The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TWELFTH YEAR MATHEMATICS

12B (Solid Geometry)

Monday, January 27, 1964 — 1:15 to 4:15 p.m., only

Name of pupil..... Name of school.....

Name and author of textbook used.....

Name of teacher.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 A lune with an angle of 72 degrees is drawn on a sphere of radius 10 inches. Express the number of square inches in the area of the lune. 1.....

2 The dimensions of the base of a rectangular parallelepiped are 2 and 5, and a diagonal of the parallelepiped is 8. Find the length of the altitude of the parallelepiped. 2.....

3 The altitude of a right circular cylinder is equal to the radius of the base. Express the lateral area of the cylinder in terms of r, the radius of the base. 3.....

4 A line segment makes an angle of 59 degrees with a plane, and the length of its projection on the plane is 5.8 inches. Find to the *nearest inch* the length of the segment. 4.....

5 If the edges of the upper and lower bases of a frustum of a regular triangular pyramid are 3 and 9, respectively, and the slant height is 5, find the lateral area of the frustum. 5.....

6 The edge of a cube is 2. Find the volume of the sphere circumscribed about the cube. 6.....

7 Express in terms of s the total area of a right square prism whose lateral edge is 9 and whose base edge is s. 7.....

8 A spherical triangle has angles of 70 degrees, 84 degrees and 98 degrees. What is the ratio of the area of this triangle to the area of the sphere? 8.....

9 A regular polyhedron has 6 vertices and 12 edges. Express its total area in terms of an edge, e. 9.....

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10 The angles of a spherical triangle are 60 degrees, 102 degrees and 114 degrees. Find the number of degrees in the sum of the sides of its polar triangle. 10.....

11 A point P lies within a dihedral angle and is 5 inches from each face of the dihedral angle. If the dihedral angle contains 60 degrees, find the number of inches in the distance from point P to the edge of the angle. 11.....

12 The radius of the base of right circular cylinder A is twice the radius of the base of right circular cylinder B. The altitude of cylinder A is to the altitude of cylinder B as 2:5. Find the ratio of the volume of cylinder A to the volume of cylinder B. 12.....

13 The area of a zone of a sphere of radius 3 is 6π. Find the altitude of the zone. 13.....

14 $ABCD$ is a trapezoid with leg AD perpendicular to bases AB and DC. $AB = 5$, $DC = 7$ and $BC = 6$. $ABCD$ is rotated through 360 degrees about AD as an axis. Find the lateral area of the figure formed. 14.....

15 The altitude of a pyramid is 18 inches. Its base is a square with each side 12 inches in length. Find the number of square inches in the area of the section made by a plane parallel to the base and 3 inches from the vertex. 15.....

16 The volume of a circular cone is 21π. The altitude is 7. Find the radius of the base. 16.....

17 The base of a prism is a 5-by-6 rectangle, and each lateral edge of the prism makes an angle of 45 degrees with the base. If each lateral edge is 8, find the volume of the prism. 17.....

18 A frustum of a cone of revolution has an altitude of 4 feet and radii of the bases 2 feet and 5 feet. Express in terms of π the number of cubic feet in the volume of the frustum. 18.....

19 The polar distance of a small circle of a sphere is 50 degrees, and the radius of the sphere is 20 inches. Find to the *nearest inch* the radius of the small circle. 19.....

20 A solid sphere 6 inches in diameter weighs 81 pounds. Find the number of pounds in the weight of a solid sphere of the same material which is 2 inches in diameter. 20.....

Directions (21–26): Indicate the correct completion for *each* of the following by writing on the line at the right the number 1, 2, 3 or 4.

21 The locus of points equally distant from two given points A and B and a given distance d from the line AB is a

(1) line
(2) pair of lines
(3) pair of points
(4) circle

21.....

22 The altitude of a regular tetrahedron whose edge is e is

(1) $\frac{e}{2}\sqrt{6}$ (3) $\frac{e}{3}\sqrt{6}$

(2) $\frac{e}{3}\sqrt{3}$ (4) $\frac{e}{2}\sqrt{3}$ 22.....

23 The face angles of a trihedral angle may be

(1) 72°, 108°, 170° (3) 32°, 110°, 170°
(2) 82°, 60°, 145° (4) 72°, 110°, 178° 23.....

24 Three classifications of solid figures arranged in order so that each includes the classifications that follow are

(1) prism, prismatoid, parallelepiped
(2) prismatoid, prism, parallelepiped
(3) parallelepiped, prism, prismatoid
(4) prismatoid, parallelepiped, prism 24.....

25 If a cylinder is circumscribed about a sphere, the area of the sphere, compared to the total area of the cylinder, is

(1) one-half as great (3) the same
(2) twice as great (4) two-thirds as great 25.....

26 AB and CD are two parallel lines. The locus of points equally distant from AB and CD and also 5 inches from the plane of AB and CD is a

(1) line (3) pair of parallel lines
(2) pair of intersecting lines (4) pair of skew lines 26.....

Directions (27–30) : If the blank space in each statement below is replaced by the word *always, sometimes* (but not always) or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

27 If two planes are perpendicular to each other, a line drawn in one of them perpendicular to their intersection is ... perpendicular to the other. 27.....

28 A fixed line l is parallel to a fixed plane M. If a second line p moves, taking all possible positions in which it remains parallel to M, it will ... be parallel to l. 28.....

29 The diagonals of a cube are ... perpendicular to each other. 29.....

30 If two trihedral angles have the three face angles of one equal respectively to the three face angles of the other, the corresponding dihedral angles are ... equal. 30.....

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Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove *either* *a* or *b* but *not* both: [10]

a Two planes perpendicular to the same line are parallel.

OR

b A spherical angle is measured by the arc of a great circle described from its vertex as a pole and included between its sides, produced if necessary.

32 Prove that, if a plane is passed through one of the diagonals of a parallelogram, the perpendiculars to this plane from the end points of the other diagonal are equal. [10]

33 The slant height of a cone of revolution makes an angle θ with its projection on the base. The altitude of the cone is h.

a Express the lateral area S of the cone as a function of h and θ. [5]

b Find S to the *nearest integer* if $h = 3.21$ and $\theta = 51$ degrees. [Use the approximation $\pi = 3.14$.] [5]

34 Given: The distance between two parallel planes M and N is s. Line q lies on plane M and point A lies on line q.

a Describe fully the locus of points

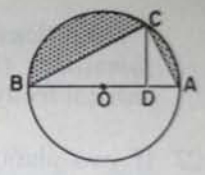
(1) equally distant from planes M and N [2]
(2) at a given distance d from line q [2]
(3) at a given distance g from point A [2]

b Name the locus of points satisfying both conditions in part *a*(1) and

(1) part *a*(2) if d is greater than $\frac{s}{2}$ [2]

(2) part *a*(3) if $g = \frac{s}{2}$ [2]

35 In the accompanying diagram, BA is a diameter of the circle. C is a point on the circumference. CD is perpendicular to BA. The lengths of CD and BA are p units and $2q$ units, respectively. The figure is revolved through 360 degrees about BA as an axis.



a Express the total volume generated by the shaded circular segments in terms of p and q. [6]

b Express this volume in terms of π if angle $ABC = 30$ degrees and $q = 6$. [4]

*36 Answer *either* *a* or *b* but *not* both:

a In spherical triangle ABC, angle $C = 90°$, side $a = 103°$ and angle $B = 47°$.

(1) Find angle A to the *nearest degree*. [8]
(2) Write an equation that could be used to find side c, given any right spherical triangle with C the right angle where a and B are known. [2]

OR

b Points A (—5,2,3), B (3,0,0), C (0,4,0) and D (0,0,5) are located with reference to three mutually perpendicular axes x, y and z.

(1) Write the coordinates of the midpoint of line segment AB. [2]
(2) Write an equation of the plane through point A parallel to the yz-plane. [2]
(3) Write an equation of the sphere whose center is at the origin and whose radius is OA. [2]
(4) Write an equation of the plane which passes through B, C and D. [2]
(5) Write an equation of the plane parallel to the z-axis and passing through points B and C. [2]

* These questions are based on optional topics in the syllabus.

FOR TEACHERS ONLY

12B

SCORING KEY
TWELFTH YEAR MATHEMATICS
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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 21–26, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) 80π

(2) $\sqrt{35}$

(3) $2\pi r^2$

(4) 11

(5) 90

(6) $4\pi\sqrt{3}$

(7) $2s^2 + 36s$

(8) 1 : 10

(9) $2e^2\sqrt{3}$

(10) 264

(11) 10

(12) 8 : 5

(13) 1

(14) 72π

(15) 4

(16) 3

(17) $120\sqrt{2}$

(18) 52π

(19) 15

(20) 3

(21) 4

(22) 3

(23) 1

(24) 2

(25) 4

(26) 3

(27) always

(28) sometimes

(29) never

(30) always

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Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely mechanical or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent depending on the relative importance of the principle in the solution of the problem.

(33) *a* $\frac{\pi h^2}{\sin\theta \tan\theta}$ *or* $\frac{\pi h^2 \cos\theta}{\sin^2\theta}$ [5]

b 34 [5]

(34) *a* (1) a plane midway between M and N and parallel to them [2]

(2) a cylindrical surface with q as axis and radius d [2]

(3) a spherical surface with center at A and radius q [2]

b (1) two parallel lines [2]

(2) one point [2]

(35) *a* $\frac{4}{3}\pi q^3 - \frac{2}{3}\pi p^2 q$ [6]

b 180π [4]

(36) *a* (1) 99° [8]

(2) $\tan c = \frac{\tan a}{\cos B}$ [2]

b (1) $\left(-1, 1, \frac{3}{2}\right)$ [2]

(2) $x = -5$ [2]

(3) $x^2 + y^2 + z^2 = 38$ [2]

(4) $\frac{x}{3} + \frac{y}{4} + \frac{z}{5} = 1$ [2]

(5) $4x + 3y = 12$ [2]