Reviewed By: .....

Percent: .....

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TWELFTH YEAR MATHEMATICS

12A (Advanced Algebra)

Wednesday, June 22, 1960 — 9:15 a.m. to 12:15 p.m., only

Name of pupil..... Name of school.....

Part I

Answer all questions in this part. Questions 1–10 count 1 credit each. Questions 11–30 count 2 credits each. No partial credit will be allowed. Write the answer to each question on the line at the right.

1–3 Questions 1–3 refer to the graph of the equation $2x + 4y + 5 = 0$.

1 Find the slope of the line. 1.....

2 Write an equation of the straight line parallel to the given line and passing through the origin. 2.....

3 Find the x-intercept of the given line. 3.....

4–6 Questions 4–6 refer to the equation $x^3 + 10x + 2 = 0$.

4 Find the sum of the roots of the equation. 4.....

5 Find the product of the roots of the equation. 5.....

6 How many rational roots does the equation have? 6.....

Directions (7–10): Indicate whether *each* of the following statements is true for
(1) all real values of x
(2) one or more, but not all, real values of x
(3) no real value of x
by writing on the line at the right the *number* 1, 2 or 3.

7 $\sqrt{x^2 + 9} = x + 3$ 7.....

8 $x^2 + 1 = 0$ 8.....

9 $(x - 1)^3 = x^3 - 3x^2 + 3x - 1$ 9.....

10 $x^2 - 6x + 9 < 0$ 10.....

[OVER]

11 Express in the form $a + bi$ the reciprocal of $2 + i$. 11.....

12 Find the numerical value of a if $x + a$ is a factor of $x^5 + 32$. 12.....

13 John travels from A to B, a distance of 30 miles, at the rate of 6 miles per hour, and then without stopping returns from B to A. What should his return rate be in miles per hour, in order that the average rate for the entire trip be 5 miles per hour? 13.....

14 If $f(x) = (2x)^0 + x^{-\frac{3}{2}}$, find the value of $f(64)$. 14.....

15 There are ten people at a conference. How many different committees of three members each can be formed from these ten people? 15.....

16 A three-digit number is to be formed using the digits from 1 to 9, inclusive. What is the probability that the number will be odd? [Repetitions of digits are permitted.] 16.....

17 Three cards (ace, king, jack) are face down on a table. If two of these cards are picked at random, what is the probability that one of them is an ace? 17.....

18 Write in *simplest form* the fourth term *only* of $(1 + i)^6$, where $i = \sqrt{-1}$. 18.....

19 The distance that a body falls from rest in t seconds is given by the formula $S = \frac{1}{2}gt^2$, and the final velocity is given by the formula $V = gt$. Express V in terms of g and S. 19.....

20 Express in *simplest form* $\dfrac{1}{1 + \dfrac{1}{1 + x}}$. 20.....

21 If $\frac{1}{2}$, $\frac{1}{x}$ and $\frac{1}{3}$ are three consecutive terms of an arithmetic progression, find the value of x. 21.....

22 Find the coordinates of the minimum point of the graph of the equation $y = x^2 - 6x + 9$. 22.....

Directions (23–30): Indicate the correct completion for *each* of the following by writing on the line at the right the *number* 1, 2, 3 or 4.

23 Ten quarts of a solution containing $x\%$ antifreeze is mixed with twenty quarts of a solution containing $y\%$ antifreeze. The fractional part of antifreeze in the resulting mixture is

(1) $\frac{x + 2y}{100}$ (2) $\frac{x + 2y}{300}$ (3) $\frac{2x + y}{100}$ (4) $\frac{2x + y}{300}$ 23.....

24 If $\log_{10} x = 1.5421$, then $10^{3.5421}$ equals (1) $2 + x$ (2) $2x$ (3) $100 + x$ (4) $100x$ 24.....

25 If $f(x)$ is divided by $x - 2$, the remainder is (1) $f(2)$ (2) 2 (3) $f(-2)$ (4) -2 25.....

26 If ${}_nC_x = {}_nC_y$, where n, x and y are positive integers such that $x \neq y$, then (1) $x = \frac{n}{2}$ (2) $y = \frac{n}{2}$ (3) $x + y = n$ (4) $x - y = n$ 26.....

27 The area of a circle varies directly as the square of its diameter. The constant of variation is (1) 1 (2) π (3) $\frac{\pi}{4}$ (4) $\frac{1}{4}$ 27.....

28 If r_1 and r_2 are real roots of the quadratic equation $x^2 + px + q = 0$ such that $r_1 > 0$, $r_2 < 0$ and p and q are integers, it is always true that (1) $q > 0$ (2) $q < 0$ (3) $p > 0$ (4) $p < 0$ 28.....

29 If $4^x = 8^y$, then x equals (1) $\frac{1}{2}y$ (2) $2y$ (3) $\frac{3}{2}y$ (4) $\frac{2}{3}y$ 29.....

30 In the equation $px^2 + qx + s = 0$, p, q and s are real numbers with $p \neq 0$. If the two roots of the equation are equal, then (1) $q^2 = 4ps$ (2) $q^2 = -4ps$ (3) $q^2 = ps$ (4) $q^2 = -ps$ 30.....

Part II

Answer ten questions from this part. Each correct answer will receive 2½ credts. No partial credit will be allowed. The question marked * is based upon an optional topic in the syllabus. Write your answer on the line at the right.

31 If two of the roots of $x^3 + px + q = 0$ are 3 and -1, find the third root. 31.....

32 If one of the roots of $x^3 - 2x^2 + x - 2 = 0$ is 2, find the other two roots. 32.....

33 The x-intercepts of the graph of the equation $y = x^2 + bx + c$ are 2 and 3. Find the value of c. 33.....

34 The points $P_1(2, 3)$, $P_2(4, 9)$, $P_3(6, k)$ are collinear. Find the value of k. 34.....

Directions (35–37): Indicate the correct completion for *each* of the following by writing on the line at the right the *number* 1, 2, 3 or 4.

35 A possible root of the equation $6x^4 + px^3 + qx^2 + rx + 4 = 0$ where p, q and r are integers is (1) $\frac{3}{2}$ (2) $-\frac{3}{2}$ (3) -3 (4) $\frac{4}{3}$ 35.....

36 If a and b are real numbers, then the product of $a + bi$ and $a - bi$ is (1) always a real number (2) sometimes, but not always, a real number (3) always imaginary (4) sometimes, but not always, imaginary 36.....

37 A rational integral function of x is (1) $x + \frac{1}{x}$ (2) $\sqrt{x} + 2$ (3) $x^2 + x^{\frac{1}{2}}$ (4) $x + \sqrt{2}$ 37.....

38 The circle whose center is $(3, -2)$ passes through the point $(5, 1)$. Find the length of the radius of the circle. 38.....

39 The first term of an arithmetic progression is x and the common difference is 2. The first, third and seventh terms form a geometric progression. Write an equation that could be used to find the value of the first term. 39.....

40 In how many ways may three pupils be seated in a row containing 5 seats? 40.....

41 Find the slope of the line tangent to the curve whose equation is $y = x^3 - 5x + 2$, at the point where the graph crosses the y-axis. 41.....

42 Find the coordinates of the point of inflection of the curve whose equation is $y = x^3 - 5x + 2$. 42.....

43 The area of a rectangle is represented by $12x - x^2$ where x is a side of the rectangle. For what value of x will the area be a maximum? 43.....

44 Find the set of values of x that satisfies the inequality $4 - 2x < 10$. 44.....

*45 Write in determinant form an equation of the straight line through the points $(3, 2)$ and $(-1, 0)$. 45.....

Part III

Answer ten questions from this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answer on the line at the right.

46 Find to the *nearest tenth* the value of $\log_2 5$. 46.....

47 Given $A = Pe^r$. Express r in terms of $\log A$, $\log P$ and $\log e$. 47.....

Directions (48–53): Indicate the correct completion for *each* of the following by writing on the line at the right the *number* 1, 2, 3 or 4.

48 The positive root of the equation $x^3 + 5x - 7 = 0$ lies between
(1) 1.0 and 1.2 (2) 1.2 and 1.4 (3) 1.4 and 1.6 (4) 1.6 and 1.8 48.....

49 The graph of $y = 3^x$ (1)intersects the x-axis only (2)intersects the y-axis only (3)intersects both coordinate axes (4)does not intersect either axis 49.....

50 If r is a positive real number and n is a postive integer, then $r^{-\frac{1}{n}}$ is equal to
(1) $\frac{1}{r^n}$ (2) $\frac{1}{\sqrt[n]{r}}$ (3) $\sqrt[n]{r}$ (4) r^n 50.....

51 If the relation, x varies inversely as y, is represented graphically, the graph will be (1)a straight line (2)an ellipse (3)a hyperbola (4)a parabola 51.....

52 If in the equation $y = 3^x$, the variable x is increased by 2, then y is (1)increased by 2 (2)multiplied by 2 (3)increased by 9 (4)multiplied by 9 52.....

53 If the roots of the equation $x^3 + x + 1 = 0$ are expressed in the form $a + bi$, then b is equal to (1) $\pm \frac{1}{2}$ (2) $\pm \frac{3}{2}$ (3) $\pm \frac{\sqrt{3}}{2}$ (4) $\pm \frac{\sqrt{3}}{4}$ 53.....

54 The area of a rectangle is represented by A, the diagonal by d and one side by s. Express d in terms of A and s. 54.....

55 In the equation $x^2 + ax + b = 0$, one root is twice the other. Express b in terms of a. 55.....

56 Express in the form $a + bi$: $2(\cos 120^\circ + i \sin 120^\circ)$ 56.....

57 Express in polar form: $-3i$ 57.....

58 Find the amplitude of the complex number $[1\ (\cos 40° + i \sin 40°)]^{\frac{1}{3}}$ which, when represented graphically, lies in the third quadrant. 58.....

*59 The polar coordinates of a point P are $\left(2, \frac{\pi}{3}\right)$. If $\left(x, \frac{4\pi}{3}\right)$ are the coordinates of the same point, find the value of x. 59.....

*60 The equation of a circle in polar form is $r = 6 \sin \theta$. Write an equation of this circle in rectangular form. 60.....

I have regularly studied twelfth year mathematics 12A (advanced algebra) for .... weeks and have had .... recitations per week under .....

Name of Teacher

at.....

School

I do so declare.....

(*Signature*)

FOR TEACHERS ONLY

12A

INSTRUCTIONS FOR RATING
TWELFTH YEAR MATHEMATICS
12A (Advanced Algebra)

Wednesday, June 22, 1960 — 9:15 a.m. to 12:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Questions 1–10 count 1 credit each. Questions 11–30 count 2 credits each. **No partial credit** will be allowed. For questions 7–10 and 23–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $-\frac{1}{2}$

(2) $x + 2y = 0$

(3) $-\frac{5}{2}$

(4) 0

(5) -2

(6) none

(7) 2

(8) 3

(9) 1

(10) 3

(11) $\frac{2}{5} - \frac{1}{5}i$

(12) 2

(13) $4\frac{3}{7}$

(14) $1\frac{1}{16}$

(15) 120

(16) $\frac{5}{9}$

(17) $\frac{2}{3}$

(18) $-20\sqrt{-1}$ *or* $-20i$

(19) $g\sqrt{\frac{2S}{g}}$ *or* $\sqrt{2Sg}$

(20) $\frac{1+x}{2+x}$

(21) $2\frac{1}{8}$

(22) (3, 0)

(23) 2

(24) 4

(25) 1

(26) 3

(27) 3

(28) 2

(29) 3

(30) 1

[OVER]

Part II

Allow 2½ credits for each of not more than 10 correct answers; allow no partial credit. If more than ten questions have been answered, only the first ten of these should be considered. For questions 35–37, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(31) -2

(32) $+i$ and $-i$

(33) 6

(34) 15

(35) 4

(36) 1

(37) 4

(38) $\sqrt{13}$

(39) $\dfrac{x}{x+4} = \dfrac{x+4}{x+12}$

(40) 60

(41) -5

(42) (0, 2)

(43) 6

(44) $x > -3$

(45) $\begin{vmatrix} x & y & 1 \\ 3 & 2 & 1 \\ -1 & 0 & 1 \end{vmatrix} = 0$

Part III

Allow 2½ credits for each of not more than 10 correct answers; allow no partial credit. If more than ten questions have been answered, only the first ten of these should be considered. For questions 48–53, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(46) 2.3

(47) $\dfrac{\log A - \log P}{\log e}$

(48) 1

(49) 2

(50) 2

(51) 3

(52) 4

(53) 3

(54) $\dfrac{\sqrt{A^4 + s^4}}{s}$

(55) $\dfrac{2a^3}{9}$

(56) $-1 + \sqrt{3}\,i$

(57) $3\,(\cos 270° + i \sin 270°)$

(58) 200°

(59) -2

(60) $x^2 + y^2 - 6y = 0$ *or* $x^2 + (y-3)^2 = 9$