ....

School ....

Your answers for part I should be recorded on this answer sheet.

Part I

Answer 25 of the 30 questions in this part.

1......

2......

3......

4......

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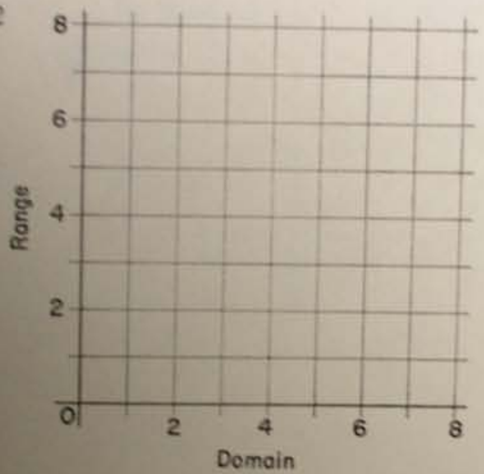
12



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Part I Score:....................
Rater's Initials:
....

Your answers to part II should be placed on paper supplied by the school.

Tear Here

385

FOR TEACHERS ONLY

SCORING KEY

12X

EXAMINATION IN EXPERIMENTAL TWELFTH YEAR MATHEMATICS

JUNE 1965

Part I

Allow 50 credits, 2 credits for each of 25 of the following:

(1) 4

(2) 5

(3) 1

(4) 4

(5) 2

(6) 5

(7) 5

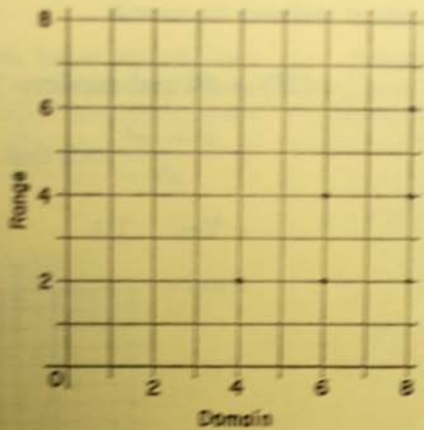
(8) 3

(9) 2

(10) 4

(11) 4

(12)



(13) (10,5)

(14) $6x - 2$

(15) $\dfrac{-3\sqrt{3}}{2} + \dfrac{3}{2}i$

(16) [illegible]

(17) $(1, -3)$

(18) $x^3 - 6x^2 + 13x - 10 = 0$

(19) $4x + y = 3$ or $y + 1 = -4(x - 1)$

(20) 1.5

(21) -2

(22) 5

(23) [illegible]

(24) $-1 \leq x \leq 2$

(25) 2

(26) $2(\cos 120° + i \sin 120°)$

(27) 2 or 4

(28) $(-1,0)$, $(1,0)$

(29) $x^2 = 8y$

(30) r

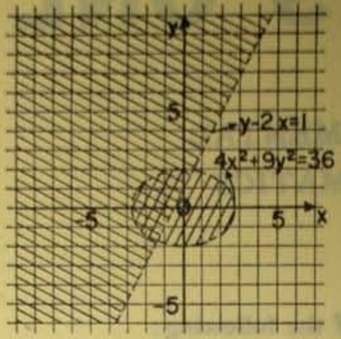
Part II

(31) 1.3 [10]

(32) $(x + \frac{1}{2})^2 + y^2 = \dfrac{13}{4}$ or $x^2 + y^2 + x - 3 = 0$ [10]

(33) $4x + 5$ [10]

(35)



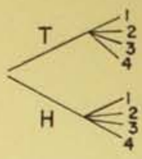
[10]

The cross-hatched shaded area is the solution set for

$$A = \{x,y \mid 4x^2 + 9y^2 < 36 \text{ and } y - 2x > 1\}.$$

The dash lines (---) of the ellipse and the straight line are the boundary lines and are *not* to be included in the solution set.

(36) *a*



[2]

b $\frac{3}{8}$ [2]

c $\frac{1}{4}$ [2]

d $\frac{3}{8}$ [2]

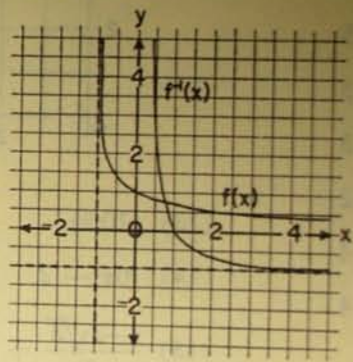
e $\frac{1}{4}$ [2]

(37) *a* All real numbers > 0 [2]

b $\dfrac{1 - x^2}{x^2}$ such that $x > 0$ [3]

c Yes [1]

d and *e*



[4]