The University of the State of New York

Regents High School Examination

TWELFTH YEAR MATHEMATICS

12B (Solid Geometry)

Monday, June 19, 1961 — 1:15 to 4:15 p.m., only

Name of pupil..... Name of school.....

Name and author of textbook used.....

Name of teacher.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 The volume of a sphere is 36π cubic inches. Find the number of inches in the radius of the sphere. 1.....

2 The lateral area of a certain cylinder of revolution is $2\pi r^2$ where r is the radius of the base. What is the ratio of the height of the cylinder to the diameter of its base? 2.....

3 Two similar pyramids have heights of 2 inches and 6 inches, respectively. If the volume of the first is 10 cubic inches, find the number of cubic inches in the volume of the second pyramid. 3.....

4 Triangle RST is inscribed in circle O whose diameter RT is 24. Line segment OQ is perpendicular to the plane of the circle. If OQ is 5, find QS. 4.....

5 Each lateral face of a regular square pyramid makes a 60° angle with the base. If the base edge is represented by m, express the total area of the pyramid in terms of m. 5.....

6 A rectangular solid has dimensions 3, 5 and 6. What is the length of a diagonal of the solid? 6.....

7 The lateral edge of a frustum of a regular triangular pyramid is 10 inches. If an upper base edge is 5 inches and a lower base edge is 17 inches, find the number of square inches in the lateral area of the frustum. 7.....

8 What is the locus of points which are equidistant from the faces of a trihedral angle? 8.....

[OVER]

9 The radius of a sphere is 12 inches and the radius of a small circle on this sphere is 8 inches. Find the distance in inches from the center of the sphere to the plane of the small circle. 9.....

10 Find the total area of a right circular cylinder circumscribed about a sphere whose radius is 5. 10.....

11 The angles of a spherical triangle are 50°, 125° and 104°. Find the number of degrees in the perimeter of its polar triangle. 11.....

12 The altitude of a zone is equal to $\frac{1}{4}$ the radius, r, of the sphere on which it is drawn. Express in terms of r the area of the zone. 12.....

13 Corresponding edges of two similar solids are in the ratio 3 : 5. If the lateral area of the smaller solid is 72, find the lateral area of the larger solid. 13.....

14 A pyramid and a prism have equal altitudes. The area of the base of the pyramid is 4 times the area of the base of the prism. What is the numerical value of the ratio of the volume of the pyramid to the volume of the prism? 14.....

15 Point P is equidistant from the faces of a 48° dihedral angle and 6 inches from the edge of the angle. Find to the *nearest tenth* the number of inches from P to either face. 15.....

16 Find in spherical degrees the area of an equilateral spherical triangle, each of whose angles is 70°. 16.....

17 The slant height l of a certain right circular cone is equal to the diameter of the base. A sphere is inscribed in the cone. Express in terms of l the radius of this sphere. 17.....

Directions (18–26) : Write on the line at the right of *each* of the following the ***number*** preceding the expression that best completes the statement.

18 Two face angles of a trihedral angle are 125° and 65°. The third face angle may be

(1) 170°
(2) 60°
(3) 190°
(4) 90°

18.....

19 If two planes R and S make equal angles with a third plane, then R and S

(1) are always perpendicular to the third plane
(2) are always parallel to each other
(3) may intersect each other
(4) must intersect each other

19.....

20 The total number of lines which can be drawn obliquely to a plane from an external point so as to make equal angles with the plane is

(1) infinite
(2) 2
(3) 3
(4) 4

20.....

21 Plane M and plane N are parallel to line AB. If M intersects N in QR, then QR and AB
(1) are parallel
(2) may intersect
(3) are skew
(4) may coincide
21.....

22 It is *not* true that the diagonals of a cube
(1) are equal
(2) are perpendicular
(3) bisect each other
(4) intersect in a point which is equidistant from the faces
22.....

23 If a regular polyhedron has 12 edges and 6 vertices, the number of faces is
(1) 12
(2) 8
(3) 6
(4) 4
23.....

24 If a line intersects a plane and is *not* perpendicular to the plane, then this line is perpendicular to
(1) only one line in the given plane
(2) two lines in the given plane
(3) every line in the given plane
(4) no line in the given plane
24.....

25 The projection of a straight line segment upon a plane can *not* be a straight line segment when the given segment
(1) intersects the plane at a 45° angle
(2) is oblique to the plane but does not intersect the plane
(3) is perpendicular to the plane but does not intersect the plane
(4) is parallel to the plane
25.....

26 The total number of lines equidistant from two intersecting planes and also at a given distance from the line of intersection of the planes is
(1) 1
(2) 2
(3) infinite
(4) 4
26.....

Directions (27–30) : If the blank space in each statement below is replaced by the word *always*, *sometimes* (but not always) or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

27 Two spherical triangles which are symmetric to a third triangle are ... congruent to each other.
27.....

28 If two great circles are drawn on the same sphere, the plane of the first great circle is ... parallel to the plane of the second.
28.....

29 A section of an oblique prism made by a plane parallel to a lateral edge of the prism is ... a rectangle.
29.....

30 The perimeter of a spherical polygon is ... equal to 360°.
30.....

[OVER]

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove *either* *a* or *b*:

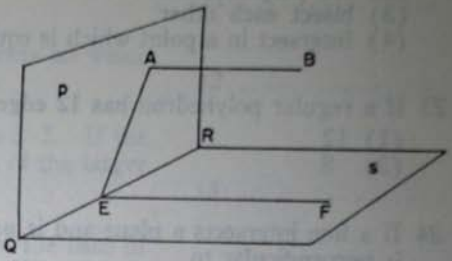
a Two lines perpendicular to the same plane are parallel. [10]

OR

b In two polar triangles each angle of one has the same measure as the supplement of the side lying opposite it in the other. [10]

32 In the accompanying diagram plane p is perpendicular to plane s, intersecting s in line QR. Line EF (in plane s) is perpendicular to QR at E. Line AB is drawn parallel to EF through any point A in p not on QR. Line AE is drawn.

Prove: Angle BAE is a right angle. [10]



33 Given plane m and point P at distance d from m. Describe fully the locus of points

a at a distance f from P [2]

b at a distance g from m [2]

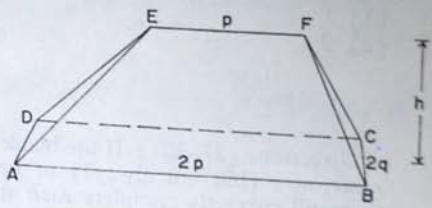
c that satisfy both conditions in *a* and *b* if

(1) $f = d$ and $g = 2d$ [3]

(2) $f = 2d$ and $g < d$ [3]

34 Each leg of an isosceles triangle is $3m$ units in length and the base is $2m$ units. A line s is drawn through the vertex of the triangle parallel to the base. The triangle is revolved through 360° about line s with the base of the triangle always remaining parallel to s. Find in terms of m the volume of the resulting solid. [10]

35 The accompanying figure represents one type of prismatoid with altitude equal to h. Line EF is parallel to side AB of rectangle $ABCD$. Let $EF = p$, $AB = 2p$ and $BC = 2q$. Using the prismatoid formula, find the volume of the solid in terms of h, p and q. [10]



*36 Answer *either* *a* or *b*:

a In spherical triangle ABC, angle $C = 90°$, side $b = 28°$ and side $c = 102°$. Find side a to the *nearest degree*. [10]

OR

b Points A $(-2, 1, 4)$ and B $(3, -3, 6)$ are located with reference to three mutually perpendicular axes: x, y and z.

(1) Write an equation of the xy-plane. [2]

(2) Write an equation of the plane through point A parallel to the yz-plane. [2]

(3) Find the length of line segment AB. [3]

(4) Find the coordinates of the midpoint of line segment AB. [3]

* This question is based on optional topics in the syllabus.

FOR TEACHERS ONLY

12B

INSTRUCTIONS FOR RATING TWELFTH YEAR MATHEMATICS

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 18–26, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) 3

(2) 1:2

(3) 270

(4) 13

(5) $3m^2$

(6) $\sqrt{70}$ *or* 8.4

(7) 264

(8) straight line

(9) $4\sqrt{5}$ *or* 8.9

(10) 150π

(11) 261

(12) $\frac{\pi r^2}{2}$

(13) 200

(14) $\frac{4}{3}$

(15) 2.4

(16) 30

(17) $\frac{l\sqrt{3}}{6}$

(18) 4

(19) 3

(20) 1

(21) 1

(22) 2

(23) 2

(24) 1

(25) 3

(26) 4

(27) always

(28) never

(29) sometimes

(30) never

[OVER]

Please refer to the Department's pamphlet Suggestions on the Rating of Regents Examination Papers in Mathematics. Care should be exercised in making deductions as to whether the error is purely mechanical or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent depending on the relative importance of the principle in the solution of the problem.

Part II

(33) There are many ways of describing these loci. Each description should include shape and position. For instance, phrases such as the following should be allowed credit as indicated:

a a spherical surface with center at P and radius f [2]

b two planes parallel to m, one on either side and at a distance g from m [2]

c (1) the point of tangency between the spherical surface described in *a* and one of the planes described in *b* [3]

(2) two circles, the intersections formed by the spherical surface described in *a* and the two planes described in *b* [3]

(34) $\frac{32}{3}\pi m^3$ [10]

(35) $\frac{5}{3}hpq$ [10]

(36) *a* 104° [10] [Note: 3 credits should be deducted if 76° is given as the answer.]

b (1) $z = 0$ [2]

(2) $x = -2$ *or* $x + 2 = 0$ [2]

(3) $3\sqrt{5}$ [3]

(4) $(\frac{1}{2}, -1, 5)$ [3]