

TWELFTH YEAR MATHEMATICS
12A (Advanced Algebra)

Thursday, June 21, 1962—9:15 a.m. to 12:15 p.m., only

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet.

1. If $f(x)$ is $x^{-3/2} - 2x^0$, find the value of $f(4)$.
2. Find the radius of the circle $x^2 - 2x + y^2 + 8y = 4$.
3. The repeating decimal $.7666 \dots$, in which the digit 6 repeats endlessly, is rational. Show that it is rational by writing it in the form $\frac{p}{q}$ where p and q are integers.
4. Find the coordinates of the point at which the graph of $y = 2^x$ intersects the y -axis.
5. In the equation $x^3 + px^2 - qx = 0$, p and q are real numbers. If one root is $2 - i$, find the value of p .
6. Write in simplest form the tenth term *only* of the expansion of $\left(2 - \frac{1}{x}\right)^{12}$.
7. Write in the form $r(\cos \theta + i \sin \theta)$ the root of $x^3 + 8 = 0$ which, when represented graphically, lies in quadrant I.
8. If $\frac{x^{13} + 17}{x + 1}$ is expressed in the form $Q(x) + \frac{R}{x + 1}$, where R is a constant find the value of R .
9. The graph of $y = x^2 - 8x + k$ is tangent to the x -axis. Find the value of k .
10. Find the slope of a straight line which is tangent to the graph of $y = 2x^2 - 7x + 5$ at $(-1, 14)$.
11. If $y = 3x^3 - 7x$, find the average rate of change of y with respect to x for the interval from $x = -1$ to $x = 1$.
12. What is the nonintegral rational root of the equation $2x^3 - 11x^2 + 8x + 7 = 0$?
13. Between what two consecutive integers does the smaller positive irrational root of $2x^3 - 11x^2 + 8x + 7 = 0$ lie?

14. The fourth term of an arithmetic progression is $3c$ and the ninth term is $-5c$. Find the first term.

15. If each signal is to consist of five flags arranged in sequence on a pole, how many different signals may be constructed from three identical red flags and two identical blue flags?

16. Four faces of a die are painted red and the remaining two faces are painted green. If the die is tossed once, what is the probability that it lands with a green face up?

17. If the distance from $(2, b)$ to $(-3, 5)$ is $\sqrt{74}$, find a value of b .

Directions (18-23): Indicate the correct completion for each of the following by writing the number 1, 2, 3 or 4 in the space provided on the separate answer sheet.

18. The sum of $-2 + 3i$ and $3 - 4i$ is a complex number which, when represented graphically, lies in quadrant (1) I (2) II (3) III (4) IV.

19. The equation of the axis of symmetry of the parabola $y = 2x^2 - kx + c$ is $x = 1$. The value of k is (1) -2 (2) 2 (3) -4 (4) 4 .

20. A general equation of the family of lines perpendicular to $2x - 5y = 7$ is (1) $5x + 2y = k$ (2) $5x - 2y = k$ (3) $2x + 5y = k$ (4) $2x - 5y = k$

21. The expression $xy = z$ represents (1) direct variation of x with respect to y when z is constant (2) inverse variation of x with respect to y when z is constant (3) inverse variation of x with respect to z when y is constant (4) inverse variation of z with respect to the product of x and y

22. A line $kx + y = 8$ is drawn parallel to the line $4x + 5y = 7$. The value of k is (1) $-\frac{4}{5}$ (2) $-\frac{5}{4}$ (3) $\frac{4}{5}$ (4) $\frac{5}{4}$.

23. If $y = -x^2 + 1$ and $9x^2 + 4y^2 = 36$ are graphed on the same axes, the total number of points of intersection is (1) 1 (2) 2 (3) 3 (4) 4.

24. On the diagram on the answer sheet, darken the portion of the real axis which is the graph of the nonnegative solution of the inequality $3x - 4 \leq 2$.

Part II

*Answer sixteen questions from this part, 25-48. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answers on the separate answer sheet.*

25. Express the number $\frac{6 + i\sqrt{3}}{2 - i\sqrt{3}}$ as an equivalent fraction with a real denominator.

26. From a mixture of vinegar and water which is 20% vinegar, 2 quarts are removed and replaced by pure vinegar. The resulting mixture is 30% vinegar. Find the number of quarts in the original mixture.

27. A point moves on a straight line through a fixed point P so that the equation $s = 2t^2 - 7t + 4$ represents its distance s from P as a function of the time t . Find the velocity of the point when $t = 3$.

28. A root of $f(x) = x^3 - 7x^2 + 2x - 3 = 0$ is between 6.7 and 6.8; $f(6.7)$ is -3.067 and $f(6.8)$ is 1.352 . Find this root to the nearest tenth.

29. Find to the nearest tenth $\log_3 8$.

30. Write $(\sqrt{3} + i)^3$ in the form $r(\cos \theta + i \sin \theta)$.

31. Six men are candidates in an election in which two are to be chosen. If a voter is instructed to mark his ballot either for one candidate or for two candidates, in how many ways can he carry out these instructions?

32. Express as a single fraction in simplest form: $2 - \frac{2 - a}{2 - \frac{a}{2}}$.

*33. The area of a certain triangle is 12. If the area expressed in determinant form is

$$\frac{1}{2} \begin{vmatrix} a & 3 & 1 \\ -1 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix} \text{ find the value of } a.$$

Directions (34-36): For each of those chosen, write in the space provided on the separate answer sheet, the number preceding the expression that best completes the statement.

34. If $f(x) = -x^2 + 7x$, then $f\left(\frac{1}{x}\right)$ is

(1) $\frac{7x+1}{x^2}$ (2) $\frac{7x-1}{x^2}$ (3) $\frac{7-x^3}{x}$ (4) $\frac{7+x^3}{x}$

35. If $\log x^2 - \log 2a = \log 5a$, then the value of $\log x$ in terms of a is

(1) $\frac{2 \log a + 1}{2}$ (2) $\frac{2 \log a + 7}{2}$ (3) $\frac{1}{2}$ (4) $\frac{7 \log a}{2}$

36. The solution set of $2x^2 - x - 3 < 0$ is the set of all real numbers x such that

(1) $x > \frac{3}{2}$ or $x < -1$ (2) $x < \frac{3}{2}$ or $x < -1$
 (3) $-\frac{3}{2} < x < 1$ (4) $-1 < x < \frac{3}{2}$

37. A line whose equation is $\frac{x}{3} - \frac{y}{7} = 1$ intersects the axes at $(a, 0)$ and $(0, b)$. Find the value of b .

38. Solve $8^{2x} = (1/16)^{x-1}$ for x .
39. The sum of an infinite geometric progression is $2\sqrt{2} + 2$. If the common ratio is $\frac{1}{\sqrt{2}}$, find the first term.
40. If the only roots of $x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_n = 0$ are $3 - i$, $3 + i$ and -4 , find the value of a_2 .
41. If $x^{2.5} = 94.3$, find x to the nearest tenth.
42. The sum of an arithmetic progression of 8 terms is S , the first term is a and the eighth term is $\frac{a}{2}$. Express a as a function of S .
43. Ten balls numbered 1 to 10 are placed in a bag, and two of the balls are drawn at random. What is the probability that balls numbered 3 and 7 are drawn?
- *44. Transform $2x^2 + 5y^2 = 1$ into an equation in polar coordinates.
45. Using k as the constant of variation, write an equation which represents the following relationship: x varies directly as the square of y and inversely as z .

Directions (46-48): For each of those chosen, write in the space provided on the separate answer sheet, the number preceding the expression that best completes the statement.

- *46. The graph of $r = 2 \cos \theta$ is a (1) straight line parallel to the y -axis (2) circle passing through the pole (3) straight line passing through the pole (4) circle with center at the pole
47. The roots of $x^4 + 3x^2 - 28 = 0$ are (1) $\sqrt{7}$, $-\sqrt{7}$, 2, -2 (2) 7, 7, 2, 2 (3) $+7i$, $-7i$, 2, -2 (4) $+i\sqrt{7}$, $-i\sqrt{7}$, 2, -2.
48. The equation $\sqrt{2+x} - x = 0$ has (1) no root (2) 2 as its only root (3) -1 as its only root (4) the two roots 2 and -1