

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION
TWELFTH YEAR MATHEMATICS
12B (Solid Geometry)

Thursday, January 24, 1963 — 1:15 to 4:15 p.m., only

Name of pupil.....Name of school.....

Name and author of textbook used.....

Name of teacher.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

- 1 The dimensions of a rectangular parallelepiped are b , $2b$ and $3b$. Express the length of a diagonal of the parallelepiped in terms of b . 1.....
- 2 A lune with an area of 8π square inches is drawn on a sphere of radius 3 inches. Find the number of degrees in the angle of the lune. 2.....
- 3 Find to the *nearest cubic foot* the number of cubic feet of metal needed for 1,000 feet of solid metal rodding 1 inch in diameter. [Use the approximation $\pi = \frac{22}{7}$.] 3.....
- 4 Express the volume V of a cone of revolution in terms of the slant height l and the radius r of the base. 4.....
- 5 One of the regular polyhedrons has 12 edges and 6 vertices. How many faces does it have? 5.....
- 6 The projection of line segment RS of length a upon plane m is equal to $\frac{1}{2}a$. Find the number of degrees in the angle which the line determined by R and S makes with plane m . 6.....
- 7 A geometric figure consists of triangle ABC , segment RA perpendicular to the plane of triangle ABC , and segment RB . Name the line segment which is skew to RA . 7.....
- 8 The total area of a right circular cylinder is 250π square feet and the altitude is 20 feet. Find the number of feet in the radius of a base of the cylinder. 8.....
- 9 Find to the *nearest cubic foot* the volume of a frustum of a square pyramid whose base edges are 20 feet and 10 feet, respectively, and whose altitude is 6 feet. 9.....

- 10 The area of a sphere of radius a is numerically equal to the volume of a sphere of radius b . Express a in terms of b . 10.....
- 11 Point P is located in the interior of a dihedral angle whose faces are m and n . If P is 5 inches from m , $5\sqrt{2}$ inches from n and 10 inches from the edge of the dihedral angle, find the number of degrees in the dihedral angle. 11.....

In questions 12-14, P denotes a right prism which has an altitude h equal to 6 feet; the length of side s of its square base is equal to 12 feet. Prism P' has an altitude h' , and the length of side s' of its square base is equal to 10 feet.

- 12 If prism P' is equal in volume to prism P , find h' to the nearest foot. 12.....
- 13 If prism P' is similar to prism P , find h' to the nearest foot. 13.....
- 14 If a third prism Q is congruent to prism P , what is the total area of Q ? 14.....
- 15 Each base of a zone on a sphere has a radius of 2 inches. The radius of the sphere is 4 inches. Find the number of square inches in the area of the zone. 15.....
- 16 The ratio of the volumes of two similar cylinders is 27:8. If the total surface of the larger cylinder is 180, find the total surface of the smaller. 16.....
- 17 The three sides of a spherical triangle are 90° , 90° and 30° . What is the number of spherical degrees in the area of the triangle? 17.....
- 18 A spherical plastic ball has a concentric metal core of diameter 4 inches. If the diameter of the plastic ball is 10 inches, find the number of cubic inches of plastic. 18.....
- 19 If an angle of a spherical triangle is 75° , find the number of degrees in the side lying opposite it in its polar triangle. 19.....

Directions (20-23): Indicate the correct completion for each of the following by writing on the line at the right the number 1, 2, 3 or 4.

- 20 If two face angles of a trihedral angle are 135° and 100° , the third face angle may be
 (1) 20° (3) 125° 20.....
 (2) 120° (4) 170°
- 21 The locus of points which are equidistant from two intersecting planes and 4 units from a point on the intersection of the planes consists of
 (1) one circle (3) two parallel lines 21.....
 (2) two circles (4) four parallel lines

- 22 The distance of point A from plane m is 4 inches. The locus of points at a distance 3 inches from point A and also 3 inches from plane m is a
 (1) point (3) circle
 (2) straight line (4) plane 22.....
- 23 The base of a pyramid is a square. If each side of the base is doubled and the altitude of the pyramid is divided by two, the volume of the pyramid is
 (1) multiplied by 1 (3) multiplied by 4
 (2) multiplied by 2 (4) divided by 2 23.....
- 24 A right triangle whose legs are 5 and 12 is rotated through 360° about the longer leg as an axis. What is the lateral area of the surface generated? 24.....
- 25 A pyramid is 12 inches high, and the area of its base is 48 square inches. Find the number of square inches in the area of a section parallel to the base and 9 inches from it. 25.....
- 26 On a sphere point A lies on a small circle whose poles are P and P' . If chord AP is 8 inches and chord AP' is 15 inches, find the number of inches in the length of the diameter of the sphere. 26.....
- Directions (27-30):* If the blank space in each statement below is replaced by the word *always*, *sometimes* (but not always) or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.
- 27 If, in a dihedral angle with faces M and N and edge q , line m in face M forms an angle of 60° with line n in face N and line m is perpendicular to line q , then the dihedral angle ... contains 60° . 27.....
- 28 Two pyramids which have congruent square bases and equal altitudes ... have equal lateral areas. 28.....
- 29 The locus of points equidistant from three given noncollinear points is ... a plane. 29.....
- 30 The intersection of three planes is ... a single point. 30.....

Part 11

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove either a or b: [10]

a If each of two intersecting planes is perpendicular to a third plane, their intersection is also perpendicular to that plane.

OR

b The locus of points equally distant from two given points is the plane perpendicular to the line segment joining them at its midpoint.

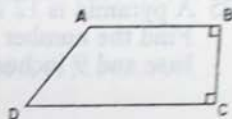
32 Points A , B and C are midpoints of the three edges of a cube which meet at vertex P of the cube. The length of an edge of the cube is represented by e . Find in terms of e the

a volume of the tetrahedron $P-ABC$ [5]

b total surface area of the tetrahedron $P-ABC$ [5]

33 Prove that a section of an oblique triangular prism made by a plane parallel to one of its lateral edges is a parallelogram. [10]

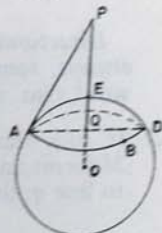
34 In the accompanying figure, $ABCD$ is a trapezoid with bases AB and DC in the ratio 5:8, and angles B and C are right angles. Let the longer base be represented by $8x$ and the altitude by y .



a Write in terms of x and y a formula for the volume of the solid formed by rotating the figure through 360° about the longer base as an axis. [4]

b Using the formula obtained in answer to part a, find to the nearest integer the volume of the solid when $x = 1.63$ and $y = 0.92$. [Use the approximation $\pi = 3.14$.] [6]

35 In the accompanying diagram, P denotes the position of a light source which illuminates a zone on sphere O . The base of this zone is a circle ABD with center Q . Line PQO intersects the sphere at E . Let $AO = r$, $PE = m$,



$EQ = h$ and $Z =$ the area of the zone. Show that $Z = \frac{2\pi r^2 m}{r + m}$. [10]

*36 Answer either a or b:

a Points $A(2, 3, -4)$ and $B(-1, 5, 6)$ are located with reference to three mutually perpendicular axes, x , y and z .

(1) Find the coordinates of the midpoint of line segment AB . [2]

(2) Write an equation of the plane through point B parallel to the xy -plane. [2]

(3) Determine the distance of point A from the plane described in part (2). [2]

(4) Point A lies on a sphere with center at the origin. Write an equation of this sphere. [2]

(5) With reference to the sphere described in part (4), point B lies (1) inside it, (2) on it, (3) outside it. [Write the number preceding the correct answer on your answer paper after the number (5).] [2]

OR

b In spherical triangle ABC , angle $C = 90^\circ$, side $a = 118^\circ$ and angle $B = 12^\circ$.

(1) Find angle A to the nearest degree. [8]

(2) Using the given data, write an equation that could be used to find side b . [2]

* This question is based on optional topics in the syllabus.

FOR TEACHERS ONLY

SCORING KEY

12B

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 20–23, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $b\sqrt{14}$

(2) 80

(3) 5

(4) $\frac{1}{3}\pi r^2 \sqrt{l^2 - r^2}$

(5) 8

(6) 60

(7) BC

(8) 5

(9) 1,400

(10) $b \sqrt{\frac{b}{3}}$ or $\frac{b}{3} \sqrt{3b}$

(11) 75

(12) 9

(13) 5

(14) 576

(15) $32\pi\sqrt{3}$

(16) 80

(17) 30

(18) 156π

(19) 105

(20) 2

(21) 2

(22) 3

(23) 2

(24) 65π

(25) 3

(26) 17

(27) sometimes

(28) sometimes

(29) never

(30) sometimes

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely mechanical or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent depending on the relative importance of the principle in the solution of the problem.

$$(32) \quad a \quad \frac{e^3}{48} \quad [5]$$

$$b \quad \frac{(3 + \sqrt{3})e^2}{8} \quad [5]$$

$$(34) \quad a \quad 6\pi xy^2 \quad [4]$$

$$b \quad 26 \quad [6]$$

$$(36) \quad a \quad (1) \quad \left(\frac{1}{2}, 4, 1\right) \quad [2]$$

$$(2) \quad z = 6 \quad [2]$$

$$(3) \quad 10 \quad [2]$$

$$(4) \quad x^2 + y^2 + z^2 = 29 \quad [2]$$

$$(5) \quad 3 \quad [2]$$

$$b \quad (1) \quad 96 \quad [8]$$

$$(2) \quad \tan b = \sin a \tan B \quad [2]$$