Reviewed By: .....

Percent: .....

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TWELFTH YEAR MATHEMATICS

12A (Advanced Algebra)

Wednesday, January 27, 1960 — 9:15 a.m. to 12:15 p.m., only

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answer on the line at the right.

1 Express $\frac{5}{2-i}$ in the form of $a + bi$. 1.....

2 Write an equation of the line that passes through the point $(3, -2)$ and that is parallel to the line whose equation is $2x - 3y = 4$. 2.....

3 For what value of k will the graph of $y = x^2 - 8x + k$ be tangent to the x-axis? 3.....

4 Write in simplest form the fourth term *only* in the expansion of $\left(x^2 + \frac{2}{x}\right)^6$. 4.....

5 Find the remainder when $3x^9 + 2x^6 - 3$ is divided by $x + 1$. 5.....

6 Solve for x: $4^x = \frac{1}{8}$ 6.....

7 Find the real root of the equation $x^{\frac{3}{2}} = \frac{8}{27}$. 7.....

8 Find the x-intercept of the graph of $y = \log_5 x$. 8.....

9 Find to the *nearest hundredth* the value of $\sqrt[3]{0.257}$. 9.....

10 Express in simplest form $\dfrac{1 + \dfrac{2}{a-2}}{1 - \dfrac{a-5}{a-2}}$. 10.....

11 Two of the roots of the equation $2x^3 - 3x^2 + px + q = 0$ are 3 and -2. Find the third root. 11.....

12 If $f(x) = x^3 + 4x$, express $f(a - 2)$ as a product of two binomials. 12.....

13 Find the value of $3x^0 + \left(\frac{4}{x}\right)^{-2} + \sqrt[3]{x^2}$ if $x = 8$. 13.....

14 If $f(x)$ is identically equal to $(x - 8)\,Q(x) + 7$, find the numerical value of $f(8)$. 14.....

15 Solve for x: $a^2 = \frac{1-x}{1+x}$ 15.....

16 An algebra examination contains 8 questions. Each student is to answer the first question and any 4 others. How many different selections of 5 questions may be made? 16.....

17 How many *odd* numbers of 3 digits each can be written with the digits 1, 2, 3, 4, 5 if no digits are to be repeated? 17.....

18 A bag contains 3 black balls and 4 white balls. If 2 balls are drawn from the bag, what is the probability that both will be black? 18.....

19 Using k as a constant of variation, write an equation that represents the relationship: x varies directly as y and inversely as the square of z. 19.....

20 The slope of the line that passes through the points $(-2, 3)$ and $(5, y)$ is $-\frac{4}{7}$. Find the value of y. 20.....

21 Express the repeating decimal 0.636363.... as a common fraction. 21.....

Directions (22–25): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement.

22 If the graphs of $4x^2 + y^2 = 16$ and $y = 2x^2 - 2$ are drawn on the same set of axes, the number of points they will have in common is (1) 1 (2) 2 (3) 3 (4) 4 22.....

23 The equation of the axis of symmetry of the parabola $y = ax^2 - 4x + 1$ is $x = 1$. The value of a is (1) -2 (2) $+2$ (3) -4 (4) $+4$ 23.....

24 The equation $\sqrt{x+6} - x = 0$ has (1) no root (2) -2 as its only root (3) 3 as its only root (4) the two roots 3 and -2 24.....

25 One machine can complete a job in p minutes and a second machine can do the job in q minutes. If both machines work together, the time in minutes required to complete the job is

(1) $\frac{pq}{p+q}$ (2) $\frac{p+q}{2}$ (3) $\frac{p+q}{pq}$ (4) $p+q$ 25.....

Part II

Answer ten questions from this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answer on the line at the right.

26 Solve the inequality $x + 8 < 4x - 1$. 26.....

27 Write an equation of the line that passes through the point $(0, 3)$ and is perpendicular to the line whose equation is $y = 2x - 1$. 27.....

28 Find the radius of the circle whose equation is $x^2 - 6x + y^2 + 2y - 6 = 0$. 28.....

29 Find the rational root of $3x^3 + 7x^2 + 8x + 2 = 0$. 29.....

30 Between what two consecutive integers does the positive root of the equation $x^3 + 3x - 20 = 0$ lie? 30.....

31 A positive root of the equation $x^3 + 3x^2 + 8x - 4 = 0$ lies between 0.4 and 0.5. Find this root to the *nearest tenth.* 31.....

32 Find all the roots of the equation $x^4 + 8x^2 - 9 = 0$. 32.....

33 If $y = x^2 - 3x$, express x in terms of y. 33.....

34 [Write the *number* preceding the correct answer in the space provided.] Which one of the following is a rational integral function in x?

(1) $2x^2 - \sqrt{x} - 1$ (2) $\frac{2}{3}x^2 - x\sqrt{3} - 5$ (3) $x^2 - \frac{3}{x} - 5$

(4) $x^{\frac{2}{3}} - x - 5$ 34.....

35 An arrow is shot vertically upward. Its height h in feet after t seconds is given by the formula $h = 128t - 16t^2$. After how many seconds will it reach its maximum height? 35.....

36 Find the abscissa of the point of inflection of the graph of $y = x^3 - 3x^2 - 9x + 2$. 36.....

37 Find the slope of the line tangent to the graph of $y = x^2 - 3x - 1$ at the point $(4, 3)$. 37.....

38 Multiply $2(\cos 30° + i \sin 30°)$ by $3(\cos 10° + i \sin 10°)$. 38.....

*39 Transform $xy = 6$ from rectangular to polar coordinates. 39.....

*40 Transform $r^2 + 2r \sin \theta = 8$ from polar to rectangular coordinates. 40.....

Part III

Answer ten questions from this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answer on the line at the right.

41 The first three terms of an arithmetic progression are x, $(2x + 1)$ and $(5x - 4)$. Find the value of x. 41.....

42 Find $\log_6 15.4$ to the *nearest tenth*. 42.....

43 Find to the *nearest tenth* the value of $(26.3)^{1.4}$. 43.....

44 If $y = 3^x$ and $x = \log_a y$, find the value of a. 44.....

45 If $a^x = b^{x+1}$, express x in terms of the logarithms of a and b. 45.....

46 The graphs of $y = mx - 1$ and $(x - 2)^2 + (y + 3)^2 = 25$ are drawn on the same set of axes. If the graph of the straight line passes through the center of the circle, find the value of m. 46.....

47 [Write the *number* preceding the correct answer in the space provided.]
The probability that Tom will hit a target is $\frac{3}{5}$. The probability that Tom will *not* hit the target is

(1) $\frac{5}{8}$ (2) $\frac{5}{3}$ (3) $\frac{2}{5}$ (4) $\frac{3}{8}$ 47.....

48 A man went on a trip of m miles traveling by a train whose average speed was r miles per hour. He returned by a plane whose average speed was three times that of the train. If the return trip was also a distance of m miles and the total traveling time for the round trip was 8 hours, express m in terms of r. 48.....

49 A man engages in a shooting contest. Each time he hits the target, he receives 10 cents and each time he misses, he pays 5 cents. If after 20 shots the man has lost 10 cents, how many times did he hit the target? 49.....

50 Find to the *nearest degree* the amplitude of $2 + 5i$. 50.....

51 Express $4(\cos 150° + i \sin 150°)$ in $a + bi$ form. 51.....

52 Express *one* of the roots of $x^3 + 8 = 0$ in polar form. 52.....

*53 Evaluate the determinant $\begin{vmatrix} 3 & 0 & 2 \\ 4 & -2 & 1 \\ 1 & 5 & 3 \end{vmatrix}$. 53.....

Continued on next page.

*54 Write in determinant form an expression for the area of the triangle whose vertices are $(-2, -1)$, $(3, 2)$ and $(2, 3)$. 54

*55 [Write the *number* preceding the correct answer in the space provided.] The two straight lines whose equations are $4x + y = 10$ and $3x + 2y = 5$ intersect in a point whose abscissa is

(1) $\dfrac{\begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix}}{\begin{vmatrix} 10 & 1 \\ 5 & 2 \end{vmatrix}}$ (2) $\dfrac{\begin{vmatrix} 10 & 1 \\ 5 & 2 \end{vmatrix}}{\begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix}}$ (3) $\dfrac{\begin{vmatrix} 4 & 10 \\ 3 & 5 \end{vmatrix}}{\begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix}}$ (4) $\dfrac{\begin{vmatrix} 1 & 2 \\ 10 & 5 \end{vmatrix}}{\begin{vmatrix} 10 & 1 \\ 5 & 2 \end{vmatrix}}$ 55.....

I have studied twelfth year mathematics 12A for ..... weeks and have had ..... recitations per week under .....
Name of Teacher

I do so declare.....
(Signature)

FOR TEACHERS ONLY

12A

INSTRUCTIONS FOR RATING TWELFTH YEAR MATHEMATICS 12A (Advanced Algebra)

Wednesday, January 27, 1960 — 9:15 a.m. to 12:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 22–25, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $2+i$
(2) $2x-3y=12$
(3) 16
(4) $160x^3$
(5) -4
(6) $-\frac{3}{2}$
(7) $\frac{4}{9}$
(8) 1
(9) .64
(10) $\frac{a}{3}$
(11) $\frac{1}{2}$
(12) $(a-2)(a+2)$
(13) 11
(14) 7
(15) $\frac{1-a^2}{1+a^2}$
(16) 35
(17) 36
(18) $\frac{1}{7}$
(19) $xz^2=ky$
(20) -1
(21) $\frac{7}{11}$
(22) 2
(23) 2
(24) 3
(25) 1

[OVER]

Part II

Allow 2½ credits for each of not more than 10 correct answers; allow no partial credit. If more than ten questions have been answered, only the first ten of these should be considered. For question 34, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(26) $x > 3$
(27) $x + 2y = 6$
(28) 4
(29) $-\frac{1}{3}$
(30) 2 and 3
(31) 0.4
(32) $+1, -1, +3i, -3i$
(33) $\frac{3 \pm \sqrt{9 + 4y}}{2}$
(34) 2
(35) 4
(36) 1
(37) 5
(38) $6\,(\cos 40° + i \sin 40°)$
(39) $r^2 \cos\theta \sin\theta = 6$
(40) $x^2 + y^2 + 2y = 8$

Part III

Allow 2½ credits for each of not more than 10 correct answers; allow no partial credit. If more than ten questions have been answered, only the first ten of these should be considered. For questions 47 and 55, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(41) 3
(42) 1.7
(43) 97.3
(44) 3
(45) $\frac{\log b}{\log a - \log b}$
(46) -1
(47) 3
(48) $6r$
(49) 6
(50) 68°
(51) $-2\sqrt{3} + 2i$
(52) $2\,(\cos 60° + i \sin 60°)$
or $2\,(\cos 180° + i \sin 180°)$
or $2\,(\cos 300° + i \sin 300°)$
(53) 11
(54) $\frac{1}{2}\begin{vmatrix} -2 & -1 & 1 \\ 3 & 2 & 1 \\ 2 & 3 & 1 \end{vmatrix}$
(55) 2