

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION
TWELFTH YEAR MATHEMATICS
12B (Solid Geometry)

Friday, June 17, 1960 — 1:15 to 4:15 p.m., only

Name of pupil..... Name of school.....

Name and author of textbook used.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

- 1 The lateral area of a prism is 90. If a right section is a regular pentagon whose side is 3, find a lateral edge. 1.....
- 2 Two edges of a rectangular solid are 7 and 9 and its diagonal is 12. Find the third edge. 2.....
- 3 The altitude of a regular square pyramid is 3 times a base edge. If the volume of the pyramid is 1 cubic inch, find the number of inches in the altitude. 3.....
- 4 The area of the base of a pyramid is 80, the altitude of the pyramid is 12 and the area of a section parallel to the base is 45. Find the distance of the section from the vertex. 4.....
- 5 A base edge of a regular square pyramid is 8 and the altitude is 3. Find the lateral area. 5.....
- 6 The radius of the base of a right circular cylinder is 4. If the lateral area is equal to the sum of the areas of the two bases, find the altitude. 6.....
- 7 An equilateral triangle is revolved through 180° about an altitude as an axis. If a side of the triangle is s , find the lateral area of the cone formed in terms of s . 7.....
- 8 The lateral area of a frustum of a regular triangular pyramid is 84. If the base edges are 4 and 3, find the slant height. 8.....

- 9 The lateral areas of two similar cones are in the ratio of 4:9. Find the ratio of the volume of the smaller cone to the volume of the larger. 9.....
- 10 An equilateral spherical triangle is equal to a lune whose angle is 30° . Find the number of degrees in one angle of the triangle. 10.....
- 11 The number of square units in the area of a sphere is equal to the number of cubic units in its volume. Find the radius of the sphere. 11.....
- 12 On a sphere whose radius is 10 inches, the plane of a small circle is 8 inches from the center. Find the number of inches in the radius of the small circle. 12.....
- 13 The number of degrees in each angle of an equilateral spherical triangle is a . Find the number of degrees in the perimeter of the polar triangle in terms of a . 13.....
- 14 If the altitude of a right circular cone is 10 and the radius of the base is 4, find to the *nearest degree* the angle that an element makes with the base. 14.....
- 15 A line segment 12 inches long is inclined at an angle of 60° to a plane. Find in inches the length of its projection on the plane. 15.....
- 16 The altitude of a prism is $\sqrt{3}$ and its base is an equilateral triangle. If the volume of the prism is 48, find a base edge. 16.....
- 17 The altitude of a circular cylinder is twice the diameter of the base. Express the volume of the cylinder in terms of d , the diameter of the base. 17.....

Directions (18-24): Indicate the correct completion for *each* of the following by writing on the line at the right the *number* 1, 2, 3 or 4.

- 18 The radii of the bases of a frustum of a right circular cone are 7 and 2. The slant height may be (1) 5 (2) 2 (3) 7 (4) 4 18.....
- 19 Two face angles of a trihedral angle are 100° and 140° . The third face angle may be (1) 20° (2) 40° (3) 100° (4) 120° 19.....
- 20 The Northern Hemisphere is divided into two equal zones by the parallel of latitude (1) 30° N (2) 45° N (3) 50° N (4) 60° N 20.....

- 21 The locus of points at a given distance from a given line consists of
(1) a cylindrical surface (2) two parallel lines (3) a spherical
surface (4) two parallel planes 21.....
- 22 The locus of points equally distant from two intersecting planes and also
at a given distance from a point on their line of intersection is (1) one
circle (2) two circles (3) two points (4) four points 22.....
- 23 The projection of a circle on a plane can *never* be (1) a line segment
(2) a circle (3) an ellipse (4) a parabola 23.....
- 24 Two planes are always parallel if they are (1) parallel to the same line
(2) tangent to the same sphere (3) perpendicular to the same plane
(4) perpendicular to the same line 24.....
- Directions (25–30):* If the blank space in each statement below is replaced by the word
always, sometimes (but not always) or *never*, the resulting statement will be true. Select the
word that will correctly complete *each* statement and write this word on the line at the right.
- 25 If two small circles of a sphere have the same poles, their planes are ...
parallel. 25.....
- 26 If two sides of a spherical triangle are each 135° , the third side is ...
obtuse. 26.....
- 27 Plane P intersects plane M in line a and plane N in line b . If a is parallel
to b , then M is ... parallel to N . 27.....
- 28 An exterior angle of a spherical triangle is ... equal to the sum of the
two remote interior angles. 28.....
- 29 The diagonals of a parallelepiped ... bisect each other. 29.....
- 30 Two lines that are skew to a third line are ... skew to each other. 30.....

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove either *a* or *b* :

a If two lines are parallel, every plane containing one of these lines, and only one, is parallel to the other. [10]

OR

b A spherical angle is measured by the arc of the great circle described from its vertex as a pole and included between its sides produced if necessary. [10]

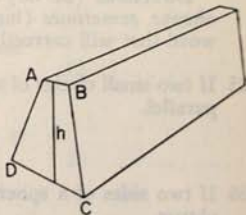
32 Plane *M* and line *a* outside plane *M* are both perpendicular to plane *P*. Prove that line *a* is parallel to plane *M*. [10]

33 The number of degrees in the angles of a spherical triangle are in the ratio of 3 : 4 : 5. The area of the triangle is equal to the area of a zone of altitude 3 on the same sphere. If the radius of the sphere is 15, find each angle of the triangle. [10]

34 An edge of a regular tetrahedron is *s*. Show that the volume of the inscribed cone is

$$\frac{\pi s^3 \sqrt{6}}{108}. \quad [10]$$

35 A wooden form for a small dam is in the shape of a right prism whose base is an isosceles trapezoid as shown in the adjacent figure. *AB* is 3 feet, *CD* is 5 feet, *h* is 8 feet and the dam is 44 feet long. The form has been filled with concrete to a depth of 2 feet. Find, to the nearest cubic yard, the additional number of cubic yards of concrete needed to fill the form. [10]



*36 Answer either *a* or *b* :

a A pyramid *V-ABCD*, shown in the adjacent figure, has the vertices *V* (2, 1, 6), *A* (0, 0, 0), *B* (4, 0, 0), *C* (5, 4, 0) and *D* (0, 3, 0).

(1) Write an equation of the plane through *V* that is parallel to the base *ABCD*. [2]

(2) Write an equation of the plane that passes through *B* and *D* and is parallel to the *z*-axis. [3]

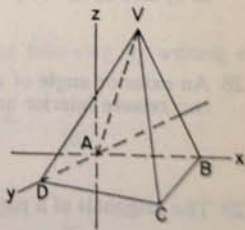
(3) Find the coordinates of the midpoint of lateral edge *VC*. [2]

(4) Find the length of lateral edge *VC*. [3]

OR

b (1) In spherical triangle *ABC*, angle *A* = 101°, angle *B* = 58° and angle *C* = 90°. Find side *a* to the nearest degree. [8]

(2) Using the given data, write an equation that could be used to find side *c*. [2]



* These questions are based on optional topics in the syllabus.

FOR TEACHERS ONLY

INSTRUCTIONS FOR RATING TWELFTH YEAR MATHEMATICS 12B (Solid Geometry)

12B

Friday, June 17, 1960—1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 18–24, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

- | | | |
|-------------------------|--------------------------|----------------|
| (1) 6 | (9) 8:27 | (21) 1 |
| (2) $\sqrt{14}$ or 3.7 | (10) 80 | (22) 2 |
| (3) 3 | (11) 3 | (23) 4 |
| (4) 9 | (12) 6 | (24) 4 |
| (5) 80 | (13) $540 - 3a$ | (25) always |
| (6) 4 | (14) 68 | (26) never |
| (7) $\frac{\pi s^2}{2}$ | (15) 6 | (27) sometimes |
| (8) 8 | (16) 8 | (28) never |
| | (17) $\frac{\pi d^3}{2}$ | (29) always |
| | (18) 3 | (30) sometimes |
| | (19) 3 | |
| | (20) 1 | |

[OVER]

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely mechanical or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent depending on the relative importance of the principle in the solution of the problem.

Part II

33 $63^\circ, 84^\circ, 105^\circ$ [10]

35 37 [10]

36 a (1) $s = 6$ [2]

(2) $3x + 4y = 12$ [3]

(3) $(3\frac{1}{2}, 2\frac{1}{2}, 3)$ [2]

(4) $\sqrt{54}$ or $3\sqrt{6}$ [3]

b (1) 103° [8]

(2) $\cos c = \cot A \cot B$ [2]