June 18, 1958

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answer on the line at the right.

1. Express $\frac{5}{3+2i}$ as an equivalent fraction with a real denominator. 1_____

2. If $(3 + i\sqrt{2})(5 - i\sqrt{2})$ is expressed in the form $x + iy$, in which x and y are real numbers, find the value of x. 2_____

3. Write in *simplest form* the fourth term *only* in the expansion of $(\sqrt{x} + 1)^7$. 3_____

4. Find the real value of $\left\{\frac{3^0}{27^{\frac{2}{3}}}\right\}^{-1}$ 4_____

5. Express in terms of k the discriminant of the equation $x^2 + kx + k - 5 = 0$. 5_____

6. Solve for x: $9^x = \frac{1}{27}$ 6_____

7. If $f(x) = x^2 + 2x$, find $f(1 - b)$. 7_____

8. Find the remainder when $6x^{11} + 5x^6 - 8$ is divided by $x + 1$. 8_____

9. If $\log \frac{x}{3} = 9.4874 - 10$, find x to the *nearest hundredth*. 9_____

10. Find the real value of x which satisfies the equation $x^{3/2} = 64$. 10_____

11. Solve the following set of equations for x:

$$\frac{3}{x} + \frac{1}{y} = \frac{1}{4}$$

$$\frac{2}{x} - \frac{1}{y} = \frac{3}{8}$$

11_____

12. Using k as the constant of variation, write an equation representing the relationship: x varies inversely as the product of r and s. 12_____

13. Find the sum of the infinite geometric series

$$2 + \frac{2}{3} + \frac{2}{9} + \ldots .$$ 13_____

14. The first three terms of an arithmetic progression are represented by $2x - 14$, x and $5x - 1$, respectively. Find x. 14_____

15. The sum of the first *five* terms of a geometric progression is 341. If the common ratio is 2, find the first term of the progression. 15_____

16. An article sells for d dollars at a profit of $r\%$ of the selling price. Express in terms of d and r the cost of the article. 16_____

17. Find the sum of the roots of the equation $3x^4 - 5x^3 + 7x - 9 = 0$. 17_____

18. Express in *simplest form* the complex fraction

$$\frac{2x}{1 - \dfrac{4 - x}{4 + x}}$$. 18_____

19. Points $(-2, 3)$ and $(1, 6)$ lie on a straight line L_1, and points $(7, 2)$ and $(a, 5)$ lie on a second line L_2. If the lines are drawn on the same set of axes, for what value of a will L_1 be parallel to L_2? 19_____

20. Write an equation of the straight line that passes through the origin and makes an angle of $45°$ with the positive direction of the x-axis. 20_____

21. The equation of the axis of symmetry of the graph of $y = x^2 - 5x + 8$ is $x = k$. Find the value of k. 21_____

22. If $V = \pi r^2 h$ and $S = 2\pi rh$, express V in terms of r and S. 22_____

23. How many distinct permutations can be made from the letters of the word *WEEKEND* if they are all used every time? 23_____

24. A man has 3 striped and 4 solid-color ties. If he selects one of the ties at random, what is the probability that it will be a striped one? 24_____

25. From a group of 4 boys and 7 girls, how many different committees can be formed, each consisting of 1 boy and 2 girls? **25**_____

Directions (26-29): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b*, *c* or *d*.

26. Given the equation $2x^4 + px^2 + qx + 6 = 0$ in which p and q are integers. Which of the following can *not* be a root of the equation? (a) ½ (b) ⅔ (c) 3 (d) 3/2 26_____

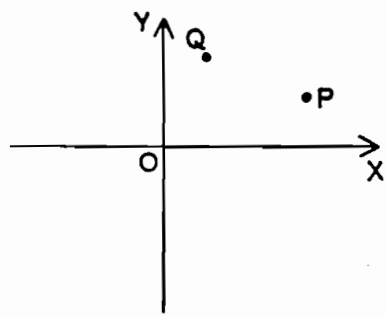
27. The equation $\sqrt{8 - x^2} + x = 0$ has (a) one positive root only (b) one negative root only (c) one positive and one negative root (d) no roots 27_____

28. When drawn on the same set of axes, the graphs of $x^2 + 4y^2 = 16$ and $y = 3x^2$ (a) do not intersect (b) intersect in one point (c) intersect in two points (d) intersect in four points 28_____

29. Which of the following is a rational number?

(a) $-\frac{5}{7}$ (b) $\sqrt{2}$ (c) π (d) $\sqrt{-4}$ 29_____

30. In the drawing, OX is the real axis and OY is the imaginary axis. The points P and Q represent $a + bi$ and $c + di$, respectively. *On the drawing*, indicate by the letter S the point which represents $(a + bi) + (c + di)$.



Part II

*Answer ten questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answer on the line at the right.*

31. The inequality $6 + x < 3x - 2$ is satisfied by *all* values of x greater than a certain number n, and by *no* other values. Find n. 31_____

32. Write an equation of the circle whose center is the point (0, 4) and which passes through the origin. 32_____

Directions (33-34) : Indicate the correct completion for each of the following by writing on the line at the right the letter *a*, *b*, *c* or *d*.

33. The slope of a line perpendicular to the line which passes through the two points (4, 3) and (2, — 1) is (*a*) 2 (*b*) — 2 (*c*) ½ (*d*) — ½ 33_____

34. The roots of the equation $x^3 - 6x^2 - x + 30 = 0$ are (*a*) 3, 1, — 10 (*b*) 3, — 2, 5 (*c*) 3, 2, — 5 (*d*) 3, — 3, 6 34_____

35. Find the rational fractional root of the equation $3x^3 + 4x^2 + 13x + 4 = 0$. 35_____

36. Between what two consecutive integers does the positive root of the equation $x^3 - 5x - 3 = 0$ lie? 36_____

37. A positive root of the equation $x^3 - 3x + 1 = 0$ lies between 0.3 and 0.4. Find this root to the *nearest tenth*. 37_____

38. If a function of x is divided by $x - 2$, the quotient is $2x^2 - 3$ and the remainder is 7. Find the function. 38_____

39. The volume V of a box is given by the equation $V = 12x^2 - x^3$ where x represents the height in inches of the box. Find the value of x for which V is a maximum. 39_____

40. The distance S in feet through which a body moves in t seconds is given by the formula $S = 4t^2 - 3t + 10$. Find the velocity of the body in feet per second when $t = 5$. 40_____

41. If $y = 4x^3 - 5x^2 + 3$, find the second derivative of y with respect to x. 41_____

42. Express in polar form the product of $4(\cos 50° + i \sin 50°)$ and $5(\cos 80° + i \sin 80°)$. 42_____

*43. Using polar coordinates, the graph of $r = 2 \sin \theta$ is (*a*) a straight line (*b*) a circle (*c*) a parabola (*d*) an ellipse 43_____

*44. Transform $xy = 4$ from rectangular to polar coordinates. 44_____

*45 Transform $r \cos \theta = 2$ from polar to rectangular coordinates. 45_____

Part III

*Answer ten questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answer on the line at the right.*

46. Find to the *nearest tenth* the value of $(17.3)^{1.2}$. 46_____

47. Log $x^2y = 0.3457$ and log $x = 0.2815$. Find log y. 47_____

48. Given log 2.6 = 0.4150 and log 7.5 = 0.8751. Solve the equation $2.6^x = 7.5$ for x to the *nearest tenth*. 48_____

49. Solve for x the equation $\log x - \log (x - 4) = \log 3$. 49_____

50. Two roots of the equation $x^3 + px^2 + qx + r = 0$ are 3 and $1 + i$. If p, q and r are real numbers, find the value of q. 50_____

51. It takes b bricklayers d days to build a wall. If c more bricklayers had been put on the job and they all worked at the same rate, how many days would it have taken to build the wall? 51_____

52. The rate of a boat upstream is two-thirds the rate of the boat downstream. Find the ratio of the rate of the stream to the rate of the boat in still water. 52_____

Directions (53-54) : Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b*, *c* or *d*.

53. The graph of $y = \log_{10} x$ (*a*) intersects both coordinate axes (*b*) intersects the y-axis only (*c*) intersects the x-axis only (*d*) does not intersect either axis 53_____

54. When drawn on the same set of axes, the graphs of $y = x^2 - 2x - 3$ and $y = -x^2 + 2x + 3$ (*a*) have no points in common (*b*) are tangent (*c*) have the same y-intercept (*d*) have the same x-intercepts 54_____

55. Find the modulus of $-2 + 3i$. 55_____

56. If the number $2(\cos 330° + i \sin 330°)$ is expressed in the form $a + bi$, find the value of b. 56_____

57. Find the average rate of change of y with respect to x for the function $y = x^2 + 2x$ over the interval from $x = 2$ to $x = 5$. 57_____

*58. Write in determinant form an expression for the area of the triangle whose vertices are (—2, 3), (6, 0) and (2, 5). 58_____

*59. Evaluate the determinant $\begin{vmatrix} 1 & -2 & 0 \\ -4 & 5 & -1 \\ 3 & 2 & 0 \end{vmatrix}$ 59_____

*60. The straight line whose equation is $\begin{vmatrix} x & y & 1 \\ 2 & 3 & 1 \\ -4 & 5 & 1 \end{vmatrix} = 0$ passes through the point (*a*) (2, 3) (*b*) 2, —4) (c) (3, 2) (d) (—4, 2) 60_____