

January 25, 1966

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Write your answers in the spaces provided.

- Find the value of $x^2 - 2x$ when $x = 1 + \sqrt{2}$. 1_____
- Find all values of x for which $3 - x > 2x + 1$. 2_____
- Express $\frac{5}{3 + i}$ with a real denominator. 3_____
- Given the equation $3x + 2y = 7$. Find the smallest integral value of x for which the corresponding value of y is negative. 4_____
- When $f(x)$ is divided by $x - 2$, the quotient is $x + 3$ and the remainder is 7. Find $f(0)$. 5_____
- The solution of the inequality $x^2 - 2x - 8 > 0$ has the form $x > a$ or $x < b$. Find the value of a . 6_____
- A straight line has a slope of $\frac{3}{2}$ and a y -intercept of -6 . Find the abscissa of the point where the line crosses the x -axis. 7_____
- Calculate the number of distinct arrangements of the six letters of the word RECEDE. 8_____
- In how many ways may a committee of 3 students be selected from a group of 7 students? 9_____
- A bowl contains 15 red balls, 15 black balls and 45 white balls. All the balls are identical except for color. If one ball is drawn at random, what is the probability that it will be red? 10_____
- Solve for the positive value of x : $25^{2x} = 5x^3 - 12$ 11_____
- Solve the following set of equations for x :

$$\frac{4}{x} + \frac{9}{y} = 5$$

$$\frac{8}{x} - \frac{3}{y} = 3$$
 12_____
- A particle moves in a line so that its velocity at any time t is given by the formula $v = 5t^2 - 2t$. For what value of t is the acceleration zero? 13_____
- The graph of the function $y = x^3 + ax^2 + bx - 6$ intersects the x -axis at $(1, 0)$, $(2, 0)$, and $(3, 0)$. Find the value of b . 14_____
- Find the value of $\log_8 \left(\frac{1}{81} \right)$. 15_____

16. One root of the equation $x^3 - 3x^2 - x + 4 = 0$ lies between 2.8 and 2.9 Find this root to the nearest tenth. 16_____

17. For what value of k will the point $(k, k + 1)$ lie on the line $y = 3x - 1$? 17_____

18. Find the sum of the infinite geometric progression $2, -1, \frac{1}{2}, \dots$ 18_____

Directions (19-24): Write in the space provided the number preceding the expression that best completes each statement or answers each question.

19. One root of the equation $x^4 - 3x^3 + 2x + 4 = 0$ is
(1) 1 (2) 2 (3) -1 (4) 0 19_____

20. Which may be a root of the equation $3x^5 + ax^4 + bx^3 + cx^2 + dx + 1 = 0$, where a, b, c and d are integers? (1) $\frac{1}{2}$ (2) $\frac{1}{3}$
(3) 3 (4) $\frac{1}{4}$ 20_____

21. When $x^3 - 3x^2 + kx + 4$ is divided by $x + 2$, the remainder is -2 . The value of k is (1) 0 (2) -1 (3) -7 (4) -8 21_____

22. The centripetal force on an object moving around a circle varies directly as the square of its speed and inversely as the radius of the circle. If both the speed and the radius are halved, then the centripetal force on the object is (1) halved (2) doubled
(3) divided by 8 (4) unchanged 22_____

23. A man travels m miles per hour for t hours and then changes his rate and travels s miles per hour for h hours. His average rate of speed for the entire trip is (1) $\frac{mt + sh}{2}$ (2) $\frac{m + s}{2}$
(3) $\frac{mt + sh}{m + s}$ (4) $\frac{mt + sh}{t + h}$ 23_____

24. A general equation of the family of lines perpendicular to $6y = 4x - 3$ is (1) $2y = -3x + k$ (2) $2y = 3x + k$
(3) $3y = 2x + k$ (4) $3y = -2x + k$ 24_____

Part II

*Answer sixteen questions from this part, 25-48. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answers in the spaces provided.*

25. Write the reciprocal of $\frac{\sqrt{5} - 1}{3}$ as a fraction with a rational denominator. 25_____

26. Given: $2^{x+2} = 4^{y-1}$. Express x in terms of y . 26_____

27. Write in simplest form the 4th term only of the expansion of $(x^2 + x^3)^8$. 27_____

28. Find the value(s) of x which will satisfy the equation $\sqrt{x+3} = 3-x$. 28_____

29. If $f(x) = x^5 + x^3 + x^0 + x^{-2} + x^{-4}$, determine the value of $f(i)$ where $i = \sqrt{-1}$. 29_____

30. What is the ordinate of the point where the graph of the equation $y = 5^x + 2$ crosses the y -axis? 30_____

31. Express in simplest form:
$$2 - \frac{5b}{a}$$

$$4 - \frac{20b}{a} + \frac{25b^2}{a^2}$$
 31_____

32. Find the radius of the circle whose equation is $x^2 + 4x + y^2 - 6y = 23$. 32_____

33. Find the value of x for which $\log 2 + \log(4x-7) = \log x$. 33_____

34. Express in the form $a + bi$ the quotient obtained when $12(\cos 90^\circ + i \sin 90^\circ)$ is divided by $3(\cos 60^\circ + i \sin 60^\circ)$. 34_____

35. Given the equation $x^2 - 6x + k = 0$. Find the value of k such that one root of the equation is twice as large as the other root. 35_____

36. Determine the three roots of $x^3 - x^2 + x - 1 = 0$. 36_____

37. Find the slope of the line tangent to the curve whose equation is $y = x^3 - 12x + 20$ at the point where the curve crosses the y -axis. 37_____

38. In 3 years a man will be half as old as his father will be. If the sum of their present ages is 69 years, find the present age of the father. 38_____

39. If a , $-a$ and b represent roots of the equation $x^3 + 2x^2 + px + q = 0$, find the value of b . 39_____

40. The arithmetic mean between two numbers is 3 and their positive geometric mean is $2\sqrt{2}$. Find the smaller of the two numbers. 40_____

41. Given $x^2 - 2ax + a^2 - 5 = 0$. Express x in terms of a . 41_____

42. A root of $x^3 + 8 = 0$ lies in quadrant IV. Express this root in the form $r(\cos \theta + i \sin \theta)$. 42_____

*43. Find a root of the equation $\begin{vmatrix} 1 & 2 & 3 \\ 1 & x & 3 \\ 1 & 3 & x \end{vmatrix} = 0$. 43_____

Directions (44-48): For each of the following *which you select*, write in the space provided the *number* preceding the expression that best completes the statement.

44. If $2^n + 2^n$ is written in the form 2^n , then n equals (1) $2a$
 (2) $a + 1$ (3) $a + 2$ (4) a^2 44_____

45. The graph of $y = \log_2 x$ intersects (1) the x -axis only
 (2) the y -axis only (3) both coordinate axes (4) neither
 coordinate axis 45_____

46. A formula from the Theory of Relativity that shows the effect
 of velocity on time is given by $T = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$

If the formula is solved for v , then the positive value of v equals

(1) $\frac{T}{c} \sqrt{T^2 + t^2}$	(2) $\frac{T}{c} \sqrt{T^2 - t^2}$
(3) $\frac{c}{T} \sqrt{T^2 + t^2}$	(4) $\frac{c}{T} \sqrt{T^2 - t^2}$ 46_____

47. When drawn on the same set of axes, exactly how many distinct points do the graphs of $y = x^2 + 2$ and $x^2 + 4y^2 = 16$ have in common? (1) 1 (2) 2 (3) 3 (4) 4 47_____

*48. The curve whose equation in polar form is $r^2 \sin \theta \cos \theta = 1$ is (1) a circle (2) an ellipse (3) a parabola (4) a hyperbola 48_____