

The University of the State of New York

Regents High School Examination

TWELFTH YEAR MATHEMATICS

12A

12A (Advanced Algebra)

Thursday, June 18, 1959—9:15 a.m. to 12:15 p.m., only

Part I

Answer all questions in this part. Questions 1-10 count 1 credit each. Questions 11-30 count 2 credits each. No partial credit will be allowed. Write the answer to each question on the line at the right.

Directions (1-10): Indicate the correct completion for each of the following by writing the letter, a , b , c or d on the line at the right.

- 1-5 Questions 1-5 refer to the graph of the equation $y = 2x - 2$.
- The slope of this line is (a) -2 (b) -1 (c) 1 (d) 2 1....
 - The x -intercept is (a) -2 (b) -1 (c) 1 (d) 2 2....
 - The y -intercept is (a) -2 (b) -1 (c) 1 (d) 2 3....
 - The graph of this line does *not* pass through quadrant (a) I (b) II (c) III (d) IV 4....
 - An equation of the line parallel to the given line and passing through the origin is (a) $y = -2x$ (b) $x = -2y$ (c) $y = 2x$ (d) $x = 2y$ 5....
- 6-10 Question 6-10 refer to the equation $x^3 - x^2 + 2x - 2 = 0$ whose roots are r , s and t .
- The value of $r + s + t =$ (a) -2 (b) -1 (c) 1 (d) 2 6....
 - The value of $rst =$ (a) -2 (b) -1 (c) 1 (d) 2 7....
 - The value of $rs + rt + st =$ (a) -2 (b) -1 (c) 1 (d) 2 8....
 - If r represents the only integral root of this equation, then r equals (a) -2 (b) -1 (c) 1 (d) 2 9....
 - If r represents the only integral root of this equation and $r = i\sqrt{2}$, then t equals (a) $-\sqrt{2}$ (b) $-i\sqrt{2}$ (c) $1 - \sqrt{2}$ (d) $\sqrt{2}$ 10....
 - Solve for the real value of x : $x^{3/2} = 1/64$ 11....
 - Write in *simplest* form the fifth term in the expansion of $(a + \sqrt{a})^7$. 12....
 - When the fraction $\frac{1+i}{1-i}$ is expressed in $a + bi$ form, what is the value of a ? 13....
 - The complex number $a + bi$ is the sum of $2 + 3i$ and $-4 - i$. In which quadrant is the graphic representation of $a + bi$ located? 14....
 - Find the remainder when $x^{15} + 5x^8 - 1$ is divided by $x + 1$. 15....
 - The slope of the line determined by the points $(-2, -5)$ and $(4, k)$ is $4/3$. Find the value of k . 16....
 - Write an equation of the axis of symmetry of the graph of $y = -2x^2 + 6x + 5$. 17....
 - For what value of k will the graph of $y = x^2 - 10x + k$ be tangent to the x -axis? 18....
 - Solve for x : $\log 28 - \log x = \log 7$ 19....

20. Factor $3x^2 - 40x - 28$. 20....
21. If $f(a) = (a - 1)^2$, find $f(a + 1)$. 21....
22. If x varies directly as y and inversely as x^2 and if $x = 4$ when $y = 9$ and $z = 6$, find x when $y = 0.25$ and $z = 3$. 22....
23. The first, second and third terms of an arithmetic progression are a , b and c , respectively. Express c in terms of a and b . 23....
24. Solve for x : $8^{2x+3} = 4^{4x+2}$ 24....
25. Find to the nearest hundredth the value of $\sqrt[3]{0.587}$. 25....
26. From a group of five chemists and four physicists a committee of five is to be chosen so as to include three chemists and two physicists. In how many different ways may the committee be chosen? 26....
27. John, Dick and Paul are members of a club of ten boys from whom three are to be chosen by lot to receive free circus tickets. What is the probability that they will be the three chosen? 27....
- Directions (28-30):* Indicate the correct completion for each of the following by writing the letter a , b , c or d on the line at the right.
28. The equation $x + 1 = \sqrt{x + 1}$ has (a) no roots (b) one and only one root (c) the roots 0 and -1 (d) the roots 0 and 1 28...
29. If $K = p^N$, then $N =$ (a) $\frac{\log p}{\log K}$ (b) $\frac{\log K}{\log p}$ (c) $\log K - \log p$ (d) $\log p - \log K$ 29....
30. If $2/3$ is a root of the equation $mx^3 + nx^2 + px + s = 0$ in which, m , n , p and s are integers, then 3 must be a factor of (a) m (b) n (c) p (d) s 30....

Part II

Answer ten questions from this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answer on the line at the right.

31. Between what two successive positive integers does a root of the equation $x^3 - 10x - 8 = 0$ lie? 31....
32. A root of the equation $x^3 + 7x^2 - 5x - 15 = 0$ lies between 1 and 2. Find the root to the nearest integer. 32....
33. Find a rational root of the equation $x^4 + x^3 - 5x^2 + x - 6 = 0$. 33....
34. Given $x^2 + xy - 3y = 0$, express x in terms of y . 34....
35. In how many different ways may the six letters of the word *summer* be arranged? 35....
36. If ${}^nC_{n-2} = 28$, find the positive value of n . 36....
37. Find to the nearest tenth $\log_3 14$. 37....
38. The velocity of a moving body is given by the equation $v = 12t - 6t^2$. For what value of t is the velocity greatest? 38....
39. Find the slope of the tangent to the graph of $y = 2x^3 + 3x^2 + 4$ at the point $(-2, 0)$. 39....

*40. Write in determinant form an expression for the area of the triangle whose vertices are the points (2, 3), (-1, 6) and (1, -2).

40

*41. Evaluate the determinant:

$$\begin{vmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{vmatrix}$$

41....

42. Express in polar form the fourth root of 16 ($\cos 240^\circ + i \sin 240^\circ$) whose amplitude is smallest.

42....

43. Solve the inequality: $2 - 4x > x + 7$

43....

44. Simplify completely: $1 - \frac{a}{1 + \frac{a}{1 - a}}$

44....

45. Write an equation of the line through (-1, 4) and perpendicular to the line whose equation is $2x - 3y = 8$.

45....

Part III

*Answer ten questions from this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answer on the line at the right.*

46. The difference of the squares of two consecutive integers is d . Express the smaller of these integers in terms of d .

46....

47. A cement walk x feet wide surrounds a rectangular plot whose dimensions are a and b . The area of the plot equals the area of the walk. Write, in terms of a , b and x , an equation which may be solved to find x in terms of a and b .

47....

48. The units digit of a two-digit number is 3 and the tens digit is t . If the number with its digits reversed is divided by the sum of the digits, the quotient is t and the remainder is 6. Write an equation which may be used to solve for t .

48....

49. Given the set of equations $x^2 + y^2 = 10$
 $x + 3y = 0$

Find one set of values for x and y that satisfies the above equations.

49....

50. One root of the equation $x^2 + x^2 - 10x - 12 = 0$ is -3 . Write the quadratic equation which can be solved to find the other two roots.

50....

51. $\log p^8 = 0.4290$. Find $\log p\sqrt{p}$.

51....

*52. Transform the equation $x^2 + y^2 = 2x$ from rectangular to polar coordinates.

52....

53. Express $2(\cos 210^\circ + i \sin 210^\circ)$ in $a + bi$ form.

53....

54. Express in polar form one of the roots of $x^5 + 32 = 0$.

54....

Directions (55-60): Indicate the correct completion for each of the following by writing the letter, *a*, *b*, *c* or *d* on the line at the right.

55. If $f(x) = Q(x)(x - a) + R$, then $f(a)$ is equal to
 (a) *a* (b) 0 (c) *R* (d) $Q(a) + R$ 55....

56. If $K^x = y$, then K^{x+2} equals (a) $y + 2$ (b) $2y$ (c) y^2
 (d) K^2y 56....

57. If $R = 36P^{1.2t}$, then $\log R$ equals (a) $\log 36 + 1.2t \log P$
 (b) $1.2t(\log 36 + \log P)$ (c) $36(1.2t \log P)$ (d) $\log 36 + (\log 1.2 + \log t) \log P$ 57....

58. If drawn on the same set of axes, the graphs of $y = 2$ and $y = x^2$ have (a) no points in common (b) one and only one common point which lies in the first quadrant (c) two common points, one in the first and the other in the fourth quadrant (d) two common points, one in the first and the other in the second quadrant 58....

59. The radius of the circle $x^2 + 2x + y^2 - 10y = 10$ is
 (a) $\sqrt{10}$ (b) 5 (c) 6 (d) 10 59....

*60. Transform the equation $r \sin \theta = \cot \theta$ from polar to rectangular coordinates. The graph of this equation is (a) a straight line (b) an ellipse (c) a parabola (d) a hyperbola 60....

I have studied twelfth year mathematics 12A for weeks and have had recitations per week under
Name of Teacher

I do so declare
(Signature)