

## TRIGONOMETRY

Tuesday, August 20, 1963 — 12:30 to 3:30 p.m., only

Name of pupil..... Name of school.....

Name of teacher.....

## Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

- 1 Express  $\cos(-50^\circ)$  as a function of a positive acute angle. 1.....
- 2 Express  $\sin x \tan x$  in terms of  $\cos x$ . 2.....
- 3 Express in terms of  $\pi$  the number of degrees in an angle of 2 radians. 3.....
- 4 Find the number of degrees in the positive acute angle  $A$  for which  $2 \sin^2 A - 7 \sin A + 3 = 0$ . 4.....
- 5 What is the amplitude of the graph defined by  $y = 2 \cos \frac{x}{2}$ ? 5.....
- 6 What is the numerical value of  $\cos 59^\circ 26'$ ? 6.....
- 7 If  $\log \tan A = 8.9624 - 10$ , find to the *nearest minute* the positive acute angle  $A$ . 7.....
- 8 If  $\sin x = -\frac{4}{5}$  and  $x$  terminates in quadrant IV, find the value of  $\sec x$ . 8.....
- 9 In triangle  $ABC$ ,  $a = 3$ ,  $b = 4$  and  $C = 60^\circ$ . Express the length of  $c$  in radical form. 9.....
- 10 If  $\sin A = \frac{5}{13}$ ,  $\cos B = \frac{4}{5}$  and  $A$  and  $B$  are acute, find the value of  $\cos(A - B)$ . 10.....

- 11 Express as an integer the number of square units in the area of a triangle whose sides are 13, 14, 15. 11.....
- 12 Find in degrees the smallest positive value of  $x$  satisfying these two conditions:  $\cot x = 1$ ,  $\sin x$  is negative. 12.....
- 13 If  $\sin \frac{x}{2}$  is positive and  $\cos x = -\frac{1}{2}$ , find the value of  $\sin \frac{x}{2}$  in radical form. 13.....
- 14 In  $\triangle ABC$ ,  $a = 10$ ,  $b = 15$  and  $C = 100^\circ$ . Express  $\tan \frac{B-A}{2}$  in terms of  $\tan 40^\circ$ . 14.....
- 15 Express  $\tan (2\pi - A)$  in terms of  $\tan A$ . 15.....
- 16 Find the positive value of  $\sin \left( \arccos \frac{5}{13} \right)$ . 16.....
- 17 Factor:  $\cos^2 x + \cos x$  17.....
- 18 From the top of a tower 200 feet high, the angle of depression of a point  $A$  on the ground, in the same horizontal plane as the base of the tower, is  $48^\circ$ . Find to the nearest foot the distance of  $A$  from the base of the tower. 18.....

*Directions (19-26):* Indicate the correct completion for each of the following by writing on the line at the right the number 1, 2, 3 or 4.

- 19 The expression  $\cos 60^\circ + \cos 40^\circ$  is equal to  
 (1)  $\cos (60^\circ + 40^\circ)$  (3)  $2 \sin 50^\circ \sin 10^\circ$   
 (2)  $2 \cos 50^\circ \cos 10^\circ$  (4)  $2 \sin 50^\circ \cos 10^\circ$  19.....
- 20 If  $P = 100 \sin^2 A$ , then  $\log P$  may be expressed as  
 (1)  $10 + 2 \log \sin A$  (3)  $200 \log \sin A$   
 (2)  $2 + 2 \log \sin A$  (4)  $4 \log \sin A$  20.....
- 21 As  $x$  varies from 0 to  $2\pi$  radians,  $\tan x$   
 (1) increases and then decreases  
 (2) increases in every quadrant  
 (3) decreases in every quadrant  
 (4) decreases and then increases 21.....
- 22 If  $\sin 4A = \cos (A + 15^\circ)$ , then  $A$  may equal  
 (1)  $5^\circ$  (3)  $15^\circ$   
 (2)  $6^\circ$  (4)  $35^\circ$  22.....

23 If the data  $A = 30^\circ$ ,  $a = 8$  and  $b = 10$  are used

- (1) triangle  $ABC$  must be acute  
 (2) triangle  $ABC$  must be obtuse  
 (3) triangle  $ABC$  may be either acute or obtuse  
 (4) no triangle can be constructed

23.....

24 If  $\tan \theta = \sqrt{5}$ , then  $\tan 2\theta$  equals

(1)  $2\sqrt{5}$  (3)  $\frac{2\sqrt{5}}{1 + \sqrt{5}}$

(2)  $\frac{2\sqrt{5}}{1 - \sqrt{5}}$

(4)  $\frac{-\sqrt{5}}{2}$

24.....

25 The expression  $\frac{2}{\sin 2\theta}$  is equivalent to

(1)  $\csc \theta$   
 (2)  $\sin \theta \cos \theta$

(3)  $2 \csc^2 \theta$   
 (4)  $\csc \theta \sec \theta$

25.....

26 The expression  $\sin (30^\circ - A)$  is equivalent to

(1)  $\cos A - \sqrt{3} \sin A$

(3)  $\frac{\cos A - \sqrt{3} \sin A}{2}$

(2)  $\sqrt{3} \cos A - \sin A$

(4)  $\frac{\sqrt{3} \cos A - \sin A}{2}$

26.....

*Directions (27-30):* For real values of  $x$  for which the following statements are defined, indicate whether *each* of the statements is true for

- (1) all of these values of  $x$ ,  
 (2) some but not all of these values of  $x$ ,  
 (3) none of these values of  $x$ ,

by writing on the line at the right the number 1, 2 or 3.

27  $1 + \tan^2 \frac{x}{2} = \sec^2 \frac{x}{2}$

27.....

28  $\sin x + \cos x = 1$

28.....

29  $\sec 2x \cos 2x = 1$

29.....

30  $\sec x = \frac{1}{2}$

30.....



## Part II

Answer four questions from this part. Show all work unless otherwise directed.

- 31 Solve for all values of
- $x$
- between
- $0^\circ$
- and
- $360^\circ$
- , inclusive: [10]

$$\cos 2x + 2 = 2 \sin^2 x$$

- 32 a Sketch the graph defined by
- $y = 2 \cos x$
- for values of
- $x$
- from
- $-\pi$
- to
- $\pi$
- . [4]

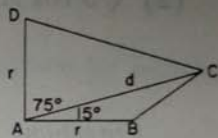
b On the same set of axes used in part a, sketch the graph defined by  $y = \sin \frac{x}{2}$  for values of  $x$  from  $-\pi$  to  $\pi$ . [4]c Determine from the graphs drawn in answer to parts a and b the number of values of  $x$  between  $-\pi$  and  $\pi$  which are solutions of  $2 \cos x = \sin \frac{x}{2}$ . [2]

- 33 a Starting with a formula for
- $\cos 2\theta$
- , derive a formula for
- $\cos \frac{x}{2}$
- . [5]

b Prove that the following equality is true for all values of  $x$  for which the members are defined (that is, prove the equality is an identity): [5]

$$\frac{1 + \sec y}{\sec y} = 2 \cos^2 \frac{y}{2}$$

- 34 In quadrilateral
- $ABCD$
- ,
- $AD = AB = r$
- ,
- $AC = d$
- ,
- $\angle DAC = 75^\circ$
- and
- $\angle CAB = 15^\circ$
- .

a Express the area of each triangle in terms of  $r$  and  $d$ . [4]b Without the use of tables, show that the area of  $ABCD$  is  $\frac{rd\sqrt{6}}{4}$ . [6]

- 35 An S O S is received by Coast Guard cutter
- $A$
- from an endangered ship. The cutter finds the bearing of the ship is
- $58^\circ 20'$
- (N
- $58^\circ 20'$
- E). Simultaneously, another Coast Guard cutter
- $B$
- , 82 miles directly north of the first cutter, finds the bearing of the ship in distress is
- $110^\circ 50'$
- (S
- $69^\circ 10'$
- E). How far is the source of the S O S from the nearer cutter, to the nearest mile? [10]

- 36 Two forces of 27 pounds and 62 pounds act on a body so that the resultant is a force of 75 pounds. Find to the nearest degree the angle between the resultant and the larger force. [10]

# FOR TEACHERS ONLY

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## SCORING KEY TRIGONOMETRY

Tuesday, August 20, 1963 — 12:30 to 3:30 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

### Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 19-30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

- |  |                                |        |
|--|--------------------------------|--------|
| (1) $\cos 50^\circ$ or $\sin 40^\circ$ | (11) 84                        | (23) 3 |
| (2) $\frac{1 - \cos^2 x}{\cos x}$      | (12) 225                       | (24) 4 |
| (3) $\frac{360}{\pi}$                  | (13) $\frac{\sqrt{3}}{2}$      | (25) 4 |
| (4) 30                                 | (14) $\frac{\tan 40^\circ}{5}$ | (26) 3 |
| (5) 2                                  | (15) $-\tan A$                 | (27) 1 |
| (6) 0.5085                             | (16) $\frac{12}{13}$           | (28) 2 |
| (7) $5^\circ 14'$                      | (17) $\cos x (\cos x + 1)$     | (29) 1 |
| (8) $\frac{5}{3}$                      | (18) 180                       | (30) 3 |
| (9) $\sqrt{13}$                        | (19) 2                         |        |
| (10) $\frac{63}{65}$                   | (20) 2                         |        |
|  | (21) 2                         |        |
|  | (22) 3                         |        |

### Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

- |   |      |         |      |
|---|------|---------|------|
| (31) $60^\circ, 120^\circ, 240^\circ, 300^\circ$  | [10] | (35) 88 | [10] |
| (32) $c$ two  | [2]  | (36) 20 | [10] |
| (34) $a$ area of $\triangle ACD = \frac{1}{2} rd \sin 75^\circ$ ;<br>area of $\triangle ABC = \frac{1}{2} rd \sin 15^\circ$ | [4]  |         |      |